$3D$ lattice cQED at $T > 0$ and monopole binding: Zero external field case

M.N. Chernodub[1], E.-M. Ilgenfritz[2], A. Schiller[3]

[1]ITEP Moscow, Russia

[2]University of Tübingen, Germany

[3]University of Leipzig, Germany

Dublin June 21, 2001

1. Motivation
2. Model and restrictions on external fields
3. Some heuristic considerations
4. Phase transition and confinement
5. Properties monopole system at
6. Summary

Recent work:

- C,I,S: hep-lat0501021, PRD accepted

Motivation

- in 3D cQED confinement is proven and understood (Polyakov, Göpfert and Mack)
- Confinement due to screening of magnetic field induced by electric current circulating along the Wilson loop,
  formation of string (polarized sheet of $M\bar{M}$ of finite thickness along minimal surface spanned by Wilson loop) leads, for non–vanishing electric current, to an excess free energy proportional to area
- study how mechanism works under special conditions: T, external fields
- $T > 0$: confinement-deconfinement phase transition studied on the lattice both analytically (Parga) and numerically (Coddington et al.)
- Using Svetitsky-Yaffe universality arguments, transition explained in terms of $U(1)$ vortex dynamics of corresponding $2D$ spin system, representing the dynamics of the Polyakov loop
  approaching T_c, vortices and anti-vortices start to form bound states, at $T > T_c$ no unbound vortices and anti-vortices are left

- We: interprete deconfinement starting from confinement picture in terms of magnetic monopoles: allows to investigate restructuring of gauge field which *does not need to project out* the non–trivial Euclidean time dependence
- Confining plasma of monopoles and anti–monopoles turns into dipole plasma at deconfinement phase transition
- The monopole deconfinement scenario raises the question whether monopole properties (such as pairing and orientation) could be influenced by external field

Model and restrictions on external fields

$$S[\theta,\theta^{\text{ext}}] = -\beta \sum_p \cos\left(\theta_p - \theta_p^{\text{ext}}\right)$$

$$\beta = \frac{1}{a\, g_3^2}, \quad \frac{T}{g_3^2} = \frac{\beta}{L_t}$$

External electric field E: field directly coupled to plaquettes lying in the 31 plane,

$$E = \theta_{31}^{\text{ext}} = \frac{2\pi n_E}{L_1\, L_3}, \quad n_E \in \mathbb{Z}\,, \quad \theta_{12}^{\text{ext}} = \theta_{23}^{\text{ext}} = 0$$

Magnetic field B: directly coupled to plaquettes lying in plane 12,

$$B = \theta_{12}^{\text{ext}} = \frac{2\pi n_M}{L_1\, L_2}, \quad n_M \in \mathbb{Z}\,, \quad \theta_{31}^{\text{ext}} = \theta_{23}^{\text{ext}} = 0$$

Quantization of external fields necessary to match with p.b.c. imposed on the lattice (Damgaard,Heller)
p.b.c. imply: $L_i \times L_j$ Wilson loop $W(L_i, L_j) = 1$
On the other hand $W(L_i, L_j) = \exp\{i\Phi\}$, Φ flux through plane ij
$\rightarrow$ $\Phi = 2\pi n_{\text{ext}}$, $n_{\text{ext}} \in \mathbb{Z}$

From form of action: number of external fluxes is restricted, $0 < n_{\text{ext}} < L_i L_j/2$
we restrict that number to $0 < n_{\text{ext}} < L_i L_j/4$

Largest considered fluxes correspond to lattice artefacts

Monte Carlo algorithm

combines local Monte Carlo step (Metropolis + microcanonical sweep) with a global refreshment step to improve ergodicity

Global refreshment step: attempt to add additional unit of flux (sign and direction chosen randomly) to dynamical gauge field $\theta_i \to [\theta_i + \tilde{\theta}_i]_{\mathrm{mod} 2\pi}$ subject to a global Metropolis acceptance check

lattice: $32^2 \times 8$

Heuristic considerations

In $3D$ cQED monopoles interact via Coulomb potentials

$$S = \frac{g_m^2}{2} \sum_{a,b} q_a\, q_b\, V_T(\vec{x}_a - \vec{x}_b)$$

V_T interaction potential eventually depends on T

$T = 0$:

$V_0(R) = -(4\pi\, R)^{-1}$, Coulomb gas of $M\bar{M}$, medium confines electric charges (Polyakov)

