Hot electroweak matter and topological defects

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available under

http://www.physik.uni-leipzig.de/~schiller/private/wien98_as.ps.gz

OUTLINE

- Motivation: Why are topological fingerprints of EWPT interesting?
- Model: 3D effective model of high temperature fundamental Higgs theory
- Defect operators: Vortices and monopoles, elementary and extended
- At strong and weak first order phase transition: Results on densities and percolation threshold
- In the crossover region: Results at $M_H^* = 100$ GeV (percolation in the would-be symmetric phase $\leftrightarrow$ finite clusters in the would-be Higgs phase)
- Z-vortices as physical objects: plaquette and ρ^2 on vortices and elsewhere
- Conclusions

Recent work:

- first order transition: hep-lat/9805016
  (Chernodub, Gubarev, Ilgenfritz, Schiller)
  Phys. Lett. (in press)
- crossover region: hep-lat/9807016
  (Chernodub, Gubarev, Ilgenfritz, Schiller)
- test case (sphaleron structure): PL B424 106 (1998) (Chernodub, Gubarev, Ilgenfritz)
- spectral properties (gauge balls): hep-lat/9807023
  (Ilgenfritz, Schiller, Strecha)
- Overview LAT'98: hep-lat/9809025
  (Chernodub, Gubarev, Ilgenfritz, Schiller)

MOTIVATION

Two reasons to be interested in the topological signature of a model describing the electroweak phase transition (EWPT)

1. $3D$ Higgs model - a toy model for QCD:
   in $3D$ confinement in the symmetric phase similar to pure Yang Mills
   $\rightarrow$ look for monopoles in pure Yang Mills:
   maximally Abelian gauge (pointlike) monopoles in $3D$ pure $SU(2)$ (Bornyakov, Grygoryev, LAT'92)
   $\rightarrow$ look for monopoles and vortices with dynamical matter: how confinement gets lost?
2. electroweak baryon genesis (BG):
   Sacharov's 3rd condition for BG: nonequilibrium
   realized in the most popular scenario:
   - strong first order phase transition
   - transition by bubble nucleation
   - BG in bubble walls
   - non-erasure by sphalerons ($E_{\text{sphal}} \propto \Phi_{\text{broken}}$)

But the standard model has no true thermal phase transition:

$$M_{H\text{lower bound}} > M_H^{\text{crit}} = 72 \text{ GeV}$$

What else to explore?

- $\rightarrow$ alternative models: MSSM
  (Laine, Rummukainen)
- $\rightarrow$ alternative mechanisms: Defect mediated BG
  (Brandenberger et al., Trodden, Vachaspati)

Two basic defect structures:

- sphaleron
  non-contractible loop in configuration space
- Z-vortex
  non-contractible sphere in configuration space

$\rightarrow$ Hindmarsh, Klinkhamer, Rupp

- unstable configurations, generated by thermal fluctuations

- can be studied on the lattice in thermal equilibrium after the percolating network becomes dissolved

MODEL

Effective $3D$ $SU(2)$ lattice Higgs model with action:

$$S = \beta_G \sum_p \left(1 - \frac{1}{2}\mathrm{Tr}U_p\right) - \beta_H \sum_{x,\mu} \frac{1}{2}\mathrm{Tr}(\Phi_x^+ U_{x,\mu} \Phi_{x+\hat{\mu}})$$

$$+ \sum_x \left(\rho_x^2 + \beta_R(\rho_x^2 - 1)^2\right)$$

Higgs self coupling $\beta_R \leftrightarrow$ Higgs mass:

$$\beta_R = \frac{\lambda_3}{g_3^2}\frac{\beta_H^2}{\beta_G} = \frac{1}{8}\left(\frac{M_H^*}{80\ \mathrm{GeV}}\right)^2 \frac{\beta_H^2}{\beta_G}$$

hopping parameter $\beta_H \leftrightarrow$ $3D$ mass m_3^2:

$$\beta_H = \frac{2(1 - 2\beta_R)}{6 + a^2 m_3^2}$$

gauge coupling $\beta_G \leftrightarrow$ $3D$ coupling g_3^2:

$$\beta_G = \frac{4}{a g_3^2}$$

$3D$ approach pioneered by Helsinki group
worked out also perturbative relation to $4D$ continuum physics

lattice studies of the phase transition (PT):

- Helsinki
- Leipzig-Berlin (Humboldt Univ.)
- Bielefeld

characteristics and endpoint of the 1^{st} order PT:

- latent heat
- surface tension
- $3D$ string tension
- universality class
- spectrum (also Teper et al.)
- endpoint $M_{H\text{crit}} = 72$ GeV

DEFECT OPERATORS

Elementary topological defects:

unit isovector (adjoint composite field)

$$n_x = n_x^a \sigma^a \,, \;\; n_x^a = -(\phi_x^+ \sigma^a \phi_x)/(\phi_x^+ \phi_x)$$

