One-loop renormalisation for the second moment of generalized parton distributions

Arwed Schiller

Leipzig University, Germany

QCDSF

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Overview

- Introduction
- Operators and mixing
- Second moment in lattice perturbation theory
- Examples for renormalisation factors
- Summary & Outlook

Recent work : hep-lat/0410009, in preparation

I do not cover:

Quark bilinears and first moment for overlap fermions with improved gauge actions

NPB693 (hep-lat/0404007), and in preparation

Introduction

- Generalised parton distributions (GPDs) very interesting objects
  They contain more information about the hadron structure than the usual structure functions:
  transverse structure, orbital angular momentum carried by quarks and gluons,. . .

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Introduction

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- GPDs well-defined QCD objects systematically studied in
  perturbation theory (e.g. Geyer, Müller, Robaschik,. . . ,
  Ji, Radyushkin,. . .)

- Limited experimental access: $ep \longrightarrow ep\gamma$, $ep \longrightarrow ep\pi^+\pi^-$
  first data from e.g. HERMES show some evidence

- Need complementary information from lattice QCD: Calculate non-forward matrix elements of local composite operators with twist $T = 2$

$$\mathcal{O}_{\mu_1\cdots\mu_n} = \left(\frac{i}{2}\right)^{n-1} \bar{\psi}\,\gamma_{(\mu_1} \overset{\leftrightarrow}{D}_{\mu_2} \cdots \overset{\leftrightarrow}{D}_{\mu_n)}\psi$$

$$\langle p'|\mathcal{O}_{\mu_1\cdots\mu_n}|p\rangle =$$

$$\bar{\psi}(p')\gamma_{(\mu_1}\psi(p) \sum_{i=0}^{\left[\frac{n-1}{2}\right]} A_{n,2i}(\Delta^2)\Delta_{\mu_2}\cdots\Delta_{\mu_{2i+1}}\overline{P}_{\mu_{2i+2}}\cdots\overline{P}_{\mu_n)}$$

$$-\frac{1}{2M}\bar{\psi}(p')\,i\Delta^{\alpha}\sigma_{\alpha(\mu_1}\psi(p) \sum_{i=0}^{\left[\frac{n-1}{2}\right]} B_{n,2i}(\Delta^2)\Delta_{\mu_2}\cdots\Delta_{\mu_{2i+1}}\overline{P}_{\mu_{2i+2}}\cdots\overline{P}_{\mu_n)}$$

$$+C_n(\Delta^2)\mathrm{Mod}(n+1,2)\frac{1}{M}\bar{\psi}(p')\psi(p)\Delta_{(\mu_1}\cdots\Delta_{\mu_n)}$$

[$\Delta = p - p'$, $\overline{P} = \frac{p+p'}{2}$, $(\cdots)$: index symmetrisation and trace subtraction]

Generalised form factors A, B, C related to moments of GPDs

$$\int_{-1}^{1} dx\, x^{n-1} H(x,\xi,\Delta^2) =$$

$$\sum_{i=0}^{\left[\frac{n-1}{2}\right]} A_{n,2i}(\Delta^2)(-2\xi)^{2i} + \mathrm{Mod}(n+1,2) C_n(\Delta^2)(-2\xi)^{2n}$$

$\xi = -n \cdot \Delta$, $n \cdot \bar{p} = 1$

Off-forward parton distribution H with: $H(x,0,0) = q(x)$

First lattice results 03: QCDSF and LHPC

recent results of QCDSF, see talk of Göckeler

- Important: relate lattice results to continuum:

  $\longrightarrow$ need renormalisation factors

- Non-perturbative determination of Z-factors preferable

  But

  - Computationally rather complicated (concerning clear signals)
  - Some aspects (explicit dependence on a, mixing, ...) can be naturally studied in lattice perturbation theory
  - Complications:

    H(4) less stringent than O(4): possibilities for mixing increase

    Mixing with operators containing external ordinary derivatives starts from $n = 3$ (2nd moment)