$T > 0$:

At small separations interaction is $T = 0$ like

At large spatial separations $\mathbf{x}$ potential essentially two-dimensional

$$V_T(\mathbf{x}, z) = -2\, T\, \ln |\mathbf{x}| + \dots\,, \qquad |\mathbf{x}| T \gg 1$$

in z direction $3D$ Coulomb type for small spatial inter–monopole distances $|\mathbf{x}|T \ll 1$

$$V_T(\mathbf{x}, z) = -(4\pi\, z)^{-1}$$

at $z = 1/(2T)$ force between $M\bar{M}$ vanishes due to periodicity

Expect

$T > 0$ monopoles form magnetically neutral states (dipoles) bounded in spatial directions

dipoles have finite average spatial size $\bar{d}$ which decreases with increasing T
$\bar{r} \approx \rho^{-1/3}$: average distance between monopoles in plasma
small T: $\bar{d} < \bar{r}$, monopole plasma
large T: $\bar{d} > \bar{r}$, dipole plasma

Use these heuristice arguments to estimate T_c

$$\bar{d} = \xi\, \rho^{-1/3}$$

factor ξ to be defined from simulation

Consider $3D$ compact $U(1)$ on $L_s^2 \times L_t$ lattice with action in Villain representation

$$\mathcal{Z} = \int_{-\pi}^{+\pi} \mathcal{D}\theta \sum_{n(c_2)\in \mathbb{Z}} \exp\Big\{-\frac{\beta_V}{2}||\mathrm{d}\theta + 2\pi n||^2\Big\}$$

θ compact $U(1)$ gauge field, n integer–valued auxiliary tensor field variable

Villain coupling β_V related to Wilson coupling β

$$\beta_V(\beta) = \left[2\log\left(\frac{I_0(\beta)}{I_1(\beta)}\right)\right]^{-1}$$

Z rewritten in *grand canonical* form (sum over monopole charges in the (dual) lattice):

$$\mathcal{Z} \propto \mathcal{Z}_{mon} = \sum_{m(c_3)\in \mathbb{Z}} \exp\Big\{-2\pi^2\, \beta_V (m, \Delta^{-1} m)\Big\}$$

$\Delta^{-1}(x; L_s, L_t)$ inverse *asymmetric* Laplacian
m_c monopole charge in cube c_3
Monopole density ρ from $\mathcal{Z}_{mon}$

$$\rho(\beta; L_s, L_t) = 2 \exp\Big\{-2\pi^2 \beta_V(\beta)\, \Delta^{-1}(0; L_s, L_t)\Big\}$$

Note: no interaction between monopoles ("bindingless")

Estimate dipole size using the *canonical* $M\bar{M}$ (dipole) partition function (easily read off from $\mathcal{Z}_{mon}$)

$$\mathcal{Z}^{(2)}_{dip} = \text{const} \cdot \sum_{\substack{x \\ x^2 \neq 0}} \exp\Big\{4\pi^2 \beta_V [\Delta^{-1}(x) - \Delta^{-1}(0)]\Big\}$$

r.m.s. dipole size $\bar{d}$

$$\bar{d}^2(\beta_V; L_s, L_t) = \frac{1}{\mathcal{Z}^{(2)}_{dip}} \sum_x x^2 \times$$

$$\exp\Big\{4\pi^2 \beta_V \Big[\Delta^{-1}(x) - \Delta^{-1}(0)\Big]\Big\}$$

Extract ξ from data on β_c from at $16^2 \times L_t$, $L_t = 4, 6, 8$ (Coddington et al.):

We take $\xi \approx 2/3$ and predict size dependence of β_c

	$L_s = 16$		$L_s = 32$		$L_s = 64$
L_t	β_c^{th}	β_c	β_c^{th}	β_c	β_c^{th}
4	1.87	1.83(2)	2.01	-	2.10
6	2.04	2.08(2)	2.26	2.18(3)	2.44
8	2.12	2.14(5)	2.39	2.30(2)	2.62

Table 1: The critical coupling constant β_c^{th} calculated for different lattices $L_s^2 \times L_t$ compared to lattice Monte Carlo results

Conclusion:

Simple heuristic arguments based on the monopole–dipole picture work surprisingly well