Abelian projection

$$V_{x,\mu}(U,n) = U_{x,\mu} + n_x U_{x,\mu} n_{x+\hat{\mu}}$$

n_x covariantly constant w.r.t. $V_{x,\mu}$

gauge invariant flux through plaquette p

$$\bar{\theta}_p(U,n) = \arg\Big(\mathrm{Tr}\left[(\mathbb{1} + n_x) V_{x,\mu} V_{x+\hat{\mu},\nu} V^+_{x+\hat{\nu},\mu} V^+_{x,\nu} \right] \Big)$$

Note $\bar{\theta}_p \in [-\pi, \pi)$ and independent on reference point x on plaquette

Gauge invariant construction of monopole charge j_c inside cube c in terms of $\bar{\theta}_p$ penetrating surface ∂c

$$j_c = - \sum_{p \in \partial c} \bar{\theta}_p / (2\pi)$$

Next construct plaquette variable χ_p in terms of Abelian links $\chi_{x,\mu} = \arg\left(\phi_x^+ V_{x,\mu} \phi_{x+\hat{\mu}} \right)$ and define vorticity (as Abelian vortices in Abelian HM)

$$\sigma_p = \left(\chi_p - \bar{\theta}_p \right) / (2\pi)$$

Z-vortex formed by links dual to plaquettes carrying non-vanishing vorticity σ_p: ${}^*\sigma_l$

Z-vortex trajectories: either closed or begin/end on Nambu (anti-)monopoles:

$$\sum_{\mu=1}^{3} ({}^*\sigma_{x-\widehat{\mu},\mu} - {}^*\sigma_{x,\mu}) = {}^*j_x$$

Note: originally Nambu monopoles topological defects of an Abelian field defined in unitary gauge

Extended topological defects

A priori, we do not know the physical size (or size distribution) of electroweak defects:

therefore construct $(ka)^2$ Z-vortices and $(ka)^3$ Nambu monopoles on decimated lattice of sites X

Extended V-links and phase angles of length-k Higgs operators

$$V^{(k)}_{X,\mu} = V_{X,\mu} V_{X+\widehat{\mu},\mu} \dots V_{X+(k-1)\widehat{\mu},\mu}$$

$$\chi^{(k)}_{X,\mu} = \arg\left(\phi^{+}_{X} V^{(k)}_{X,\mu} \phi_{X+k\widehat{\mu}}\right)$$

Construct extended vorticity and monopole charges

AT STRONG AND WEAK 1^{ST} ORDER PHASE TRANSITION

monopole density

$$\rho_m = \sum_c |j_c| / L^3$$

vortex density

$$\rho_v = \sum_p |\sigma_p| / (3L^3)$$

Vortex cluster: collection of dual links with non-zero vorticity and common sites

A connected cluster cannot be decomposed into sub-clusters without cutting a dual link with non-zero vorticity

Cluster length: number of dual links forming a connected cluster

cluster 2-point function

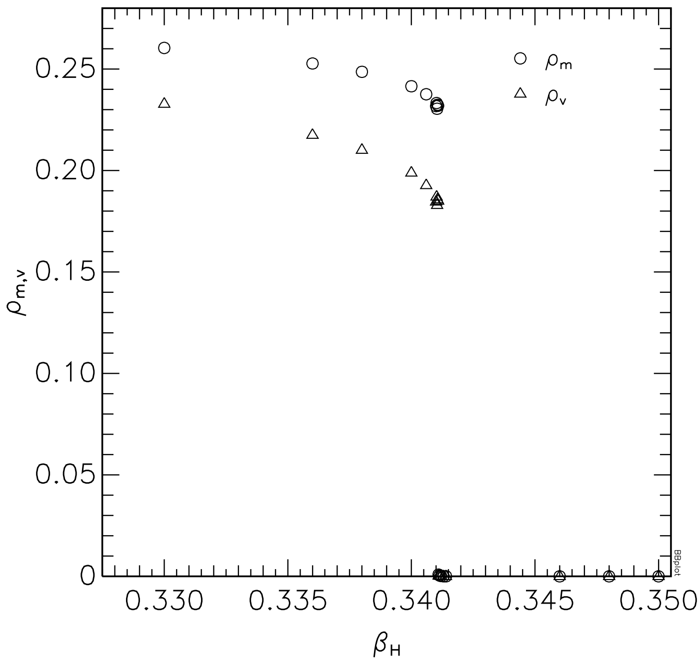
$$C(r) = \frac{\sum\limits_{x,y,i} \delta_{x \in {}^*\sigma(i)} \delta_{y \in {}^*\sigma(i)} \cdot \delta\left(|x-y| - r\right)}{\sum\limits_{x,y} \delta\left(|x-y| - r\right)}$$