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- **This talk:** Perturbative Z-factors for the 2nd moment of GPDs for Wilson and clover fermions

Operators and mixing

Well known that operators of second and higher moments mix:

One-loop result for a matrix element of a certain operator contains structures which differ from its Born structure

$\longrightarrow$ No multiplicative renormalisation of operators

Set of possible operators determined by the transformation properties under H(4) and charge conjugation

Only operators can mix which belong to the same representation and have identical charge conjugation parity (Göckeler et al., 1996)

Determine the mixing matrix of renormalisation factors

Consider dimensionally regularised vertex function of operator $\mathcal{O}_j$, $j = 1, \ldots, N$, $\bar{g}_R^2 = g_R^2 C_F/(16\pi^2)$

Renormalised vertex in one-loop in $\overline{MS}$ scheme

$$\Gamma_j^R(p', p, \mu, g_R) = \Gamma_j^{\mathrm{Born}}(p', p) + \bar{g}_R^2 \left[\sum_{k=1}^{N} (-\gamma_{jk}^V) \ln \frac{(p'+p)^2}{4\mu^2} \Gamma_k^{\mathrm{Born}}(p', p) + f_j(p', p) \right]$$

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Regularised vertex on the lattice (without possible $1/a^k$)

$$\Gamma_j^L(p', p, a, g_R) = \Gamma_j^{\text{Born}}(p', p) + \bar{g}_R^2 \left[\sum_{k=1}^{N} (-\gamma_{jk}^V) \ln \frac{a^2(p'+p)^2}{4} \Gamma_k^{\text{Born}}(p', p) + f_j^L(p', p) \right]$$

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General form of connection between Γ^L and Γ^R

$$\Gamma_j^R(p', p, \mu, g_R) \;\; = \;\; Z_\psi \sum_{k=1}^{N} Z_{jk} \Gamma_k^L(p', p, a, g_R)$$

Z_ψ relating lattice to $\overline{MS}$ is known: $Z_\psi = 1 + \bar{g}_R^2 \left(\ln(a^2\mu^2) + f_\psi \right)$

Result for the mixing matrix

$$Z_{jk} = \delta_{jk} - \bar{g}_R^2 \left[\left(-\gamma_{jk}^V + \delta_{jk} \right) \ln(a^2\mu^2) + c_{jk}^V + f_\psi \, \delta_{jk} \right]$$

The p and p' independent constants c_{jk}^V have to fulfil

$$f_j^L(p',p) - f_j(p',p) \;\; = \;\; \sum_{k=1}^{N} c_{jk}^V \Gamma_k^{\mathrm{Born}}(p',p)$$

If the last equation cannot be satisfied, mixing operators have been overlooked

Second moment in lattice perturbation theory

We consider non-forward matrix elements between off-shell quark states of the following operators:

$$\mathcal{O}^{DD}_{\mu\nu\omega} = -\frac{1}{4}\overline{\psi}\gamma_\mu \overleftrightarrow{D}_\nu \overleftrightarrow{D}_\omega \psi \qquad (1)$$

$$\mathcal{O}^{\partial\partial}_{\mu\nu\omega} = -\frac{1}{4}\partial_\nu\partial_\omega \left(\overline{\psi}\gamma_\mu\psi\right) \qquad (2)$$

$$\mathcal{O}^{\partial D}_{\mu\nu\omega} = -\frac{1}{4}\partial_\nu \left(\overline{\psi}\gamma_\mu \overleftrightarrow{D}_\omega\psi\right) \qquad (3)$$

$$\mathcal{O}^{\partial}_{\mu\nu\omega} = -\frac{i}{2}\partial_\nu \left(\overline{\psi}\left[\gamma_\mu,\gamma_\omega\right]\psi\right) \qquad (4)$$

In addition spindependent operators: $\gamma_\mu \rightarrow \gamma_\mu\gamma_5$

and transversity operators with two derivatives: $\gamma_\mu \rightarrow \sigma_{\mu\tau}$

Mixing problem for form factors studied by Shifman and Vysotsky (1981) (see also Lepage, Brodsky and Efremov, Radyushkin)

They derived mixing matrices for anomalous dimensions only between operators (1) $\leftrightarrow$ (2)

Operators (3,4) are special for lattice GPD (trafo under H(4))

How to define lattice operators with derivatives $\overleftrightarrow{D}$ at $q \neq 0$?