Phase transition and confinement

localize deconfinement transition

$$\langle|L|\rangle = \frac{1}{L_s^2}\langle|\sum_{\mathbf{x}} L(\mathbf{x})|\rangle\,, \qquad \langle|L|^2\rangle = \frac{1}{L_s^2}\langle|\sum_{\mathbf{x}} L(\mathbf{x})|^2\rangle$$

constructed from Polyakov loop $(\mathbf{x} = (x, y))$

$$L(\mathbf{x}) = \exp\left\{i\sum_{z=1}^{L_t}\theta_3(\mathbf{x}, z)\right\}$$

Susceptibility of Polyakov loop

$$\chi_L = \langle|L|^2\rangle - \langle|L|\rangle^2$$

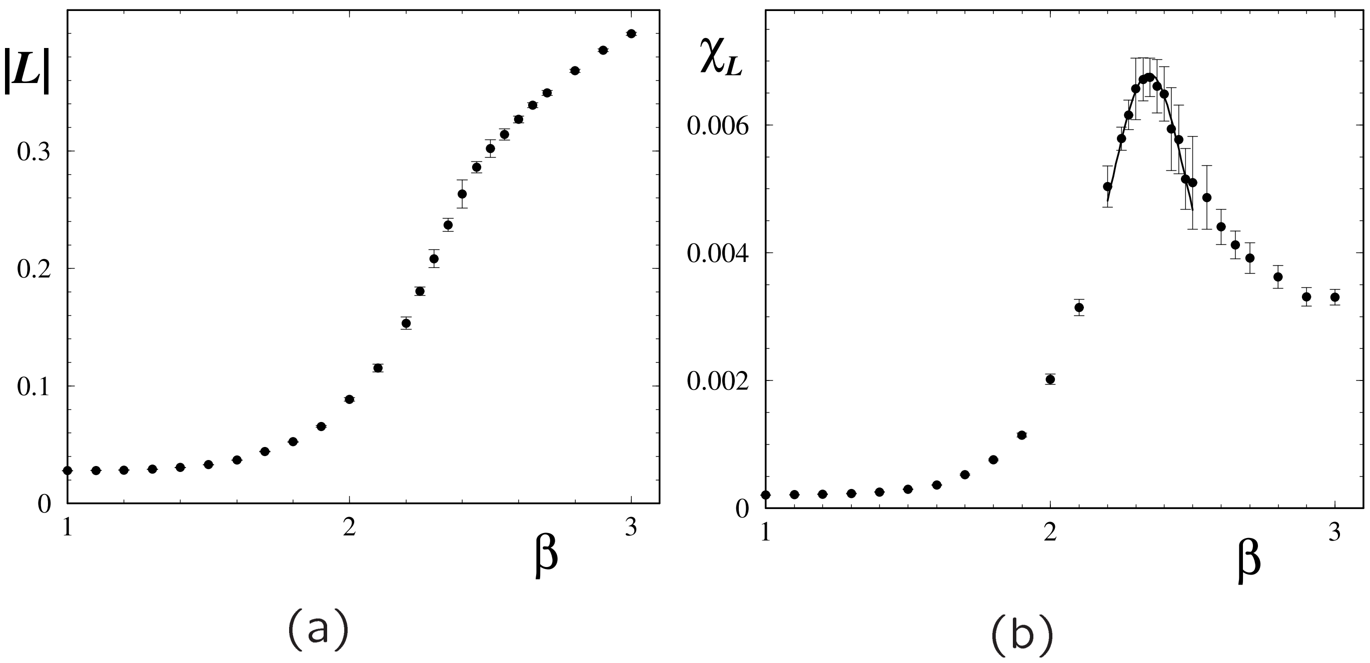


Figure 1: (a) The expectation value of the absolute value of the average Polyakov loop and (b) its susceptibility *vs.* β.

String tension:

use plane-plane correlators of two Polyakov loops and averages of temporal Wilson loops

$$L_{\mathrm{plane}}(x) = \sum_{y=1}^{L_s} L(x,y)$$

Plane-plane correlator expected to be of form:

$$\langle L_{\mathrm{plane}}(0) L^*_{\mathrm{plane}}(x)\rangle = \mathrm{const} \cdot \cosh\left[\sigma L_t \left(x - \frac{L_s}{2}\right)\right]$$