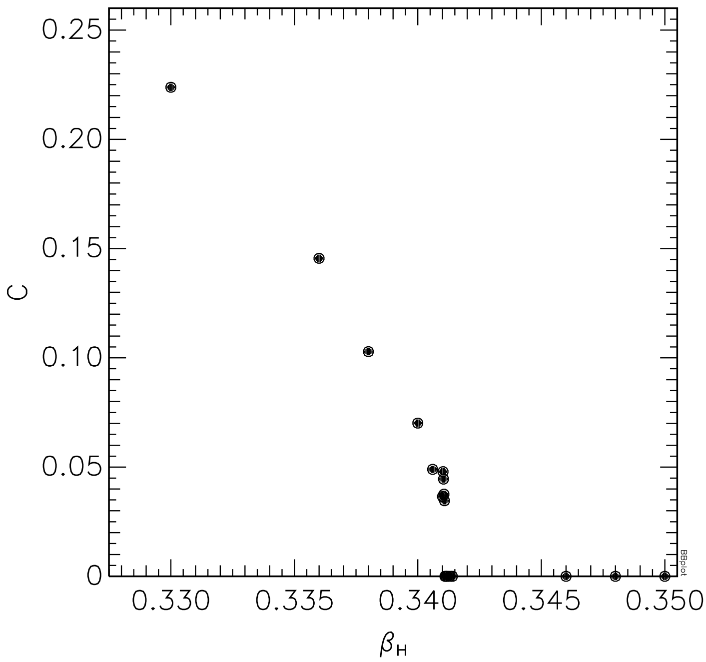
percolation probability

$$C = \lim_{r \to \infty} C(r)$$

1. Elementary defects:

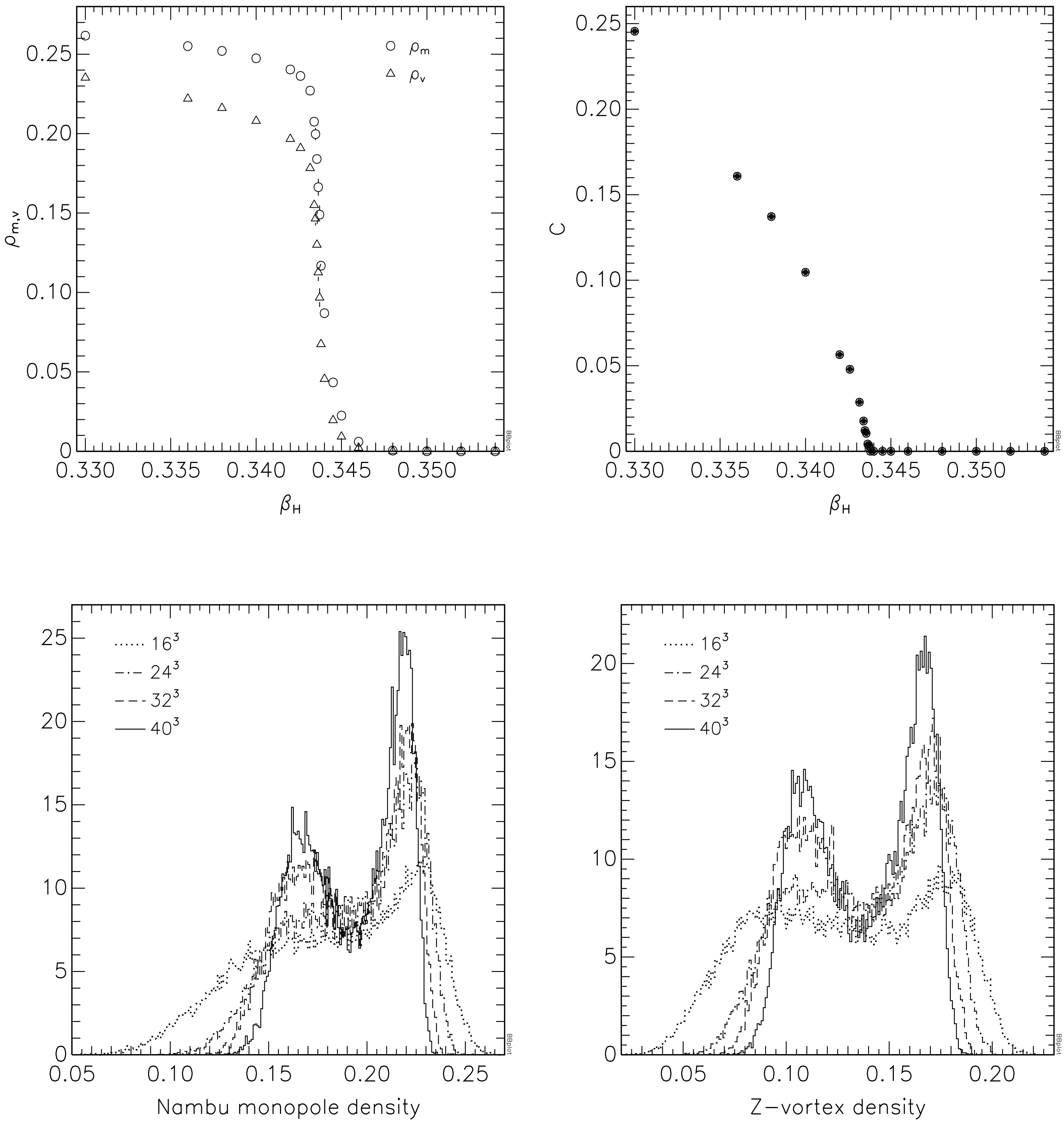
- strongly first order: $M_H^* = 30$ GeV, $\beta_G = 12$
  lattice size 16^3
  $\rightarrow$ percolation threshold, densities vanish in Higgs phase





