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Photon propagator, monopoles and the thermal phase transition in 3D compact QED

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- Introduction and Model
- Decomposition of lattice photon propagator
- Landau and Laplacian gauge
- Some results for $T > 0$ photon propagator
- Conclusions

Recent work:

- PR D64,054507, PR D64,114502, LAT01
- heplat 0112048 (PRL to appear), in preparation

Introduction

- in 3D cQED confinement is proven and understood (Polyakov, Göpfert and Mack)
  Confinement is caused by a plasma of monopoles which emerge due to the compactness of the gauge field
- We recently studied how mechanism works under special conditions: T, external fields
  Localisation of phase transition using gauge boson degrees of freedom or monopoles:

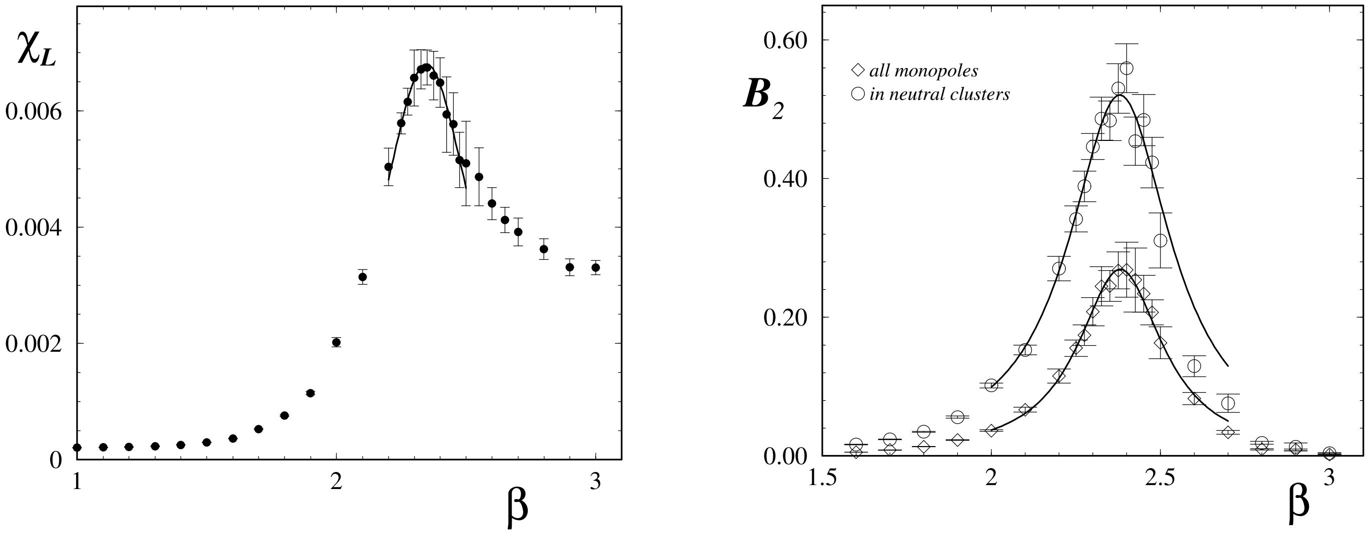

Figure 1: The expectation value of Polyakov loop susceptibility (l) and Binder second cumulant of monopole density (r) *vs.* β.

- Found: confining plasma of monopoles and anti–monopoles turns into dipole plasma at deconfinement phase transition

- Analogously in QCD_4: Studying the dynamics of the monopole current inside gluodynamics, monopole de-condensation at T_c appears as *de-percolation*, *i.e.* the decay of the infrared, percolating monopole cluster into short monopole loops
- Here we focus on possible similarities between the gauge boson propagators of both theories
- The infrared physics in QCD is governed by non–perturbative effects which should become manifest in the low p^2 behaviour of the propagator
  One school of thought predicted $1/(p^2)^2$
  A more careful, but rather general analysis: absence of a double pole in confining theories
  Gribov: gluon propagator should vanish at low momentum
  Enhancement at intermediate momenta in Landau gauge (anomalous dimension)
- We demonstrate that the p^2 dependence of the photon propagator in $cQED_3$ is also described by a Debye mass and by an anomalous dimension which both vanish at the deconfinement transition
  This mechanism can be clearly attributed to magnetic monopoles

Model

$$S[\theta] = -\beta \sum_p \cos\theta_p\,, \qquad \beta = \frac{1}{a\, g_3^2}\,, \qquad \frac{T}{g_3^2} = \frac{\beta}{L_t}$$

Decomposition of lattice photon propagator

We study the gauge boson propagator in lattice momentum space

The propagator is defined in terms of the Fourier transformed θ_l or $\sin\theta_l$

$$\left\{ \begin{array}{c} \widetilde{\theta}_{\vec{k},\mu} \\ \widetilde{\sin\theta}_{\vec{k},\mu} \end{array} \right\} = \frac{\mathrm{e}^{i\pi k_\mu \hat{\mu}}}{\sqrt{\Lambda}} \sum_{\vec{x}} \mathrm{e}^{2\pi i \sum_\rho k_\rho x_\rho / L_\rho} \left\{ \begin{array}{c} \theta_{\vec{x},\mu} \\ \sin\theta_{\vec{x},\mu} \end{array} \right\}$$

by

$$D_{\mu\nu}(p_1,\ldots,p_d) = \left\{ \begin{array}{c} \langle \tilde{\theta}_{\vec{k},\mu} \tilde{\theta}_{-\vec{k},\nu} \rangle \\ \langle \widetilde{\sin\theta}_{\vec{k},\mu} \widetilde{\sin\theta}_{-\vec{k},\nu} \rangle \end{array} \right\}$$

