Exercises in Advanced Quantum Mechanics

Due Thursday, November 6, 2014

6. We consider a system of two particles of spin $\frac{1}{2}$ in the pure state

$$|\varphi\rangle = \frac{1}{2\sqrt{2}}\big(|+\rangle \otimes |+\rangle + |-\rangle \otimes |-\rangle\big) + \frac{3}{2\sqrt{2}}\big(|+\rangle \otimes |-\rangle + |-\rangle \otimes |+\rangle\big)$$

   a) Determine the reduced density operators $\rho^{(1)}$ and $\rho^{(2)}$, as well as their eigenvalues and eigenvectors.

   b) Finde the Gram-Schmidt decomposition of $|\varphi\rangle$. Is $|\varphi\rangle$ entangled? Is it maximally entangled?

   c) Find two distinct purifications of $|\varphi\rangle$.

7. (Mandatory) We consider the tensor product $\mathcal{H}_X \otimes \mathcal{H}_A \otimes \mathcal{H}_B$ of two-dimensional Hilbert spaces. Let

$$\varphi_{ij}^{\pm} = \tfrac{1}{\sqrt{2}}\big(|+\rangle \otimes |+\rangle \pm |-\rangle \otimes |-\rangle\big), \qquad \psi_{ij}^{\pm} = \tfrac{1}{\sqrt{2}}\big(|+\rangle \otimes |-\rangle \pm |-\rangle \otimes |+\rangle\big)$$

be the Bell basis in $\mathcal{H}_i \otimes \mathcal{H}_j$, $i,j = X, A, B$. Let $\phi_X \in \mathcal{H}_X$ be given and assume that the system $\mathcal{H}_X \otimes \mathcal{H}_A \otimes \mathcal{H}_B$ is prepared in the state $\phi_X \otimes \varphi_{AB}^{+}$. Assume that we carry out a Bell state measurement on the system $\mathcal{H}_X \otimes \mathcal{H}_A$. Finde the state of the system B after this measurement for each of the possible results.