Statistical Physics II
Problem Set 12

Due: Tuesday, Juli 1, **before** the lecture

17. **Harmonic oscillator (part I)** **(6 points)**

The 1-dimensional equation of motion of a harmonic oscillator in the presence of a viscous force $\zeta\dot{x}(t)$ and an external force $\xi(t)$ is given by

$$m\ddot{x}(t) + m\omega_0^2 x(t) + \zeta\dot{x}(t) = \xi(t)\,.$$

a) Determine the characteristic equation and discuss the mode structure of the damped harmonic oscillator ($\xi = 0$). Describe the qualitative behavior of the oscillator for $\omega_0^2 > \zeta^2/(4m^2)$ and $\omega_0^2 < \zeta^2/(4m^2)$.

b) In Fourier space, the response of the system to the external force $\xi(t)$ is given by $x(\omega) = \chi(\omega)\xi(\omega)$ with $x(\omega) = \int_{-\infty}^{\infty} dt\, x(t) e^{i\omega t}$. Determine $\chi(\omega)$ from the inhomogeneous solution of the oscillator equation. Separate $\chi(\omega) = \chi_{\mathrm{re}}(\omega) + i\chi_{\mathrm{im}}(\omega)$ into its real part $\chi_{\mathrm{re}}(\omega)$ and its imaginary part $\chi_{\mathrm{im}}(\omega)$ and sketch both as a function of ω/ω_0 assuming that $\omega_0^2 > \zeta^2/(4m^2)$.

18. **Brownian motion: Diffusion** **(4 points)**

The (one-dimensional) diffusion equation,

$$\frac{\partial \rho(x,t)}{\partial t} = D\frac{\partial^2 \rho(x,t)}{\partial^2 x}\,,$$

is a deterministic equation for the probability density $\rho(x,t)$. It determines the probability to find a suspended particle at (x,t) in a homogeneous solution, where D is the diffusion coefficient.

a) Solve the equation using Fourier transforms (or Fourier-Laplace transforms) for the initial condition $\rho(x, 0) = \delta(x)$.

b) For the initial condition specified in a), $\rho(x, t)$ is self-similar, *i.e.*, it reduces to a scaling form $\rho(x, t) = C(t)\tilde{\rho}\big(x/\xi(t)\big)$ with a time-dependent characteristic length scale $\xi(t)$. Determine ξ and argue that the self-similarity is due to a specific symmetry of the diffusion equation.

Total score: 10 points