Relation between lattice and Fourier momenta

$$p_\mu = \frac{2}{a} \sin \frac{\pi k_\mu}{L_\mu}, \qquad k_\mu = 0, \pm 1, \ldots, \pm \frac{L_\mu}{2}, \qquad p^2 = p_\mu p_\mu$$

Decomposition (Euclidean space, summation over indices understood)
Case $T = 0$:

$$D_{\mu\nu}(p_1,\ldots,p_d) = P_{\mu\nu} D(p^2) + \frac{p_\mu p_\nu}{p^2} \frac{F(p^2)}{p^2}$$

d-dimensional transverse projection operator

$$P_{\mu\nu} = \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}$$

In a covariant gauge specified by F gauge invariance requires

$$p_\mu D_{\mu\nu} p_\nu = F$$

The scalar function $D(p^2)$ is obtained from

$$P_{\mu\nu} D_{\mu\nu} = (d-1) D$$

In practice we measure (imag. parts of non-diagonal $D_{\mu\nu}$ vanish after projections)

$$p^2\, D = \frac{1}{d-1} \left[p^2 D_{\mu\mu} - F \right]$$

Case $T > 0$ (asymmetric lattice, the d-th dimension defines T):
Consider two scalar functions:
D_T for $(d-1)$-dimensional transverse (P^T) operator
D_L for $(d-1)$-dimensional longitudinal (P^L) operator

$$D_{\mu\nu} = P^T_{\mu\nu} D_T(|\mathbf{p}|, p_d) + P^L_{\mu\nu} D_L(|\mathbf{p}|, p_d) + \frac{p_\mu p_\nu}{p^2} \frac{F}{p^2}$$

The projection operators (transverse in d dimensions) are given as follows $(i,j = 1, \ldots, d-1)$:

$$P^T_{ij} = \delta_{ij} - \frac{p_i p_j}{\mathbf{p}^2}\,, \qquad P^T_{dd} = P^T_{di} = P^T_{id} = 0\,, \qquad \mathbf{p}^2 = p_i p_i$$

$$P^L_{\mu\nu} = P_{\mu\nu} - P^T_{\mu\nu}$$

Scalar functions D_T and D_L are extracted from $D_{\mu\nu}$

$$P^T_{\mu\nu} D_{\mu\nu} = (d-2) D_T\,, \quad P^L_{\mu\nu} D_{\mu\nu} = D_L$$

We use the forms:

$$\mathbf{p}^2 D_T = \frac{1}{d-2}\left(\mathbf{p}^2 D_{ii} - F_s\right)$$

$$(\mathbf{p}^2 + p_d^2)\mathbf{p}^2 D_L = \mathbf{p}^2\left((\mathbf{p}^2 + p_d^2) D_{dd} - F_t\right) + p_d^2 F_s$$

with

$$F_s = p_i D_{ij} p_j\,, \quad F_t = F - F_s = p_i p_d (D_{id} + D_{di}) + p_d^2 D_{dd}$$

$$D_{\mu\mu} = D_{ii} + D_{dd}$$

The $T = 0$ case corresponds to

$$D = \frac{1}{d-1}\left(D_L + (d-2) D_T\right)\,, \quad F = F_s + F_t$$

$$p_d = 0: \quad F_t = 0\,, \quad F = F_s\,, \quad D_L = D_{dd}$$

$p_d = 0$ corresponds to the high temperature regime

Study effect of monopoles:

Divide gauge field θ_l into a regular (photon) and a singular (monopole) part
In notation of lattice forms:

$$\theta = \theta^{\mathrm{phot}} + \theta^{\mathrm{mon}}, \quad \theta^{\mathrm{mon}} = 2\pi\Delta^{-1}\delta p[j]$$

0-form ${}^*j \in \mathbb{Z}$ is nonvanishing on sites of the dual lattice occupied by monopoles
1-form ${}^*p[j]$ corresponds to the Dirac strings (living on links of the dual lattice) which connect monopoles with anti–monopoles: $\delta^* p[j] = {}^*j$

For a Monte Carlo configuration $\{\theta_l\}$:
1) fix the gauge
2) locate the Dirac strings, $p[j] \neq 0$
3) construct θ^{mon}
4) $\theta^{\mathrm{phot}} = \theta - \theta^{\mathrm{mon}}$

Photon and monopole parts contribute to the propagator

$$D = D^{\mathrm{phot}} + D^{\mathrm{mon}} + D^{\mathrm{mix}}$$

$$F = F^{\mathrm{phot}} + F^{\mathrm{mon}} + F^{\mathrm{mix}}$$

at $T > 0$

$$D_{L/T} = D_{L/T}^{\mathrm{phot}} + D_{L/T}^{\mathrm{mon}} + D_{L/T}^{\mathrm{mix}}$$

Landau and Laplacian gauge

A gauge transformation in U(1) is specified by angles $g_{\vec{x}}$ such that $\{\theta_l\} \to \{\theta_l^g\}$ is given for all links $l = \{\vec{x}, \mu\}$ by

$$\theta^g_{\vec{x},\mu} = -g_{\vec{x}} + \theta_{\vec{x},\mu} + g_{\vec{x}+\hat{\mu}} + 2\pi k \quad \theta^g_{\vec{x},\mu} \in (-\pi, \pi]$$

Landau gauge fixing: maximize the functional

$$\sum_l \cos \theta^g_l$$

starting from a MC configuration $\{\theta_l\}$ over all gauge transformations g

Our procedure: iterative mixture of overrelaxation and non-periodic gauge transformations, both applied in alternating order