- weakly first order: $M_H^* = 70$ GeV, $\beta_G = 12$
  lattice sizes 16^3, 24^3, 32^3, 40^3
  $\rightarrow$ percolation threshold, densities extend to the Higgs phase
  $\rightarrow$ two-state signal in histograms of the averaged densities, competing phases differ in monopole and vortex density

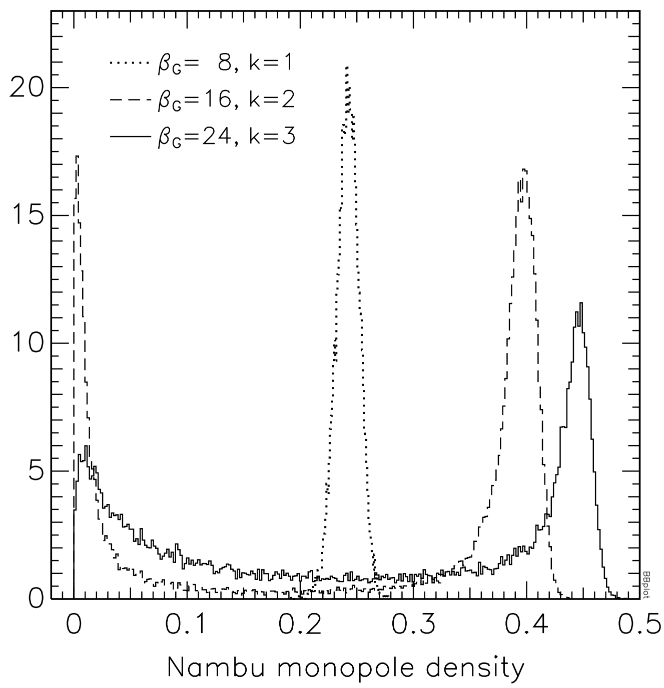
$M_H^* = 70$ GeV

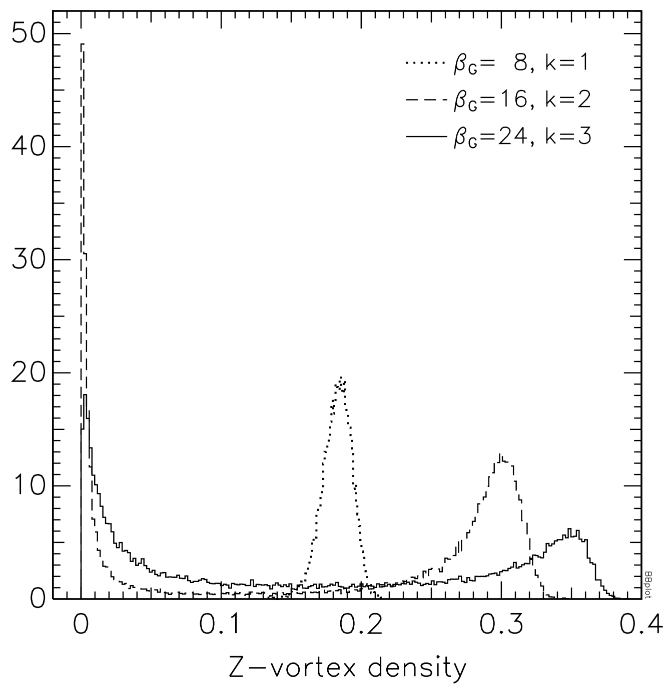


ρ_m
ρ_v
$\rho_{m,v}$
0.25
0.20
0.15
0.10
0.05
0
0.330
0.335
0.340
0.345
0.350
β_H
C
16^3
24^3
32^3
40^3
25
20
15
10
5
Nambu monopole density
Z−vortex density

2. Extended defects (continuum limit?):

- investigated only at strongly first order: $M_H^* = 30$ GeV, there expected less severe finite volume effects,
  $\rightarrow$ only one lattice size 16^3 for $\beta_G = 8 \rightarrow 16 \rightarrow 24$, in order to compare defects with extension $k = 1 \rightarrow 2 \rightarrow 3$, respectively
  $\rightarrow$ percolation threshold coincides with phase transition for all defect sizes
  $\rightarrow$ is density convergent to a continuum limit? Data so far inconclusive.

