UNIVERSITY OF LEIPZIG
INSTITUTE FOR THEORETICAL PHYSICS
Department: Theory of Elementary Particles

TP2 2017

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List of problems 7

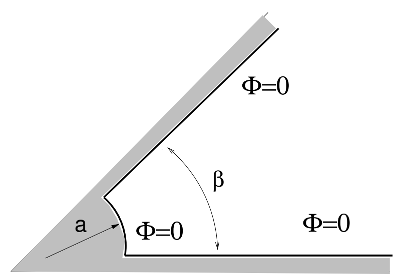
19. A hollow cube has conducting walls defined by six planes $x = 0$, $y = 0$, $z = 0$, and $x = a$, $y = a$, $z = a$. The walls $z = 0$ and $z = a$ are held at a constant potential V. The other four sides are at zero potential.

    (i) Find the potential $\Phi(x, y, z)$ at any point inside the cube (double series).

    (ii) Find the surface-charge density on the inside top surface $z = a$ and bottom surface $z = 0$.

    (iii) Sketch the electric field lines inside the hollow cube.

20. The two–dimensional region $\rho \geq a$, $0 \leq \varphi \leq \beta$, is bounded by conducting surfaces at $\varphi = 0$, $\rho = a$, and $\varphi = \beta$ held at zero potential, as indicated in the sketch. At large ρ the potential is determined by some configuration of



charges and/or conductors at fixed potentials.
(a) Write down a solution for the potential $\Phi(\rho, \varphi)$ that satisfies the boundary conditions for finite ρ.
(b) Keeping only the lowest nonvanishing terms, calculate the electric field components E_ρ and E_φ and also the surface–charge densities $\sigma(\rho, 0)$, $\sigma(\rho, \beta)$, and $\sigma(a, \varphi)$ on the three boundary surfaces.
(c) Consider $\beta = \pi$ (a plane conductor with a half–cylinder of radius a on it). Show that far from the half–cylinder the lowest order terms of part (b) give a uniform electric field normal to the plane. Sketch the charge density on and in the neighborhood of the half-cylinder.

21. (Wait for the lecture section 3.6.3 !)
A surface charge density $\sigma(\theta) = \sigma_0 \cos\theta$ is glued to the surface of a spherical shell of radius R (σ_0 is a constant and θ is the polar angle). There is a

vacuum, with no charges, both inside and outside of the shell.
Calculate the electrostatic potential and the electric field vector both inside and outside of the spherical shell.

22. Voluntary

Two halves of a long hollow conducting cylinder of inner radius b are separated by small lengthwise gaps on each side, and are kept at different potentials V_1 and V_2. Show that the potential inside is given by

$$\Phi(\rho,\varphi)=\frac{V_1+V_2}{2}+\frac{V_1-V_2}{\pi}\tan^{-1}\left(\frac{2b\rho}{b^2-\rho^2}\cos\varphi\right)$$

where φ is measured from a plane perpendicular to the plane through the gap.
Calculate the surface-charge density on each half of the cylinder.
Hint: For a complex Z

$$\sum_{k=0}^{\infty}\frac{(-1)^k}{2k+1}Z^{2k+1}=\tan^{-1}Z\,,\quad |Z|\leq 1\,,\quad \mathrm{Re}\,(\tan^{-1}Z)=\frac{1}{2}\tan^{-1}\left(\frac{2\,\mathrm{Re}Z}{1-|Z|^2}\right)$$