Exercises in Advanced Quantum Mechanics

Due Thursday, December 4, 2014

17. We consider two non-interacting identical particles of spin s in a one-dimensional potential well with infinitely high walls,

$$V(x) = \left\{ \begin{array}{ccc} 0 & | & |x| < a\,, \\ \infty & | & |x| \geq a\,. \end{array} \right.$$

  a) Show that the eigenvectors of the Hamiltonian H can be written as pure tensor products of a spatial and a spin part. Under the assumption that the spin part is prepared in the state $|m\rangle \otimes |m\rangle$ with $-s \leq m \leq s$, determine the symmetry of the spatial part depending on whether s is integer or half-integer.

  b) Under the assumption of a), determine the eigenvectors of H and their energies for both cases. Compare the ground state energies.

  Hint. You may use the results for the corresponding one-particle system from the first part of the lecture or from a textbook.

18. (Mandatory) We consider the Helium atom. Let $\hat{\vec{d}} = e\big(\hat{\vec{x}}^{\,(1)} + \hat{\vec{x}}^{\,(2)}\big)$ be the the dipole operator. Show the following.

  a) For any singlet state $|\psi\rangle$ and any triplet state $|\varphi\rangle$ one has

$$\langle \psi | \hat{\vec{d}} | \varphi \rangle = 0\,.$$

  b) If one neglects spin-spin interaction, there are no dipole transitions between singlet and triplet states.