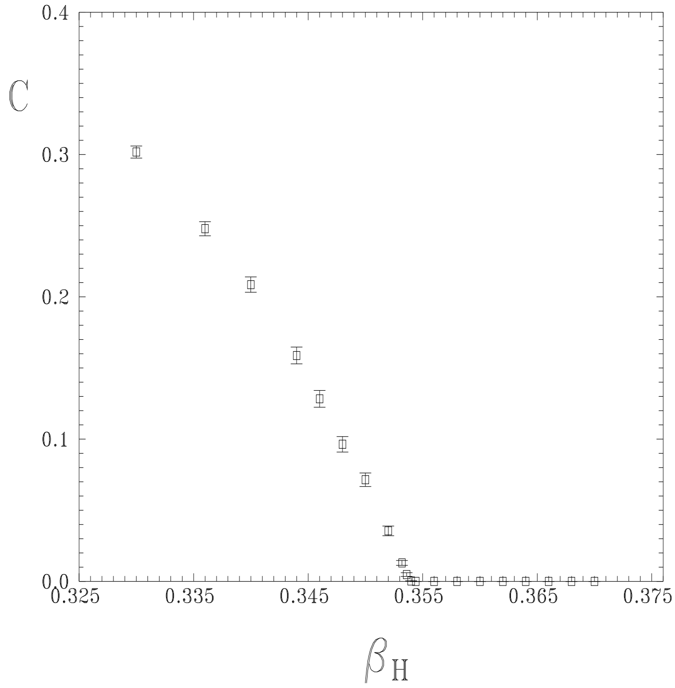




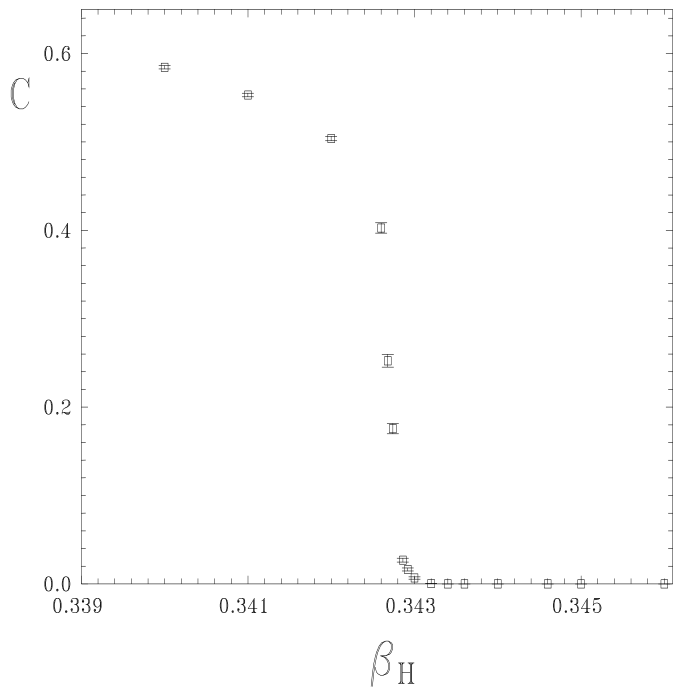
IN THE CROSSOVER REGION

1. Elementary defects:
   - smooth crossover: $M_H^* = 100$ GeV, $\beta_G = 12$
     lattice size 16^3
     $\rightarrow$ percolation threshold at appreciable densities
2. Extended defects:
   - percolation threshold: continuum behavior?
     we expect strong finite volume effects, therefore we have increased lattice $16^3 \rightarrow 32^3 \rightarrow 48^3$ for $\beta_G = 8 \rightarrow 16 \rightarrow 24$, in order to compare defects with extension $k = 1 \rightarrow 2 \rightarrow 3$, respectively
     $\rightarrow$ percolation threshold corresponds to same temperature for all defect sizes
     $T^{\rm perc} = 170$ GeV, $M_H = 94$ GeV (purely bosonic theory)
     $T^{\rm perc} = 130$ GeV, $M_H = 103$ GeV (with top)

