UNIVERSITY OF LEIPZIG
INSTITUTE FOR THEORETICAL PHYSICS
Department: Theory of Elementary Particles

TP2 2017

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List of problems 8

23. Consider a sphere of radius R centered at the origin. Suppose a point charge q is put at the origin and this is the only charge inside or outside the sphere. Furthermore the potential is $\Phi = V_0 \cos\theta$ on the surface of the sphere. What is the electrostatic potential and the electric field vector both inside and outside the sphere?
Hint: Use superposition to take into account the potential of the point charge inside the sphere.

24. A sphere of radius R is uniformly polarized with polarization vector $\mathbf{P}$ (polarization — dipole moment per unit volume) along the z-axis.
Using spherical coordinates find the potentials due to that polarization inside and outside the sphere.
Write the found potentials in a coordinate free form using the vector $\mathbf{P}$.
Calculate the corresponding electric field vectors $\mathbf{E}$ and $\mathbf{D}$ inside and outside the sphere.
Hint: Taking into account the axial symmetry, use the expansion of

$$\frac{1}{|\mathbf{x}-\mathbf{x}'|} = \frac{1}{r_>} \sum_{l=0}^{\infty} \left(\frac{r_<}{r_>}\right)^l P_l(\cos\gamma)\,, \quad r_< = \min(r, r')\,, \quad r_> = \max(r, r')$$

with $\cos\gamma = \cos\theta\,\cos\theta' + \sin\theta\sin\theta'\,\cos(\varphi - \varphi')$ in the expression of the potential [here $\mathbf{x} = (r, \theta, \varphi)$, $\mathbf{x}' = (r', \theta', \varphi')$].

25. Voluntary

A localized distribution of charge has a charge density in spherical coordinates

$$\rho(\mathbf{r}) = \frac{1}{64\pi} r^2 \mathrm{e}^{-r} \sin^2\theta\,.$$

(a) Make a multipole expansion of the potential due to this charge density and determine all the nonvanishing multipole moments. Write down the potential at *large* distances as a finite expansion in Legendre polynomials.
(b) Determine the potential of that charge distribution explicitly at *any* point in space using the expansion of $1/|\mathbf{x}-\mathbf{x}'|$ in spherical harmonics

$$\frac{1}{|\mathbf{x}-\mathbf{x}'|} = \frac{4\pi}{r_>} \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \frac{1}{2l+1} \left(\frac{r_<}{r_>}\right)^l Y^*_{lm}(\theta', \varphi')\, Y_{lm}(\theta, \varphi)\,,$$

$$r_< = \min(r, r')\,, \quad r_> = \max(r, r')\,.$$

Show that near the origin, correct to r^2 inclusive,

$$\Phi(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\left(\frac{1}{4} - \frac{r^2}{120}P_2(\cos\theta)\right)\,.$$