UNIVERSITY OF LEIPZIG
INSTITUTE FOR THEORETICAL PHYSICS
Department: Theory of Elementary Particles

TP2 2017

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List of problems 1

1. Verify the identities

$$\begin{aligned}
(\mathbf{A}\times\mathbf{B})\cdot(\mathbf{C}\times\mathbf{D}) &= (\mathbf{A}\cdot\mathbf{C})\,(\mathbf{B}\cdot\mathbf{D}) - (\mathbf{A}\cdot\mathbf{D})\,(\mathbf{B}\cdot\mathbf{C})\,,\\
(\mathbf{A}\times\mathbf{B})\times(\mathbf{C}\times\mathbf{D}) &= (\mathbf{A}\cdot(\mathbf{B}\times\mathbf{D}))\,\mathbf{C} - (\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}))\,\mathbf{D}\\
&= (\mathbf{A}\cdot(\mathbf{C}\times\mathbf{D}))\,\mathbf{B} - (\mathbf{B}\cdot(\mathbf{C}\times\mathbf{D}))\,\mathbf{A}
\end{aligned}$$

2. (i) Write in invariant vectorial form

$$\varepsilon_{inl}\,\varepsilon_{irs}\,\varepsilon_{lmp}\,\varepsilon_{stp}\,a_n\,a_r\,b_m\,c_t\,.$$

   (ii) Using the totally antisymmetric tensor ε_{ijk} write the product

$$(\mathbf{a}\cdot[\mathbf{b}\times\mathbf{c}])\,(\mathbf{a}'\cdot[\mathbf{b}'\times\mathbf{c}'])$$

   as sum of terms which contains only scalar products of the appearing vectors.

3. (i) Using Cartesian coordinates x, y, z, cylindrical coordinates ρ, φ, z and spherical coordinates r, θ, φ, calculate

$$\operatorname{grad} r\,,\quad \operatorname{div}\mathbf{r}\,,\quad \operatorname{curl}\mathbf{r}\,,\quad \operatorname{grad}(\mathbf{c}\cdot\mathbf{r})\,,\quad (\mathbf{c}\cdot\boldsymbol{\nabla})\mathbf{r}\,,$$

   where $\mathbf{r}$ is the radius vector, $\mathbf{c}$ is the same constant vector in all coordinate systems.
   Hints: The radius vector is $\mathbf{r} = x\,\mathbf{e}_x + y\,\mathbf{e}_y + z\,\mathbf{e}_z$, $\mathbf{r} = \rho\,\mathbf{e}_\rho + z\,\mathbf{e}_z$ and $\mathbf{r} = r\,\mathbf{e}_r$, respectively.
   Express the constant vector in Cartesian coordinates $\mathbf{c} = c_x\,\mathbf{e}_x + c_y\,\mathbf{e}_y + c_z\,\mathbf{e}_z = \mathbf{const}$ in the other coordinate systems using the relations between the unit base vectors.
   For the definition of the vector operations in the different coordinates use the Appendix Curvilinear coordinates – A.4 and A.5 – of the Course context (Teaching page TP2).

   (ii) Using the differential vector operator $\boldsymbol{\nabla}$ and the rules of differentiation and multiplication of vectors (without using Cartesian components) show that the following identities are valid

$$\begin{aligned}
\operatorname{grad}(\varphi\psi) &= \varphi\operatorname{grad}\psi + \psi\operatorname{grad}\varphi\,,\\
\operatorname{div}(\varphi\mathbf{A}) &= \varphi\operatorname{div}\mathbf{A} + \mathbf{A}\cdot\operatorname{grad}\varphi\,,\\
\operatorname{curl}(\varphi\mathbf{A}) &= \varphi\operatorname{curl}\mathbf{A} - \mathbf{A}\times\operatorname{grad}\varphi\,.
\end{aligned}$$