Stop criterion: change of gauge functional/per link $< 10^{-6}$

Overrelaxation done in a checkerboard fashion:

find for even and odd sublattice a suitable set of $g_{\vec{x}}$ ($\vec{x} \in \Lambda_{\text{even}}$ or Λ_{odd}) to minimize the gauge functional

Angle $g_{\vec{x}}$ is easily found:

$$\tan g_{\vec{x}} = \frac{\sum_\mu \left(\sin \theta_{\vec{x},\mu} - \sin \theta_{\vec{x}-\hat{\mu},\mu}\right)}{\sum_\mu \left(\cos \theta_{\vec{x},\mu} + \cos \theta_{\vec{x}-\hat{\mu},\mu}\right)}$$

$g_{\vec{x}} \to \omega g_{\vec{x}}$, bound new $|g_{\vec{x}}| < \pi$
perform the gauge transformation on all links
Single overrelaxation step: one even/odd pair of gauge updates (we use $\omega = 1.8$)

At a (local) extremum of the gauge functional the following condition is satisfied everywhere

$$\partial_\mu \theta_{\vec{x},\mu} = \sum_\mu \left(\sin\theta_{\vec{x},\mu} - \sin\theta_{\vec{x}-\hat{\mu},\mu} \right) = 0$$

Consequence: $F(p^2) = 0$ for total propagator with $\sin\theta_l$ definition
Non-periodic gauge transformations are used to immediately eliminate zero modes of the gauge field:
Subtract the volume average $\Theta_\mu = \sum_{\vec{x}} \theta_{\vec{x},\mu}/\Lambda$ from each link θ_l in the given direction
The subtraction

$$\theta_{\vec{x},\mu} \to \theta^g_{\vec{x},\mu} = \theta_{\vec{x},\mu} - \Theta_\mu + 2\pi k \qquad \theta^g_{\vec{x},\mu} \in (-\pi, \pi]$$

results from a gauge function $g_{\vec{x}} = -\Theta_\mu x_\mu$ (no summation!) which is in general non-periodic
This algorithm generically will not lead to the absolute maximum of the gauge functional – Gribov problem
Problem partially cured by repeating the same gauge fixing applying it to random gauge copies of the original MC configuration $\{\theta_l\}$

Example for dependence on number of Gribov copies: in most of the runs we use $N_G = 11$ and 21

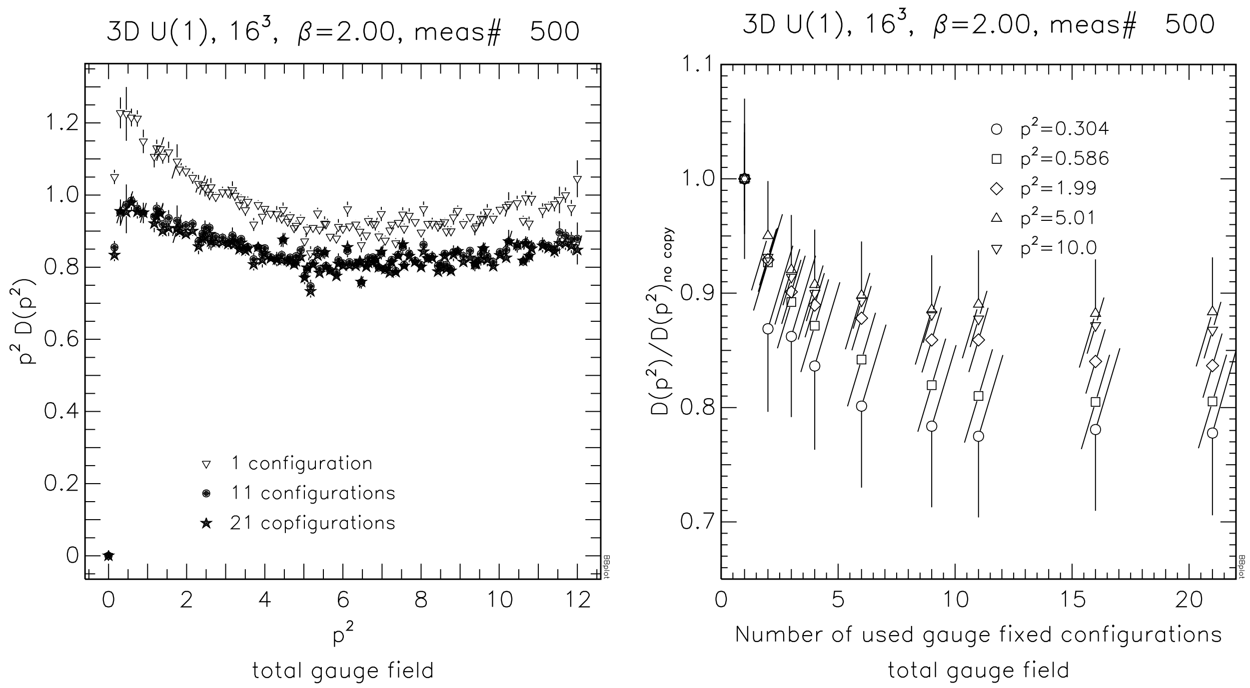

Figure 2: Study of Gribov copy dependence

Double Dirac Lines

decompose plaquette into gauge invariant flux and discrete gauge-dependent contribution representing Dirac strings.

