金沢大学
素粒子論研究室

The gauge boson propagator of cQED$_3$ in the confined and deconfined phases

M.N. Chernodub[1], E.-M. Ilgenfritz[2], A. Schiller[3]

[1]Kanazawa University, Japan

[2]RCNP, Osaka University, Japan

[3]Leipzig University, Germany

Leipzig December 13, 2001

- Introduction and Model
- Landau gauge fixing
- Gauge boson propagator, first results
- Conclusions

Recent work 2001:

- C,I,S: PR D64,054507, PR D64,114502, heplat 0110038
- C,I,S: in preparation

Introduction

- in 3D cQED confinement is proven and understood (Polyakov, Göpfert and Mack)
  Confinement is caused by a plasma of monopoles which emerge due to the compactness of the gauge field
- We recently studied how mechanism works under special conditions: T, external fields
  Localisation of phase transition using gauge boson degrees of freeedom or monopoles:

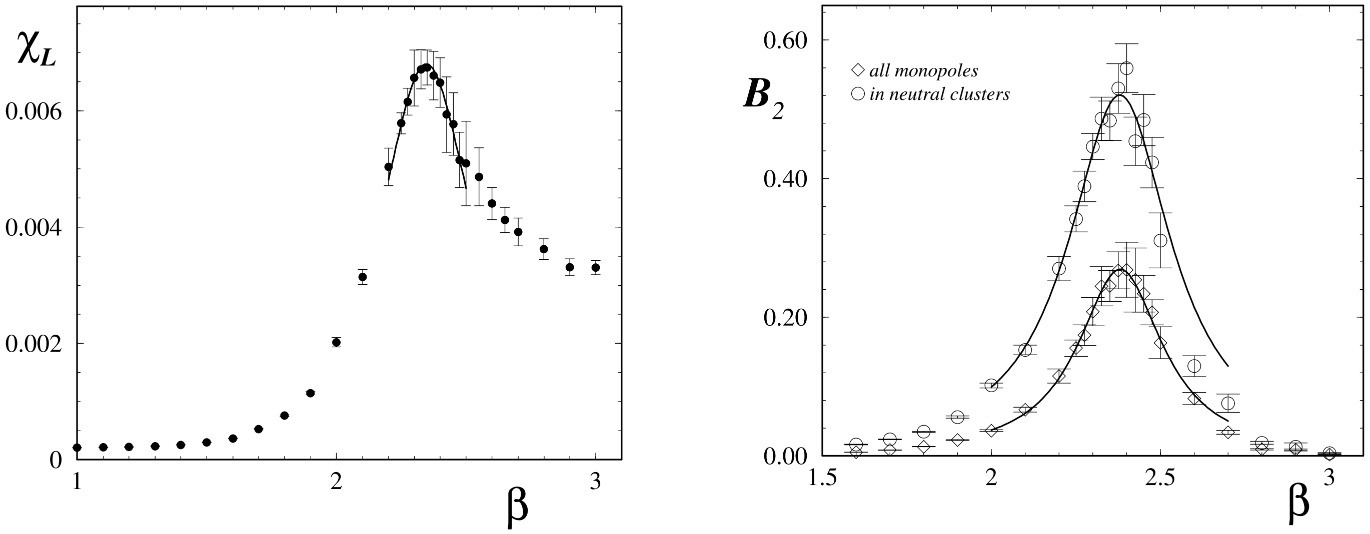

Figure 1: The expectation value of Polyakov loop susceptibily (l) and Binder second cumulant of monopole density (r) *vs.* β.

- We found: confining plasma of monopoles and anti–monopoles turns into dipole plasma at deconfinement phase transition

- Analogously in QCD_4: Studying the dynamics of the monopole current inside gluodynamics, monopole de-condensation at T_c appears as *de-percolation*, *i.e.* the decay of the infrared, percolating monopole cluster into short monopole loops
- We focus on the similarities between the gauge boson propagators of both theories
- The infrared physics in QCD is governed by non–perturbative effects which should become manifest in the low–momentum behaviour of the propagator

  One school of thought predicted a double pole $1/(p^2)^2$

  A more careful, but rather general analysis: absence of a double pole in confining theories

  Gribov: gluon propagator should vanish at low momentum
- We suggest an analogous behaviour in $cQED_3$: the monopole plasma is responsible for the infrared enhancement of the (massive) photon propagator

