UNIVERSITY OF LEIPZIG
INSTITUTE FOR THEORETICAL PHYSICS
Department: Theory of Elementary Particles

TP2 2017

Lecturer: PD Dr. A. Schiller

List of problems 10

29. Find the interaction energy between two small currents described by their magnetic moments $\mathbf{m}_1$ and $\mathbf{m}_2$ located at positions $\mathbf{x}_1$ and $\mathbf{x}_2$, respectively. What is the interaction force between those currents and the torque vectors acting on the currents?
    Discuss the special case of parallel magnetic moments.

30. An infinitely long, solid dielectric cylinder of radius a is permanently polarized so that the polarization is everywhere radially outward, with a magnitude proportional to the distance from the axis of the cylinder

$$\mathbf{P}(\rho) = \frac{1}{2}\, P_0\, \rho\, \mathbf{e}_\rho\,.$$

(a) Find the charge density in the cylinder.
(b) If the cylinder is rotated with a constant angular velocity ω about its axis without change in $\mathbf{P}$, what is the magnetic field on the axis of the cylinder?

Hints:
Rotating bounded charge densities lead to effective currents.
From those densities determine the effective volume and surface current densities in the rotating cylinder.
Using the current densities, calculate the vector of the magnetic induction on the axis.

31. Voluntary!
    A steady current I flows in a wire along the z-axis. Consider three half-planes originating from that axis in a fan-shape manner. Denote the angles between those half-planes by α_1, α_2 and α_3 with $\alpha_1 + \alpha_2 + \alpha_3 = 2\pi$. The regions between the half-planes are filled with homogeneous magnetic materials of permeabilities μ_1, μ_2 and μ_3, respectively. Denote the magnetic induction vector of the current I in vacuum by $\mathbf{B}_0$.
    Calculate the resulting magnetic fields $\mathbf{H}_i$ and $\mathbf{B}_i$ in the three regions as function of $\mathbf{B}_0$, the angles α_i and permeabilities μ_i.

32. Voluntary!
    Consider a localized current distribution $\mathbf{J}(\mathbf{x})$ that gives rise to a magnetic

induction $\mathbf{B}(\mathbf{x})$ throughout space and a sphere of radius R.
If the sphere contains all of the current, show that

$$\int_{r<R} \mathbf{B}(\mathbf{x})\, d^3x = \frac{2\,\mu_0}{3}\,\mathbf{m}$$

where $\mathbf{m}$ is the total magnetic dipole moment of the current distribution with respect to the center of the sphere.
What is the consequence for the magnetic induction of a magnetic dipole?
If, on the contrary, the current lies external to the sphere, show that the average value of the magnetic induction over the spherical volume containing no current is the value of the field at the center of the sphere

$$\int_{r<R} \mathbf{B}(\mathbf{x})\, d^3x = \frac{4\pi R^3}{3}\,\mathbf{B}(0)\,.$$

(*Hint:* See Jackson, Chapter 5.6: Magnetic fields of a localized current distribution, magnetic moment; perform the steps in the derivation, see also lecture Chapter 4.1 for a similar calculation in electrostatics.)