Universität Leipzig, Institut für Theoretische Physik Prof. Dr. Gerd Rudolph

Exercises in Advanced Quantum Mechanics

Due Thursday, November 13, 2014

8. (Mandatory) Show that $\hat{\vec{x}}, \hat{\vec{p}}, \hat{\vec{L}}$ are vector operators, that is, that they obey the transformation law

$$\hat{D}(R)\,\hat{A}_i\,\hat{D}(R)^\dagger = (R^{-1})_{ij}\,\hat{A}_j\,, \qquad \hat{A}_i = \hat{x}_i, \hat{p}_i, \hat{L}_i\,.$$

Here, $R \mapsto D(R)$ is the representation of the rotation group on the Hilbert space defined in the lecture.

9. Define the quantum mechanical analogue of a classical special Galilei transformation $\vec{x} \mapsto \vec{x}\,' = \vec{x} - \vec{v}t$ by

$$\hat{G}(\vec{v})\,|\vec{p}\rangle := |\vec{p} - m\vec{v}\rangle\,.$$

Determine the corresponding transformation law for the wave function in the momentum and the position representation. Find the tranformation laws for the position operator and the momentum operator.

10. Let $\hat{\vec{A}} = (\hat{A}_1, \hat{A}_2, \hat{A}_3)$ be a vector operator. Show that the operators

$$\hat{T}^1_1 = -\frac{1}{\sqrt{2}}(\hat{A}_1 + i\hat{A}_2)\,, \qquad \hat{T}^1_0 = \hat{A}_3\,, \qquad \hat{T}^1_{-1} = \frac{1}{\sqrt{2}}(\hat{A}_1 - i\hat{A}_2)$$

form an irreducible tensor operator of rank one.