UNIVERSITY OF LEIPZIG
INSTITUTE FOR THEORETICAL PHYSICS
Department: Theory of Elementary Particles

TP2 2017

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List of problems 5

15. voluntary, to collect an additional point

12. Derive the interaction energy from the electric fields of two charges q_1 and q_2 located at positions $\mathbf{x_1}$ and $\mathbf{x_2}$, respectively, as

$$W_{\text{int}} = \frac{q_1 q_2}{16\pi^2\varepsilon_0} \int \frac{(\mathbf{x}-\mathbf{x}_1)\cdot(\mathbf{x}-\mathbf{x}_2)}{|\mathbf{x}-\mathbf{x}_1|^3|\mathbf{x}-\mathbf{x}_2|^3}\, d^3x$$

where the integration has to be taken over the whole space (see lecture). By changing the integration variable $\mathbf{x}$ to $\boldsymbol{\rho} = (\mathbf{x}-\mathbf{x}_1)/|\mathbf{x}_1-\mathbf{x}_2|$ show that

$$W_{\text{int}} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|\mathbf{x}_1-\mathbf{x}_2|}\,.$$

In other words, check by explicit calculation that the following relation holds ($|\boldsymbol{\rho}| = \rho$)

$$\int \frac{\boldsymbol{\rho}\cdot(\boldsymbol{\rho}+\mathbf{n})}{\rho^3\,|\boldsymbol{\rho}+\mathbf{n}|^3}\, d^3\rho = 4\pi\,, \quad \mathbf{n} = \frac{\mathbf{x}_1-\mathbf{x}_2}{|\mathbf{x}_1-\mathbf{x}_2|}\,.$$

Here $\mathbf{n}$ is the unit vector in direction $\mathbf{x}_1 - \mathbf{x}_2$.

13. Two semi-infinite plane grounded aluminium sheets make an angle of 90°, 60°, 45°. A single point charge $+q$ is placed in the region between the plates. Make clear drawings for the three cases indicating the position and the size of all image charges. In a few sentences explain your reasoning.

14. The potential at position $\mathbf{x}$ of a point-like electric dipole vector $\mathbf{p}$ located at position $\mathbf{x}'$ is given by

$$\Phi(\mathbf{x}) = \frac{1}{4\pi\varepsilon_0}\frac{\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3}\,, \quad \mathbf{x} \neq \mathbf{x}'\,.$$

Suppose the dipole is fixed at a distance $z_0 > 0$ along the z-axis and at an orientation θ with respect to that axis (i.e., $\mathbf{p}\cdot\mathbf{e}_z = p\cos\theta$). Suppose the xy plane is a conductor at zero potential.
Give the charge density $\sigma(x, y)$ on the conducting plane induced by the dipole.
Calculate the total induced charge on the conducting plane.
Hint: To satisfy the boundary conditions use as image an image point-like

dipole. To find the correct orientation of that dipole vector, consider a dipole as a pair of a negative and a positive charge of same strength q in the limit of a vanishing length vector pointing from $-q$ to q.

Voluntary (not required, an additional point):
Find the positions of vanishing σ on the x-axis and y-axis and sketch the behavior of the distribution along those axes.
What happens to those positions when the dipole is orientated parallel or perpendicular to the conducting plane?
What is the curve of vanishing charge density $\sigma(x, y) = 0$ on the conducting plane for $\theta < \pi/2$ and $\theta = \pi/2$?

15. Voluntary, to collect an additional point
Two semi-infinite plane grounded aluminium sheets at zero potential make an angle of $90°$. Using Cartesian coordinates, the sheets are assumed to be located in the (x, z) and (y, z) plane, respectively and intersect at the line $z = 0$. A single point charge $+q$ is placed at the point $(D, D, 0)$ between the sheets.
Using the method of images, calculate the electric field vector and the induced surface charge densities on the sheets.
What is the surface charge density on the intersecting line (the z-axis)?
Check that the total induced charge one each of the sheets is $-q/2$.