Set of Dirac plaquettes providing a world line of Dirac strings on dual lattice is called Dirac line

DDL: two lines with opposite flux orientation wrapping around the lattice (no monopoles at the end)

Observation: for larger maximum of gauge functional less hints for DDL's

Laplacian gauge: alternative free of the Gribov problem

In that case $F(p^2)$ is an important part of the propagator

The Laplacian gauge closest to the Landau gauge is defined by a complex–valued scalar field ("Higgs" field) which forms the lowest eigenmode $\phi^{(0)}_{\vec{x}}$ of minus the lattice Laplacian

$$\sum_{\vec{y}} \left(-\Delta_{\vec{x}\vec{y}}\right) \phi^{(0)}_{\vec{y}} = \lambda_0 \, \phi^{(0)}_{\vec{x}}$$

$$\Delta_{\vec{x}\vec{y}} = \sum_{\mu=1}^{3} \left(\delta_{\vec{x}+\hat{\mu},\vec{y}} \, \mathrm{e}^{i\,\theta_{\vec{x},\mu}} + \delta_{\vec{x}-\hat{\mu},\vec{y}} \, \mathrm{e}^{-i\,\theta_{\vec{x}-\hat{\mu},\mu}} - 2 \right)$$

The gauge transformation $g_{\vec{x}}$ is defined such as to transform $\phi^{(0)}_{\vec{x}}$ into a real-valued vector everywhere

$$-g_{\vec{x}} + \arg(\phi^{(0)}_{\vec{x}}) = 0$$

Procedure well-defined up to a globally uniform phase factor (no effect on the link angles $\{\theta_l\}$)

Examples of differences:

