Statistical Physics II Problem Set 11

Due: Tuesday, June 24, **before** the lecture

16. Renormalization group transformation, part II $(10+2^* \text{ points})$

- a) Consider again the Ising model in 2 dimensions as discussed in problem 15. To find the critical point, set $L_0 = 0$, vary the initial value K_0 and iterate the RG-equations found in problem 13c) until you find an estimate for the critical value of the coupling $K_0 = K_c$ for which $K_n \to K^*$ and $L_n \to L^*$ as $n \to \infty$. Compare your result with Onsager's exact value of $K_c = 0.4407...$ Where do the non-critical starting values $K_0 \neq K_c$ for $L_0 = 0$ converge to under the iteration?
- b) Sketch the critical trajectory in a (K_0, L_0) -diagram and mark the fixed points and the flow of the coupling (K_n, L_n) on the critical trajectory and in its vicinity. Explain why the introduction of L_0 in problem 15c) was useful. What is the meaning of the critical trajectory with respect to *universality*?
- c) Calculate the critical exponent ν , which describes the divergence of the correlation length $\xi(K_0, L_0)$ near the critical point, by linearizing the RG-equations about (K_*, L_*) and calculating the eigenvalues of the linearized RGT. Which eigenvalue is the relevant one?
- d*) Discuss the reliability of the performed approximations. How can the calculation be improved?

Total score: 10 points + 2 additional points