Universität Leipzig, Institut für Theoretische Physik Prof. Dr. Gerd Rudolph

Bonus exercises in Advanced Quantum Mechanics

Due Thursday, February 5, 2015

36. (1 bonus point) We consider a one-dimensional harmonic oscillator of mass m and frequency ω driven by a weak periodic force $F(t) = F_0 \cos(\Omega t)$ which is turned on at time $t = 0$. Determine the first order perturbative correction to the expectation value of the displacement $\langle\psi(t)|\,\hat{x}\,|\psi(t)\rangle$ under the assumption that for $t < 0$ the system was in a stationary state.

37. (1 bonus point) We consider the Dirac equation.

a) In the spinor representation of the Lorentz group, a rotation in the x^μ-x^ν plane of Minkowski space about an angle ϕ is given by

$$S^{\mu\nu}(\phi) = \exp\left(-\tfrac{i}{2}\phi\sigma^{\mu\nu}\right) .$$

Calculate $S^{\mu\nu}(\phi)$ for the cases $\mu, \nu = k, l = 1, 2, 3$ (spatial rotations in the x^k-x^l plane) and $\mu = 0, \nu = k = 1, 2, 3$ (Lorentz boosts in der x^0-x^k plane).

b) Show that the helicity operator $\hat{s}(\vec{p}) = \frac{\hat{\vec{S}}\cdot\hat{\vec{p}}}{|\hat{\vec{p}}|}$ commutes with the Dirac Hamiltonian $\hat{H} = c\vec{\alpha}\cdot\hat{\vec{p}} + mc^2\gamma^0$ and that one has

$$\hat{s}(\vec{p})^2 = \frac{\hbar^2}{4} .$$

c) Determine how the operator of charge conjugation acts on an electron with negative energy and spin up relative to the rest system. Give a physical interpretation of your result.