Universität Leipzig, Institut für Theoretische Physik Prof. Dr. Gerd Rudolph

Exercises in Advanced Quantum Mechanics

Due Thursday, January 8, 2015

25. (Mandatory) We consider a Helium atom and view the Coulomb interaction between the two electrons as a perturbation.

a) Calculate the first order perturbative correction to the ground state energy.

b) Discuss qualitatively how the degeneracy of the excited states in the discrete spectrum is broken in first order of perturbation theory. For that purpose, convince yourself that there are two types of contributions to the energy correction, which for obvious reasons are called the Coulomb interaction and the exchange interaction. For your discussion, use that the Coulomb interaction is much larger than the exchange interaction and that the latter is always positive.

26. We consider the Hamiltonian of the spin-orbit interaction in an atom with Z electrons,

$$\hat{H}_3 = \sum\nolimits_{i=1}^{Z} U(r_i)\ \hat{\vec{L}}_i \cdot \hat{\vec{S}}_i \,.$$

Let $\hat{\vec{L}}$ and $\hat{\vec{S}}$ denote the total orbital angular momentum and the total spin, respectively.

a) Show that the total angular momentum $\hat{\vec{J}} = \hat{\vec{L}} + \hat{\vec{S}}$ commutes with $\hat{H}_3$ and $\hat{\vec{L}} \cdot \hat{\vec{S}}$.

b) Use the Wigner-Eckart Theorem to show that the matrix elements of $\hat{H}_3$ with respect to the orthonormal basis $\big\{\, |\{\lambda_i\}, L, L_3, S, S_3\rangle \,\big\}$, explained in the lecture, can be written in the form

$$\begin{aligned}\langle \{\lambda_i\}, L, L_3', S, S_3'|\, \hat{H}_3\, |\{\lambda_i\}, L, L_3, S, S_3\rangle \\ = A\ \langle \{\lambda_i\}, L, L_3', S, S_3'|\, \hat{\vec{L}} \cdot \hat{\vec{S}}\, |\{\lambda_i\}, L, L_3, S, S_3\rangle \ ,\end{aligned}$$

where A is a constant which does not depend on L_3, S_3 or i.

27. We consider a hydrogen atom in a static homogeneous electric field (Stark effect):

$$\hat{H} = \hat{H}_0 + \hat{H}_1 \,, \qquad \hat{H}_0 = -\frac{\hbar^2}{2m}\Delta - \frac{e^2}{4\pi\varepsilon_0 r} \,, \qquad \hat{H}_1 = -e\vec{E}\hat{\vec{x}} \,.$$

Calculate the first order perturbative corrections to the ground state energy and to the first excited energy level.