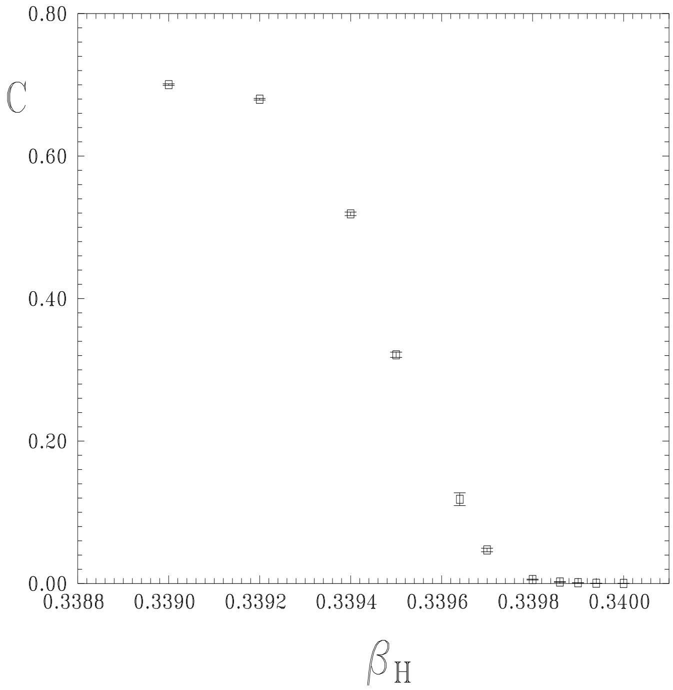
$M_H^* = 100\ \mathrm{GeV}$



$\beta_G = 8,\ L = 16,\ k = 1$



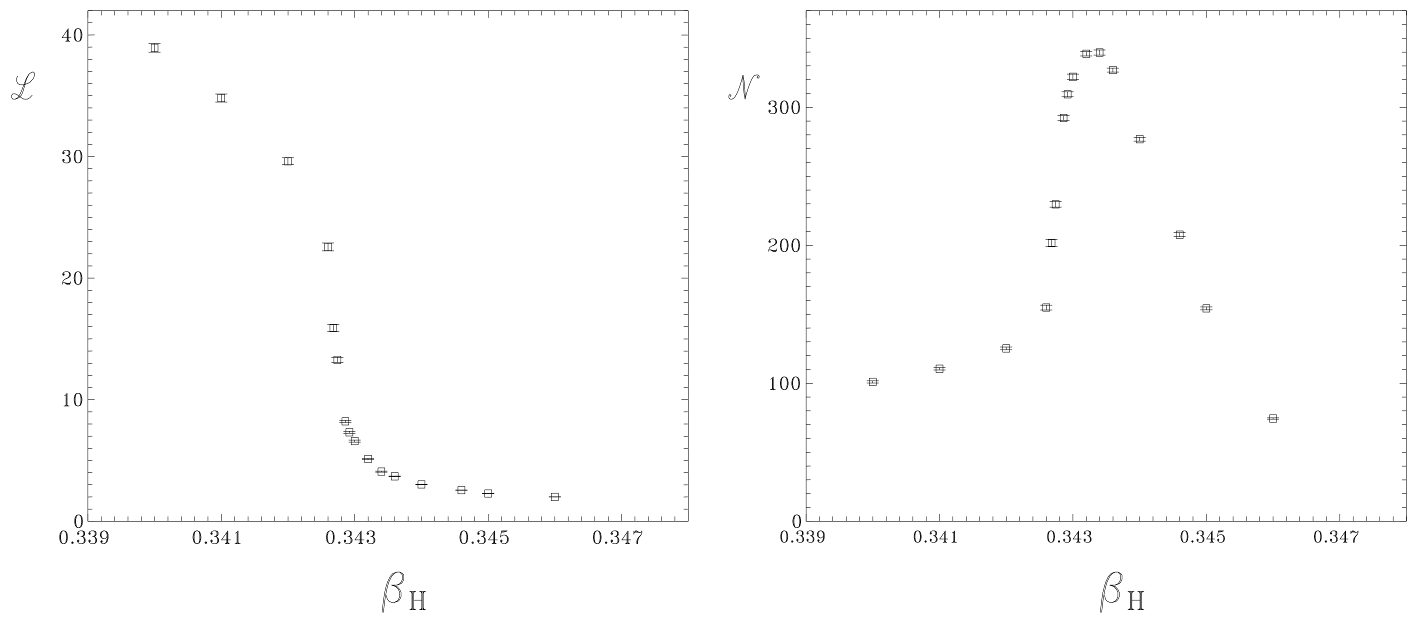
$\beta_G = 16,\ L = 32,\ k = 2$



$\beta_G = 24,\ L = 48,\ k = 3$

- change of vortex cluster structure similar for all vortex thicknesses $(ka)^2$, example $k = 2$
$\rightarrow$ average length per cluster drops to a plateau which then changes slowly with temperature
$\rightarrow$ multiplicity of separated cluster grows suddenly