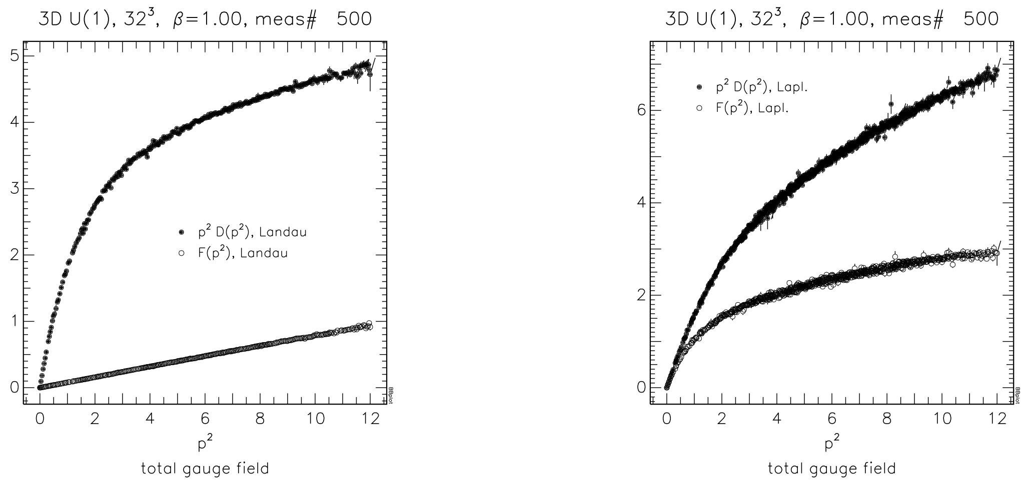

Figure 3: Landau and Laplacian gauges defining $D_{\mu\nu}$ via θ_l

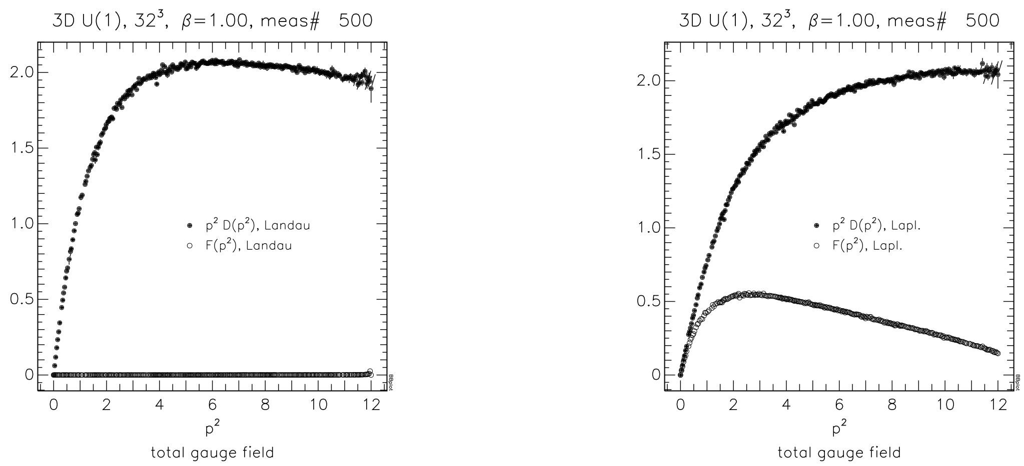

Figure 4: Landau and Laplacian gauges defining $D_{\mu\nu}$ via $\sin\theta_l$

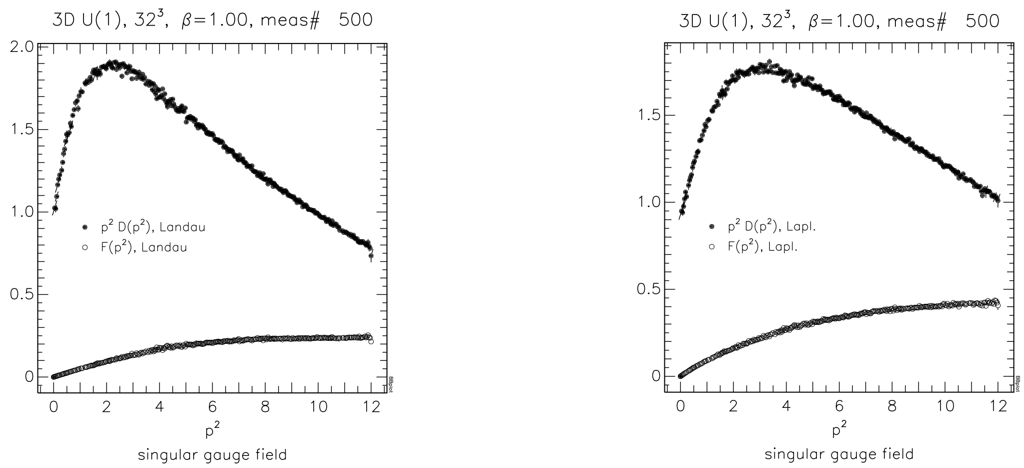

Figure 5: Monopole contribution (using the $\sin\theta_l$ definition)

Some results for $T > 0$ photon propagator

We discuss here $T > 0$ for $32^2 \times 8$ and focus on the temporal component of the propagator

$$D_{33}(\mathbf{p}^2, 0) = D_L(\mathbf{p}^2, 0)$$

$$D_{33} = D_{33}^{\text{phot}} + D_{33}^{\text{mon}} + D_{33}^{\text{mix}}$$

Remind: the confining properties of static electrically charged particles at finite T are encoded in θ_3

The regular part of the propagator has perfectly the free field form

$$D_{33}^{\text{phot}} = \frac{1}{\beta} \frac{Z^{\text{phot}}}{\mathbf{p}^2}$$

at all available β.

D^{mon} maximum in the infrared region
Mixed component D^{mix} gives a negative contribution which is growing with decreasing momentum

All contributions together do *not* give a simple massive Yukawa propagator

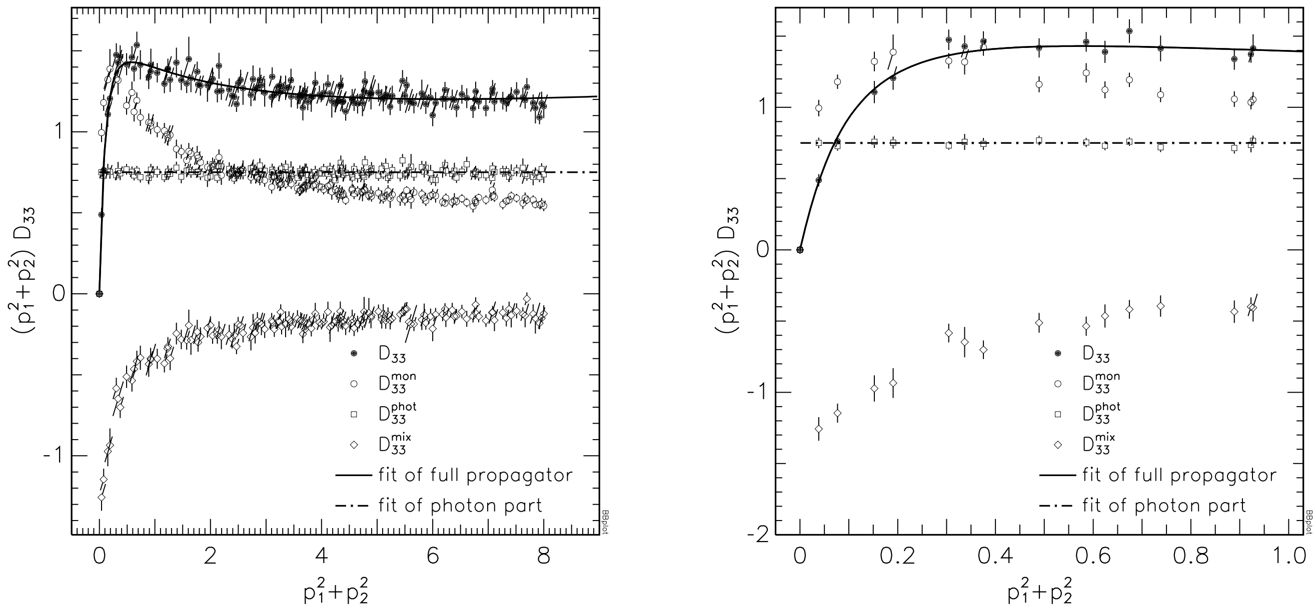

Figure 6: Different contributions to the full D_{33} propagator (multiplied by $\mathbf{p}^2$) vs $\mathbf{p}^2$ and fits for $\beta = 1.8$ on a $32^2 \times 8$ lattice.