  $\rightarrow$ The Abelian monopole condensate in the case of QCD presumably plays the same role for the gluon propagator

Model

$$S[\theta] = -\beta \sum_p \cos\theta_p \,, \quad \beta = \frac{1}{a\, g_3^2} \,, \quad \frac{T}{g_3^2} = \frac{\beta}{L_t}$$

Landau gauge fixing

Continuum: $\partial_\mu A_\mu = 0$

Lattice:
Search iteratively for global maximum of functional

$$F(\theta) = \frac{1}{V} \sum_x F_x(\theta)$$

$$F_x(\theta) = \frac{1}{2d} \sum_{\mu=1}^{d} \left(\cos\theta_{x,\mu} + \cos\theta_{x-\hat{\mu},\mu} \right)$$

with respect to (local) gauge transformation

$$U_{x,\mu} \to \Lambda_x U_{x,\mu} \Lambda^*_{x+\hat{\mu}} \,, \qquad \Lambda_x = e^{i\Omega_x} \in U(1)$$

Standard procedure: find (local) maximum via over-relaxation

Problems:
1) Gribov copies:
no unique solution, existence of local maxima
(in continuum: problem for theory on torus)

Solution: search for global maximum providing the best gauge copy
Practically: perform 10 additional random gauge transformation on original gauge fields

choose gauge field with maximal gauge functional to measure the gauge field propagator

2) Zero momentum modes of gauge fields

$$\Phi_\mu = \frac{1}{V}\sum_x \theta_{x,\mu}$$

ZMM do not contribute to gauge field action
for gauge configurations representing small fluctuations around constant modes global maximum requires $\Phi_\mu \equiv 0$
latter condition can be achieved by non-periodic gauge transformation

$$\theta_{x\mu} \to \theta^c_{x\mu} = [c_\mu + \theta_{x,\mu}]_{\mathrm{mod}2\pi}\ , \qquad c \in (-\pi, \pi]$$

removing of ZMM's leads to larger maxima (!)

3) Double Dirac Lines
decompose plaquette into gauge invariant flux and discrete gauge-dependent contribution representing Dirac strings.
Set of Dirac plaquettes providing a world line of Direc strings on dual lattice is called Dirac line
DDL: two lines with opposite flux orientation wrapping around the lattice (no monopoles at the end)
less hints for DDL: larger maximum of gauge functional

Procedure of Landau gauge fixing:
1) iteratively maximize functional by overrelaxation
2) remove ZMM in each iteration
3) count for possible DDL
3) new gauge field by random gauge transformations
4) choose gauge field from all copies with largest gauge field maximum (no ZMM, minimal danger of DDL)
5) check how good the gauge fixing condition is fulfilled.

lattice: 32^3, $32^2 \times 8$, $32^2 \times 4$

Gauge boson propagator, first results

Tensor structure of gauge boson propagator $\langle\theta_\mu(\mathbf{x})\theta_\nu(\mathbf{0})\rangle$ in Landau gauge in momentum space

$$D_{\mu\nu}(p^2) = \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) D(p^2)$$

$p_\mu = 2\sin(\pi k_\mu / L_\mu)$ $(k_\mu = 0, \ldots, L_\mu/2)$ is μ^{th} component of lattice momentum

Discuss here only temporal component $D_{33}(\vec{p}\,^2, 0)$ as function of *spatial* momentum

Remind: the confining properties of static electrically charged particles at finite T are encoded in θ_3

To study the effect of monopoles:

divide gauge field θ_μ into a regular (photon) and a singular (monopole) part

In notation of lattice forms:

$$\theta = \theta^{\mathrm{phot}} + \theta^{\mathrm{mon}}, \quad \theta^{\mathrm{mon}} = 2\pi\Delta^{-1}\delta p[j]$$

0-form ${}^*j \in \mathbb{Z}$ is nonvanishing on sites of the dual lattice occupied by monopoles

1-form ${}^*p[j]$ corresponds to the Dirac strings (living on the links of the dual lattice) which connect monopoles with anti–monopoles: $\delta^* p[j] = {}^*j$

For a Monte Carlo configuration:
1) fix the gauge
2) locate the Dirac strings, $p[j] \neq 0$
3) construct θ^{mon}
4) $\theta^{\text{phot}} = \theta - \theta^{\text{mon}}$

