Universität Leipzig, Institut für Theoretische Physik Prof. Dr. Gerd Rudolph

Exercises in Advanced Quantum Mechanics

Due Thursday, January 15, 2015

28. Determine the ground state and the first excited state of the harmonic oscillator in one dimension by means of the Ritz variational method. Use the ansatz $\psi_A(x) = e^{-Ax^2}$ for the ground state and the ansatz $\psi_{AB}(x) = (B + x)e^{-Ax^2}$ for the first excited state. Compare your results with the exact solutions.

29. (Mandatory) Use the Ritz variational method to find the optimal values for A and the corresponding approximate ground state and ground state energy for the hydrogen atom for each of the following 3 ansatzes and compare the results:

a) $\psi_A(\vec{r}) = \mathrm{e}^{-Ar}$,

b) $\psi_A(\vec{r}) = r\mathrm{e}^{-Ar}$,

c) $\psi_A(\vec{r}) = \frac{1}{A^2+r^2}$.

30. Use the Ritz variational method and the ansatz

$$\psi_A(\vec{x}_1, \vec{x}_2) = e^{-A(|\vec{x}_1|+|\vec{x}_2|)} , \qquad A > 0 ,$$

to find the approximate ground state energy of the Helium atom. Compare your result with that obtained by first order perturbation theory (Exercise 24).

31. After separation of the motion of the center of mass, the Hamiltonian of the deuteron (consisting of a proton and a neutron) is

$$H = -\frac{\hbar^2}{2\mu}\Delta + V(r) ,$$

where $\vec{r}$ is the relative vector and $V(r)$ is given by the Yukawa potential

$$V(r) = -V_0 \, \frac{e^{-\frac{r}{c}}}{\frac{r}{c}} , \qquad V_0 > 0 .$$

Use the Ritz variational method and the ansatz

$$\psi(\vec{x}; A) = e^{-A\frac{r}{c}} , \qquad A > 0 ,$$

to find the approximate ground state energy.