$$M_H^* = 100 \text{ GeV}$$

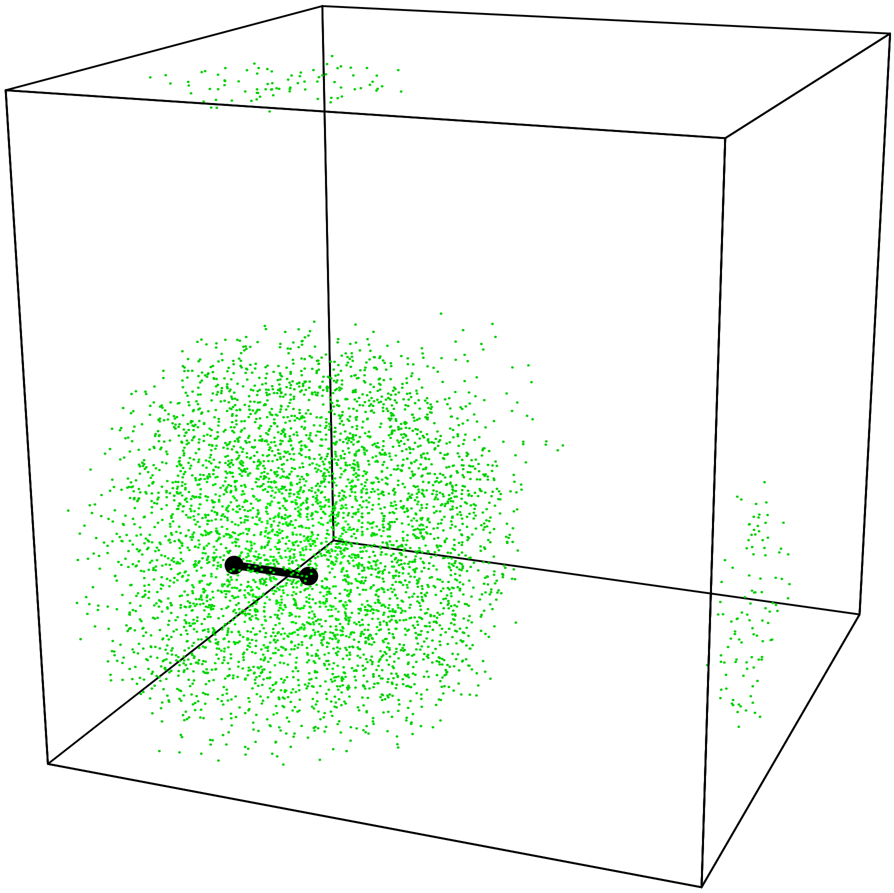


- Suggested scenario: percolating clusters decay into smaller ones (vortex rings and monopolium) $\rightarrow$ kinetic approach to crossover
at still smaller T rings disappear but pairs still survive
tail: interpret $\rho_v/2$ as sphaleron density on broken side

conjecture supported by signatures of classical sphaleron

Classical lattice sphaleron with Nambu $M\bar{M}$-pair connected by Z-vortex string, density of clouds $\propto$ to decrease in Higgs field modulus

from PL B424



Z-VORTICES AS PHYSICAL OBJECTS

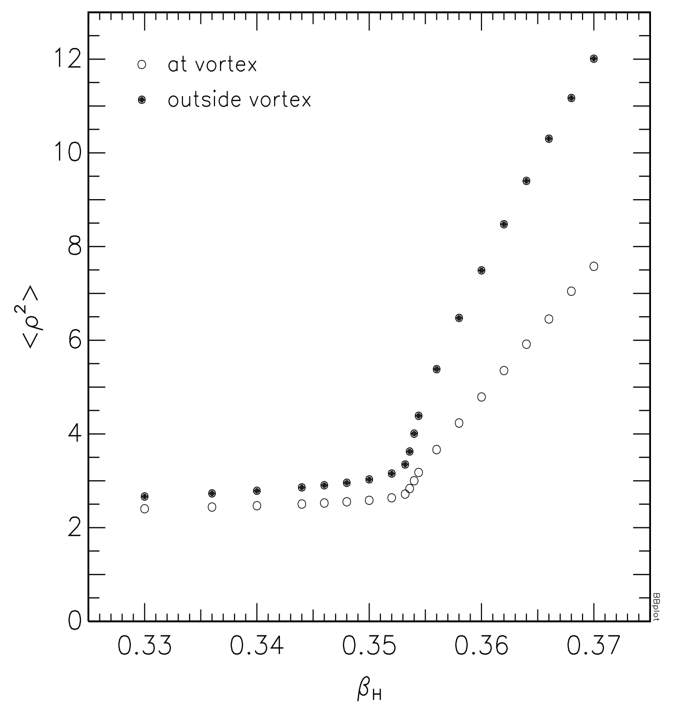
Expect in continuum: at center of Z-vortex zero Higgs field modulus, maximal energy density
$\rightarrow$ if vacuum populated by vortex-like configurations find lines of Higgs field zeroes and maximal energy density

First step: elementary defects (no profile yet)
use non-vanishing vorticity as trigger

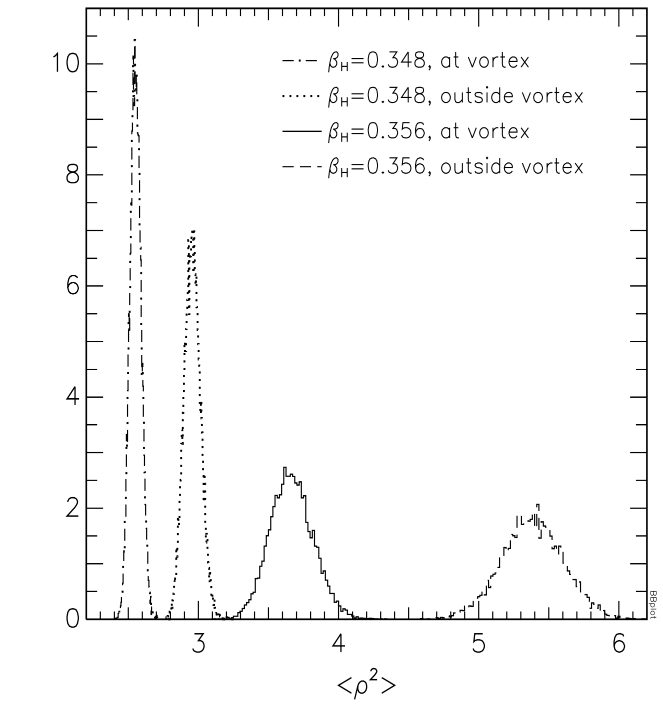
Data: $M_H^* = 100$ GeV, 16^3, $\beta_G = 8$ vs. β_H and histograms of plaquette for plaquettes carrying vorticity and of ρ^2 for corners of plaquettes carrying vorticity