Photon and monopole parts contribute to the propagator

$$D = D^{phot} + D^{mon} + D^{mix}$$

Regular part: free field form $D^{\text{phot}} = C^{\text{phot}}/(\beta\ \vec{p}^{\ 2})$
D^{mon} maximum in the infrared region
Mixed component D^{mix} gives a negative contribution which is growing with decreasing momentum

All contributions together do *not* give a simple massive Yukawa propagator

Preliminary fit to quantify the difference ($32^2 \times 8$)

$$D_{33}^{\text{fit}}(\vec{p}^{\ 2}, 0) = \frac{C_1}{\beta} \frac{1}{\vec{p}^{\ 2} + m^2} + \frac{C_2}{\beta} \frac{m^2}{(\vec{p}^{\ 2} + m^2)^2}$$

First part: "perturbative" propagator: photon acquires some Debye mass m_D due to screening (Polyakov)
Second part: "non–perturbative": can be interpreted as two propagators meeting at a pointlike monopole vertex
at large momenta the non–perturbative part vanishes quickly

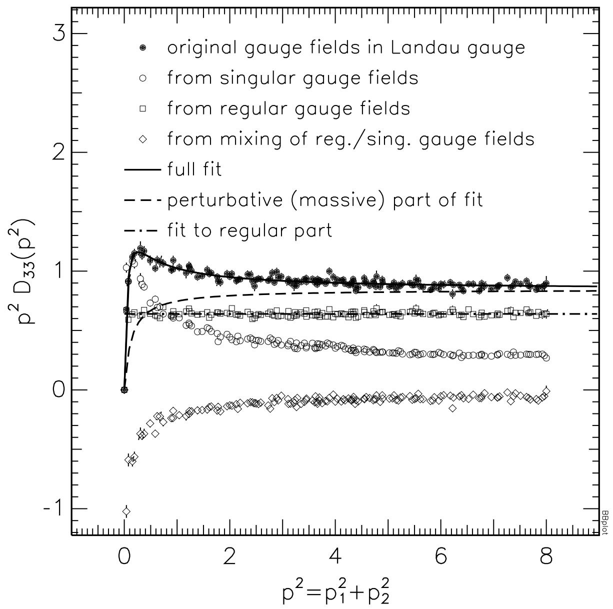

Figure 2: Different contributions to the full D_{33} propagator for $\beta = 2.0$. Solid line: fit of the $\vec{p}^{\,2}$ dependence of the propagator as described in the text; dashed line: the perturbative part of the fit; dash-dotted line: fit of the measured photon part.

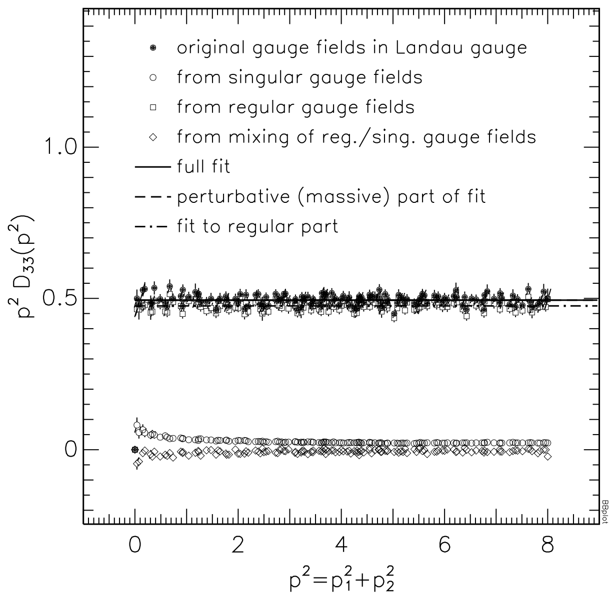

Figure 3: Same in deconfinement phase, $\beta = 2.5$

Conclusions

Data show non-perturbative *enhancement* of gauge boson propagator in the infrared region and to a depletion at $p^2 \approx 0$

The non-perturbative part of the propagator vanishes at the deconfinement transition caused by pairing of monopoles into dipoles

At present: we work on improved fits and interpretation of the results at zero and finite T, consider also spatial components of the propagator

Possible work in future:
Influence of external fields in $cQED_3$
Relation to Josephson junctions and high T_c superconductors
Role of monopoles on gauge boson propagator in SU(2) and external fields
Monopoles and gluon propagator in full QCD including dynamical fermions