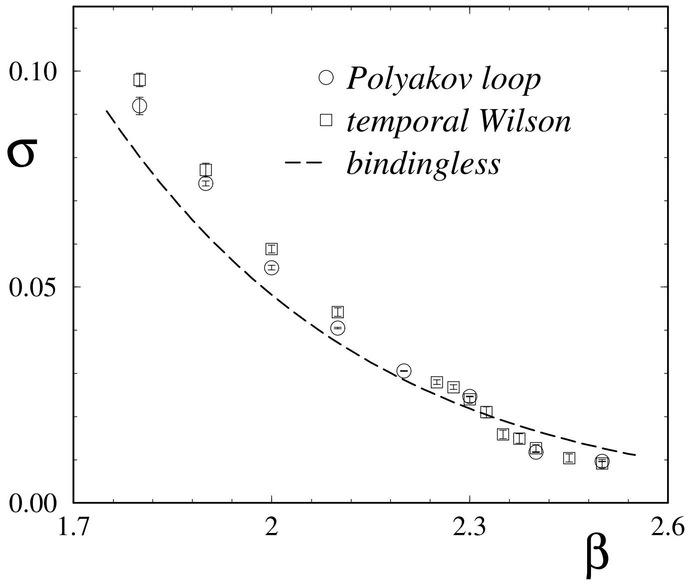


Figure 2: String tensions as functions of β compared with the bindingless theoretical result

Agreement between prediction (Banks et al., Ambjorn et al.)

$$\sigma(\beta) = \frac{4\sqrt{2}}{\pi\sqrt{\beta_V(\beta)}} \exp\left\{-\pi^2 \beta_V(\beta)\, \Delta^{-1}(0; L_s, L_t)\right\}$$

and numerical results only near $\beta \approx 2.3$

Properties of monopole system

Basic quantity: monopole density

$$\rho = \frac{1}{L_s^2 L_t} \sum_c |m_c|$$

m_c integer valued monopole charge inside cube c defined in standard way

$$m_c = \frac{1}{2\pi} \sum_{P \in \partial c} (-1)^P \, [\theta_P]_{\mathrm{mod}\, 2\pi}$$