clearly seen different behaviour near to vortex trajectory and outside
larger effect on Higgs side since vortices are dilute

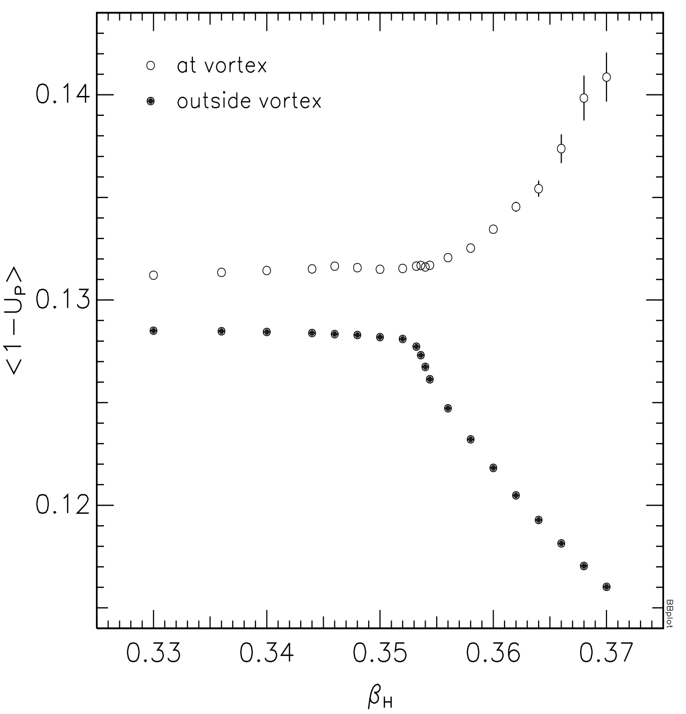
localization of Higgs zeroes from extended vortex construction?



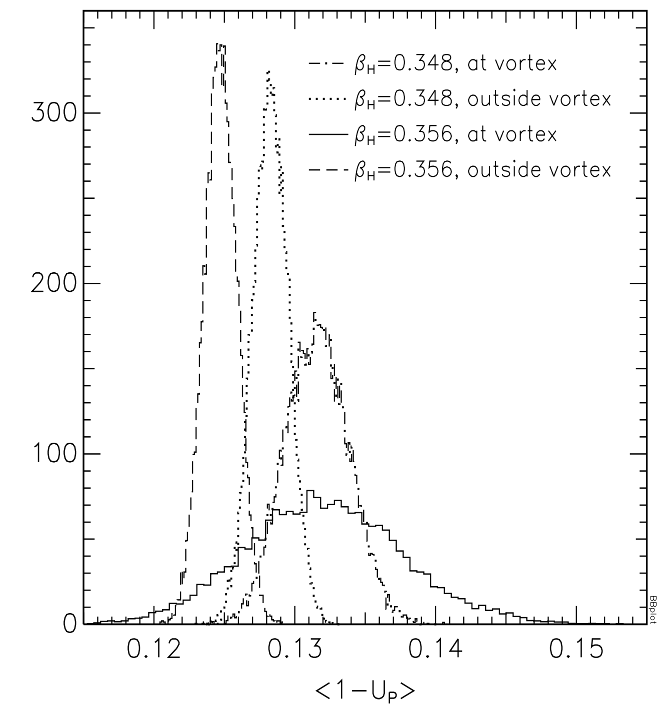
at vortex
outside vortex
$<\rho^2>$
β_H



β_H=0.348, at vortex
β_H=0.348, outside vortex
β_H=0.356, at vortex
β_H=0.356, outside vortex
$<\rho^2>$



at vortex
outside vortex
$<1-U_P>$
β_H



β_H=0.348, at vortex
β_H=0.348, outside vortex
β_H=0.356, at vortex
β_H=0.356, outside vortex
$<1-U_P>$

CONCLUSIONS

1. Study of topological defects in lattice models developed for EWPT is an interesting new topic
2. Topic is not so new for people outside the lattice community
3. New field of overlapping interest with cosmologists and solid state physicists
4. It gives fresh inspiration for the search for the ultimate confinement mechanism
5. Possible basis for an alternative scenario of baryon genesis, within the standard model or beyond
6. Near future:
   - more systematic exploration of continuum behavior
   - more realistic models with non-zero Weinberg angle (Z and W vortices)