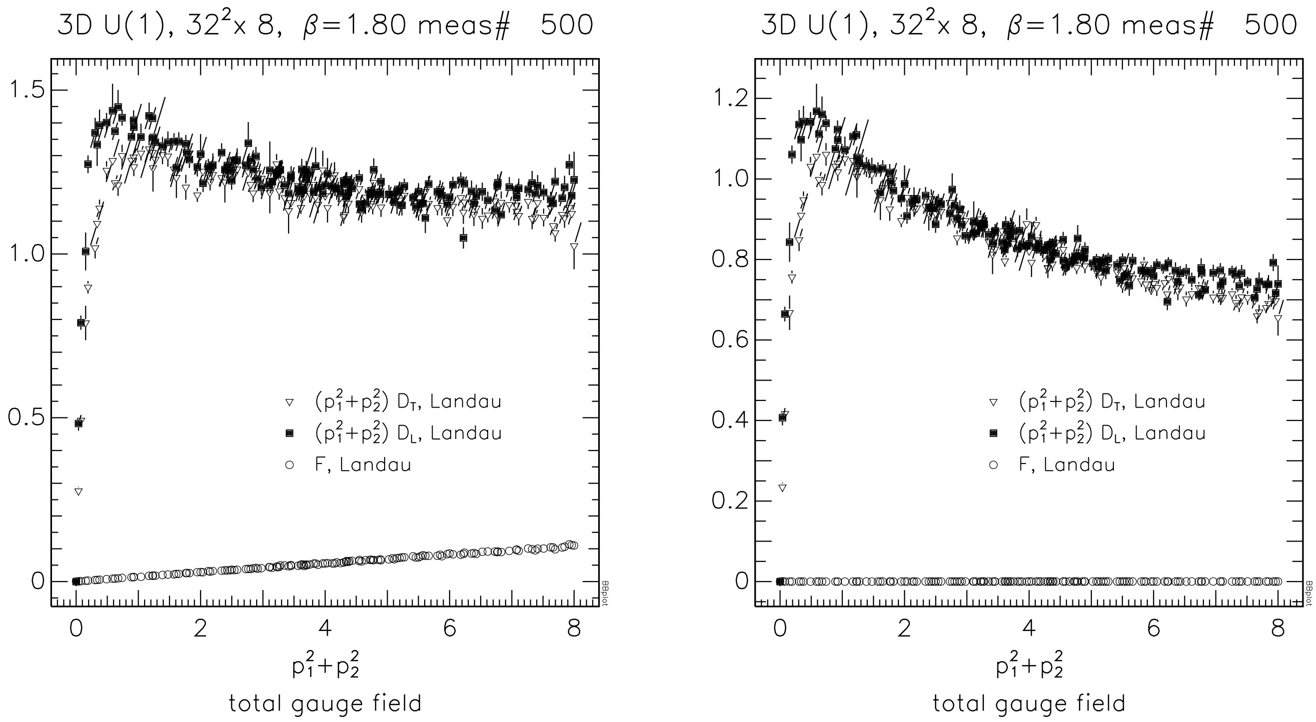

Figure 7: Use of the two different definitions, (l) θ_l, (r) $\sin\theta_l$

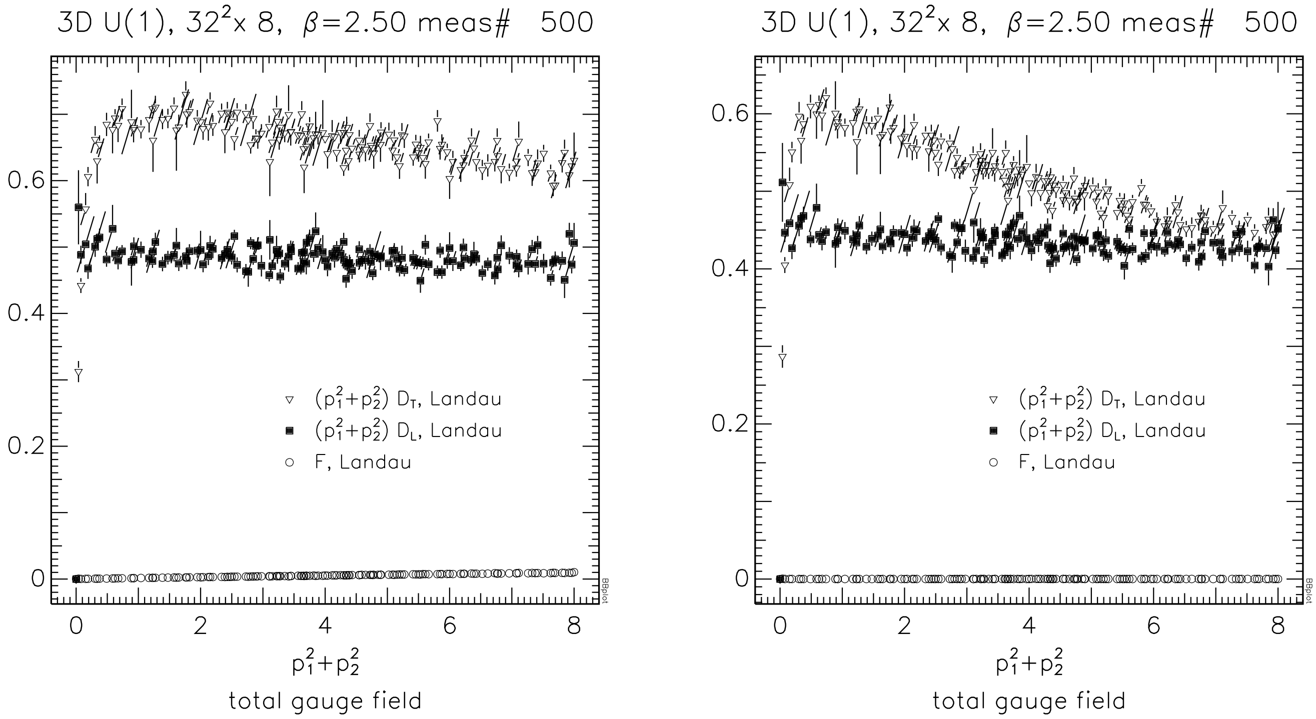

Figure 8: In deconfinement phase: (l) θ_l, (r) $\sin\theta_l$

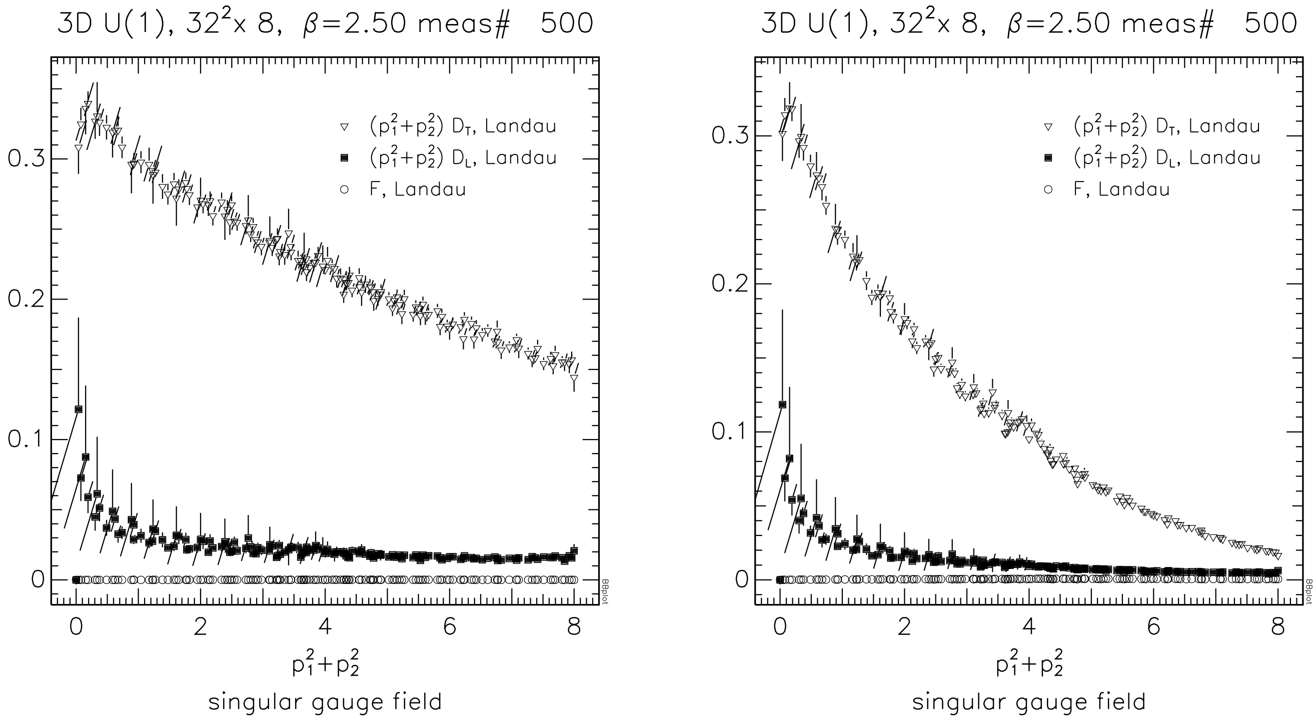

Figure 9: In deconfinement phase, "Monopole" contributions

To quantify the difference to a Yukawa–type behaviour we use the four–parameter model function

$$D_{33}(\mathbf{p}^2, 0) = \frac{Z}{\beta} \frac{m^{2\alpha}}{\mathbf{p}^{2(1+\alpha)} + m^{2(1+\alpha)}} + C \, ,$$

Z, α, m and C are fitting parameters (similar to some studies of the gluon propagator)

This ansatz implies:
Photon acquires a Debye mass m (due to screening) together with an anomalous dimension α
Z describes the renormalization of the photon wave function due to quantum corrections
C represents a δ–function–like interaction in coordinate space

Here we study the "angle" propagator

Before fitting we average the propagator over all lattice momenta at same $\mathbf{p}^2$ to improve rotational invariance

