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# Statistical Physics II
# Problem Set 1

Due: Friday, April 11

1.* **Heat capacity of graphene** **(5 points)**

Graphene consists of a monolayer of carbon atoms arranged in a two-dimensional hexagonal lattice. The band structure is such that the spectrum of single-particle excitations at low energies can be approximated by

$$\mathcal{E}_{\pm}(\mathbf{k}) = \pm\hbar v|\mathbf{k}|,$$

reminiscent of relativistic Dirac fermions. The spin degeneracy is $\mathfrak{g} = 2$, and $v \approx 10^6\,\mathrm{m\,s^{-1}}$ (see *Nature* **438**, 197 (2005)).

a) Determine the chemical potential $\mu(T)$ in the case that at $T = 0$ all negative energy states are occupied and all positive ones are empty, that is $\mu(T = 0) = 0$.

b) Show that the mean excitation energy of this system at finite temperature satisfies

$$E(T) - E(0) = 4A \int \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2} \frac{\mathcal{E}_+(\mathbf{k})}{\exp(\beta\mathcal{E}_+(\mathbf{k})) + 1}.$$

c) Calculate $E(T)$ by using b).

d) Calculate the heat capacity $C_A$.

e) Explain qualitatively the contribution of phonons (lattice vibrations) to the heat capacity of graphene. The typical sound velocity is on the order of $2\times10^4\,\mathrm{m\,s^{-1}}$. Is the low temperature heat capacity of graphene controlled by phonon or electron contributions?