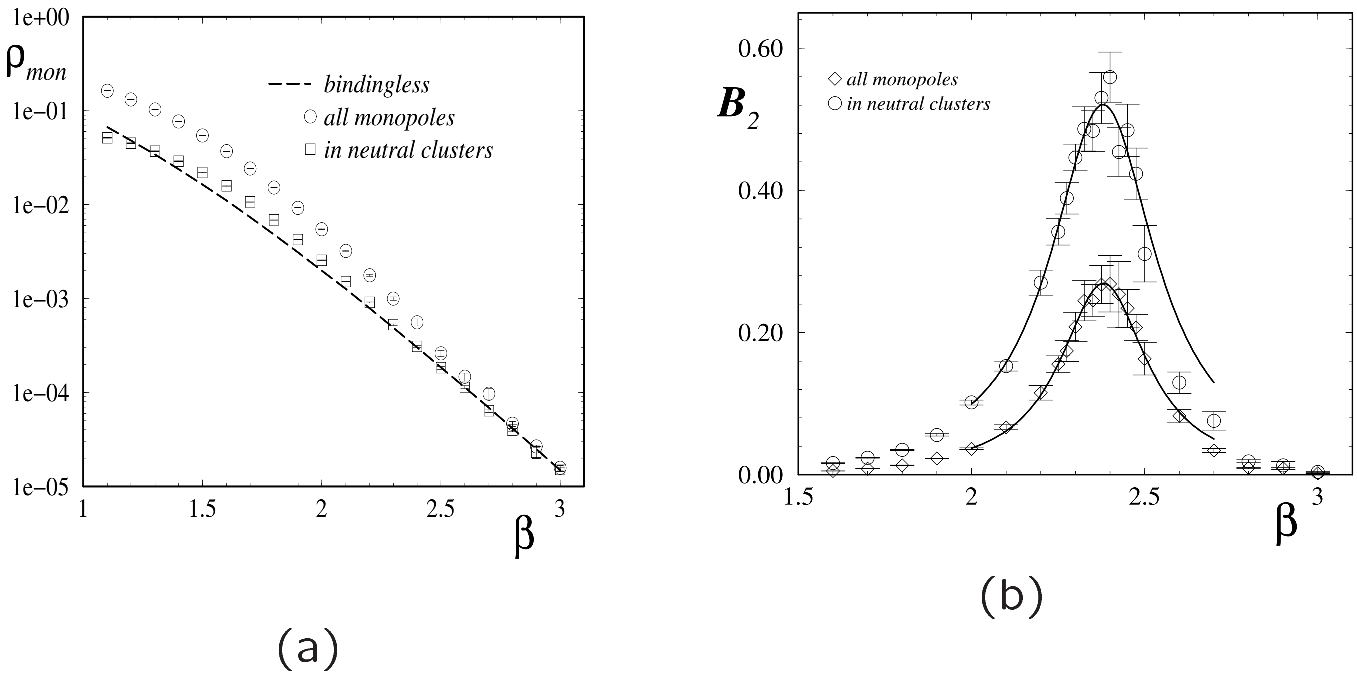
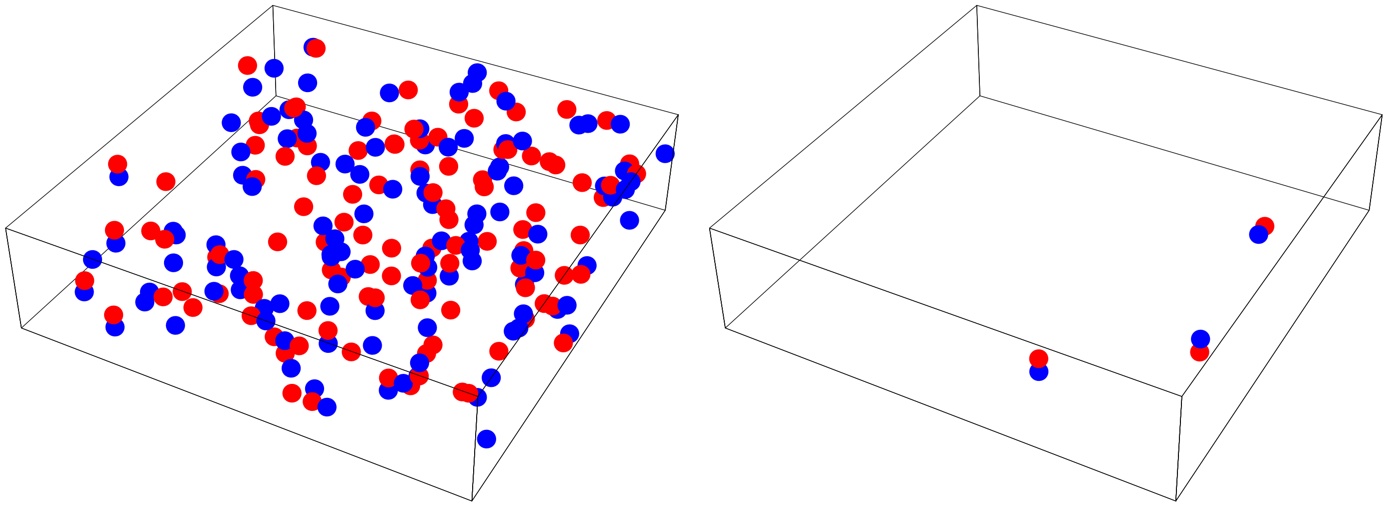


Figure 3: (a) Density of all monopoles and of monopoles in neutral clusters and (b) Binder second cumulants *vs.* β.

predicted "bindingless" monopole density in agreement with numerical data only near β_c

Typical monopole configurations in both phases



(a) (b)

Figure 4: (a) confinement phase ($\beta = 1.6$) and (b) deconfinement phase ($\beta = 2.5$).

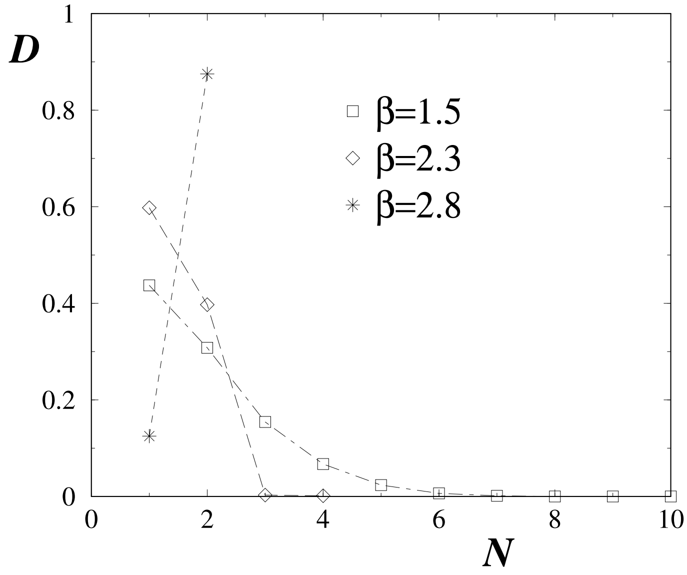
study monopole pairing from cluster structure of monopole ensemble extracted from Monte–Carlo configurations