Fits: used standard Mathematica packages combined with search for global minimum in $\chi^2/d.o.f.$
Stability check: study several possibilities of averaging and thinning out the data sets

Outcome: The model function works perfectly for all $\mathbf{p}^2$ and couplings β

Fits:

We set $m = 0$ and $\alpha = 0$ for $\beta \geq 2.37$ to improve the quality of the fit of Z and C

Inclusion of C is crucial for obtaining good fits in the confinement phase for "angle" propagator, despite the fact that it is very small

Z and Z^{phot}:

$Z \approx Z^{\text{phot}}$ in the deconfined phase while it is much larger in the confined phase: The photon wave function gets strongly renormalized by the monopole plasma

In contrast, Z^{phot} smoothly changes crossing transition at $\beta_c \approx 2.35$.

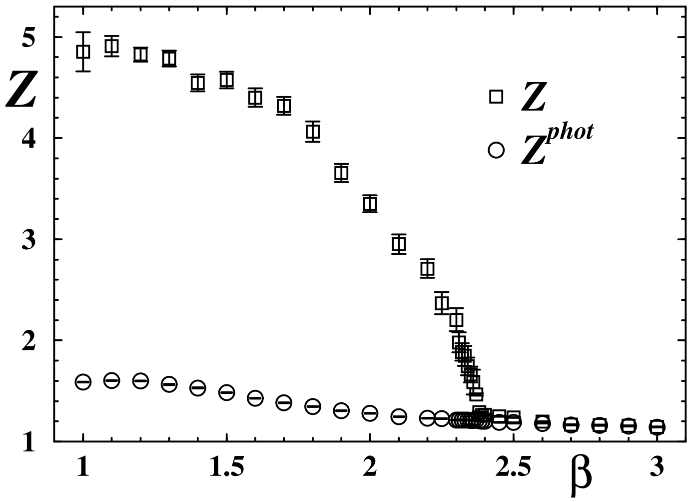

Figure 10: Coefficients Z of fit for full propagator and Z^{phot} for photon contribution vs β,

