Prof. Dr. K. Kroy SS 2014

Statistical Physics II
Problem Set 2

Due: Tuesday, April 22, **before** the lecture

2. **Structure of dilute gases** **(13 points)**

Calculate the second virial coefficient $B(T)$ and from $B(T = T_B) = 0$ the Boyle-temperature T_B for the following examples. What is the meaning of T_B? Calculate and sketch the corresponding radial distribution functions $g(r)$ in the low-density limit. Furthermore, calculate and sketch the structure factor $S_{\mathbf{q}}$ in the low-density approximation for both examples.

a) Hard spheres

$$\nu(r) = \begin{cases} \infty\,, & r < \sigma \\ a(r)\,, & r \geq \sigma \end{cases}\,,$$

with the Debye-Hückel attraction

$$a(r) = -\varepsilon e^{-\varkappa r}/(\varkappa r)$$

as a simple model for ions in solution. Use $\varkappa\sigma \ll 1$ and $\beta\varepsilon/\varkappa\sigma \ll 1$. Establish the relation to the parameters of the van der Waals gas via the equation of state. How are the two parts of the potential reflected in the structure factor?

b) Dilute non-interacting bosons with the „statistical potential"

$$\mathcal{V}_{\text{eff}}(\mathbf{r}_i - \mathbf{r}_j) = -k_B T \ln[1 + \exp(-2\pi(\mathbf{r}_i - \mathbf{r}_j)^2/\lambda_T^2)]\,.$$

Calculate the spinodal and compare the result with the exact result for the BEC transition. In which (obvious) sense is the interpretation of BEC as spinodal decomposition flawed?

3. **Moments and cumulants** **(4 points)**

If the moments $\langle x^n \rangle$ of a normalized distribution $p(x)$ exist, the generating functions

$$Z(k) = \langle e^{ikx} \rangle = \sum_{n=0}^{\infty} \frac{(ik)^n}{n!} \langle x^n \rangle$$

and

$$\ln Z(k) = \sum_{n=1}^{\infty} \frac{(ik)^n}{n!} \kappa_n$$

can be used to determine in principle all moments $\langle x^n \rangle$ and cumulants κ_n of $p(x)$.

a) Express the moments in terms of the derivatives of $Z(k)$.

b) For the Gaussian distribution

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-x_0)^2/(2\sigma^2)},$$

calculate the first two moments and *all* cumulants.

c) Assume f to be an external field perturbing a closed system in a heat reservoir. Then, the expansion of moments of $Z(k = if/(k_B T))$ can be considered as a *perturbation expansion* of the canonical partition sum in powers of f. Which thermodynamic potential corresponds to the cumulant expansion?

Total score: 17 points