Clusters:

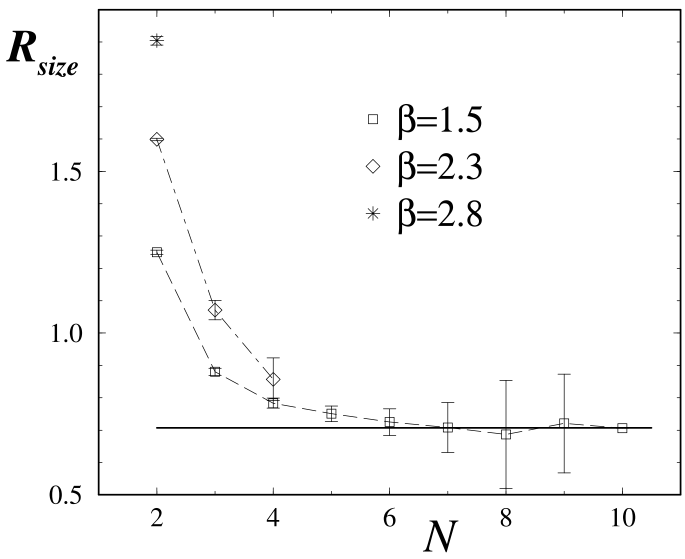
connected groups of monopoles and anti-monopoles, where each object separated from at least one neighbour belonging to the same cluster by distance $\leq R_{\max}(= \sqrt{3}a)$

Neutral monopole cluster: charges of constituent monopoles sum up to zero

Some details on cluster structure:
Confinement: 45 % isolated (anti-)monopole
Transition: mostly monopoles and dipoles (monopoles: dipoles of larger size?)
Deconfinement: dipoles dominate



(a)



(b)

Figure 5: (a) Cluster distribution and (b) cluster shape function as function of constituent monopole number N inside clusters for various β.

Expect: spatial force > "temporal " force
to be analysed using cluster sphericity

$$R_{\text{size}}(N) = \frac{\langle|\Delta z|\rangle_N}{\sqrt{\langle|\Delta x|\rangle_N^2 + \langle|\Delta y|\rangle_N^2}}$$

$\langle|\Delta x|\rangle_N$ average x-distance from cluster center
Small clusters: directed predominantly along z; the larger β the stronger the elongation
Large clusters: direction is random, random limit marked by solid line

Check monopole density fits vs. string tension:

$$\sigma^{\text{th}} = \frac{4}{\pi}\sqrt{\frac{\rho(\beta)}{\beta_V(\beta)}}$$

Study

$$R_\sigma = \frac{\sigma}{\sigma^{\text{th}}}$$

using as input *measured* monopole density ρ in σ^{th}

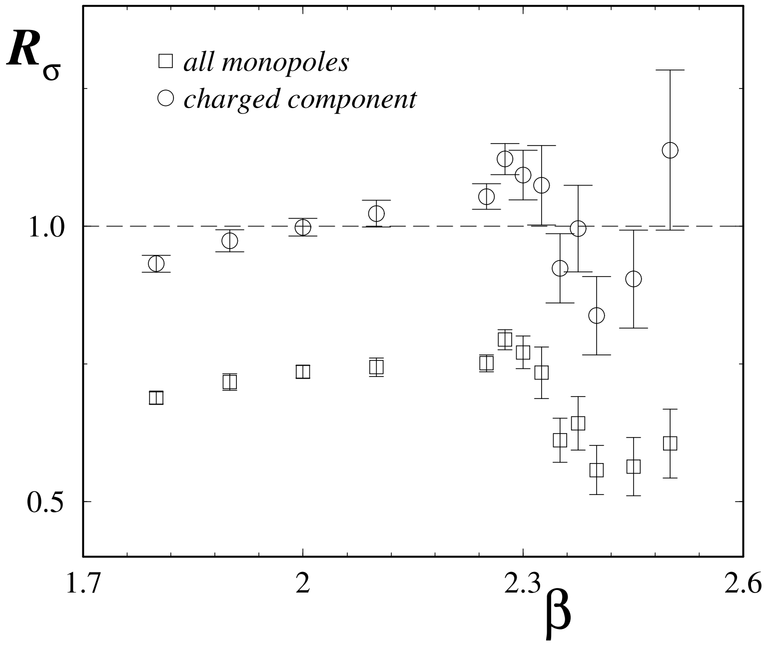


Figure 6: Ratio of the string tensions *vs.* β.