α: In the confinement phase the monopole plasma causes the anomalous dimension growing to $\alpha \approx 0.25$

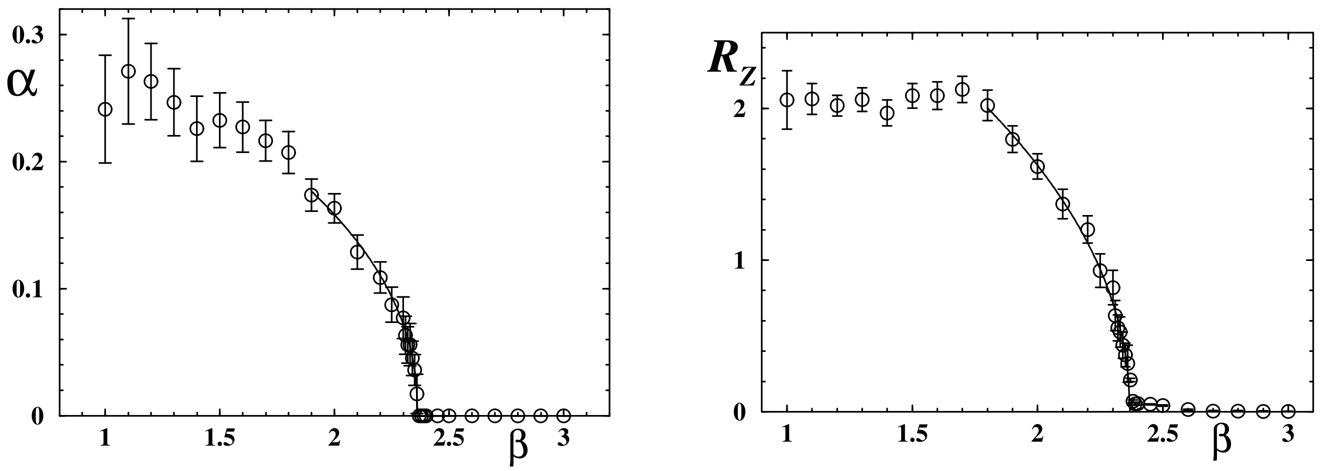

Figure 11: Best fits for α and R_Z near β_c.

To characterize the properties of Z and α approaching the phase transition we fit

$$R_Z(\beta) = \frac{Z(\beta)}{Z^{\mathrm{phot}}(\beta)} - 1\,,$$

and the anomalous dimension α in the following form:

$$f_i(\beta) = h_i\,(\beta_c^{(i)} - \beta)^{\gamma_i}\,, \quad \beta < \beta_c^{(i)}\,, \quad (i = \alpha, Z)$$

where $i = Z, \alpha$. The $\beta_c^{(\alpha,Z)}$ are the pseudo–critical couplings

Best fits f_α and f_Z are shown in the Figures

$\beta_c^{(\alpha)}$ and $\beta_c^{(Z)}$ are in agreement with previous numerical studies giving $\beta_c = 2.346(2)$.

i	$\beta_c^{(i)}$	γ_i
α	2.363(3)	0.50(2)
Z	2.368(5)	0.48(3)

Table 1: Best parameters for the fits.

Debye mass m:

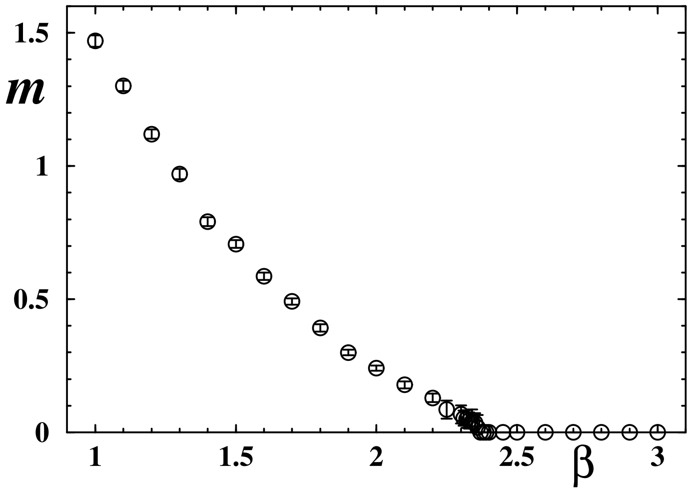

Figure 12: The mass m vs β.

The mass scale generated is non–vanishing in the confinement phase due to presence of the monopole plasma

It vanishes at the deconfinement transition point when the very dilute remaining monopoles and anti–monopoles form dipoles.

Conclusions

The presence of the monopole plasma leads to the appearance of an $\alpha > 0$ in the boson propagator of $cQED_3$ in the confinement phase
$\rightarrow$ stimulate an analytical explanation?

We conjecture that in the case of QCD the Abelian monopoles defined within the Abelian projection may be responsible for the anomalous dimension of the gluon propagator observed (Martinelli group)

We found that α and the ratio of the photon wave function renormalization constants R_Z represent alternative, also non–local order parameters characterizing the confinement phase

At present: we work on fits and interpretation of the results at zero and finite T, discussing both Landau and Laplacian gauges

Possible work in future:
Influence of external fields in $cQED_3$
Role of monopoles on gauge boson propagator in SU(2) and external fields
Monopoles and gluon propagator in full QCD including dynamical fermions