Outcome:

Charged monopoles provide major contribution to string tension

Neutral fraction of monopoles bound in small dipoles does not contribute to σ

Dipole formation due to Coulomb forces also happens at $T = 0$
effect increases monopole density compared to that in "bindingless" world

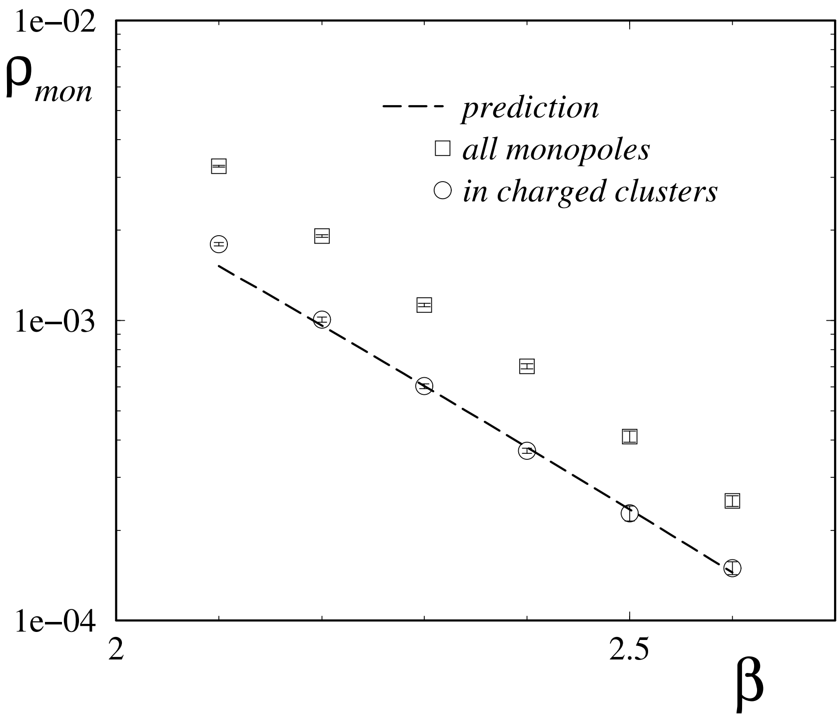


Figure 7: The density of all monopoles and of monopoles in charged clusters *vs.* β for a 32^3 lattice compared to the "bindingless" prediction.

Charged monopoles comprise around 55% of total monopole density
Only monopoles from charged clusters (including separate monopoles) contribute to the string tension
The binding effect leads to a large fraction of inactive neutral clusters

Summary

- we have considered a mechanism of the finite temperature deconfinement phase transition in $3D$ lattice cQED based on monopole binding (similar to Agasian, Zarembo in continuum)
- predict pseudocritical coupling as function of the lattice size
- we demonstrated that monopoles are sensitive to the phase transition despite the fact that the monopole density itself behaves smoothly across the transition
- Both monopole density and string tension differ from predictions based on a "bindingless" model
  The measured ratio between these two quantities derived in that model remains valid in the confinement
- Dipole formation happens both in the confinement and deconfinement phases
- In the deconfinement phase tightly bound dipoles — safely identified by a cluster algorithm — dominate in the vacuum
  The dipoles are elongated dominantly in the temporal direction

- At the confinement phase transition we observe mostly clusters with two constituents or single monopoles and anti–monopoles
  Decreasing further T (or β), the monopoles become dense and form connected clusters (on a coarser and coarser lattice)
  The largest clusters are more and more spherical

- Whether the observed properties of the dipole gas formation survives in the continuum limit deserves an additional study

- When the phase transition is mediated by charged objects, one could expect that external fields will influence the phase transition
  For non–Abelian theories in 3+1 dimensions it was recently concluded (Cea,Cosmai) that confinement seems to become restored under influence of an external chromomagnetic field

- Poster:
  study of our model under such external conditions, concerning the influence of confinement and relevant properties of the monopole system