Apply lattice momentum transfer q to lattice position x or to the "position centre"

$$\left(\bar{\psi} \overleftrightarrow{D}_\mu \psi\right)(q) = \sum_x \times$$

$$\frac{1}{2a}\left[\bar{\psi}(x)U_{x,\mu}\psi(x+a\widehat{\mu}) - \bar{\psi}(x+a\widehat{\mu})U^\dagger_{x,\mu}\psi(x)\right] \begin{Bmatrix} \mathrm{e}^{iq\cdot x} + \mathrm{e}^{iq\cdot(x+a\widehat{\mu})} \\ 2\mathrm{e}^{iq\cdot(x+a\widehat{\mu}/2)} \end{Bmatrix}$$

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Feynman rules for operators

$O(g^0)$:

$$\mathcal{O}^{DD}_{\mu\nu\omega}(p',p) = \bar{\psi}(p')\gamma_\mu\psi(p)\,\frac{1}{a}\sin\frac{a(p+p')_\nu}{2}\,\frac{1}{a}\sin\frac{a(p+p')_\omega}{2}\left\{\begin{matrix}\cos\frac{a(p-p')_\nu}{2}\,\cos\frac{a(p-p')_\omega}{2} \\ 1\end{matrix}\right\}$$

$O(g^1)$:

$$\mathcal{O}^{DD}_{\mu\nu\omega}(p',p,k) = g\sum_\sigma \bar{\psi}(p')\gamma_\mu A_\sigma(k_1)\psi(p)\,\cos\frac{a(p+p')_\sigma}{2}\,\times$$

$$\frac{1}{a}\left[\delta_{\nu\sigma}\sin\frac{a(p+p'-k)_\omega}{2} + \delta_{\omega\sigma}\sin\frac{a(p+p'+k)_\nu}{2}\right]\left\{\begin{matrix}\cos\frac{a(p-p'+k)_\nu}{2}\,\cos\frac{a(p-p'+k)_\omega}{2} \\ 1\end{matrix}\right\}$$

...

Clover fermions:

reduce cut-off effects in the fermion action from $O(a)$ to $O(a^2)$ choosing coefficient c_{sw} properly (Jansen, Sommer)

Action:

$$S_F = S_F^{\text{Wilson}} + c_{sw}\, iga^4 \sum_{x,\mu\nu=\pm} \frac{r}{4a} \bar{\psi}(x)\sigma_{\mu\nu} F_{\mu\nu}^{\text{clover}}(x)\psi(x)$$

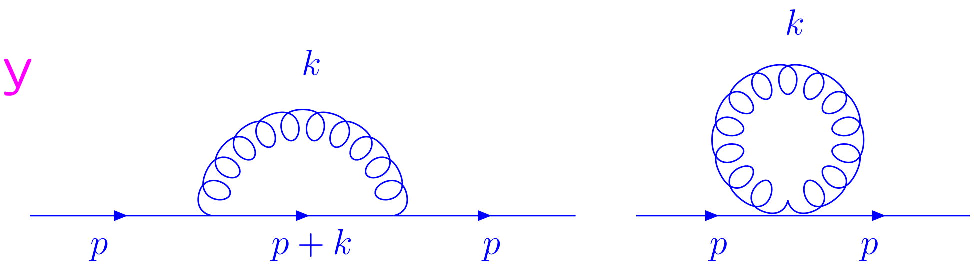
$F_{\mu\nu}$ standard "clover-leaf" form of lattice field strength

Clover contributions result in additional qqg vertex contributions vanishing in the continuum limit

Contributions to matrix elements: $O(c_{sw}^0)$, $O(c_{sw}^1)$ and $O(c_{sw}^2)$

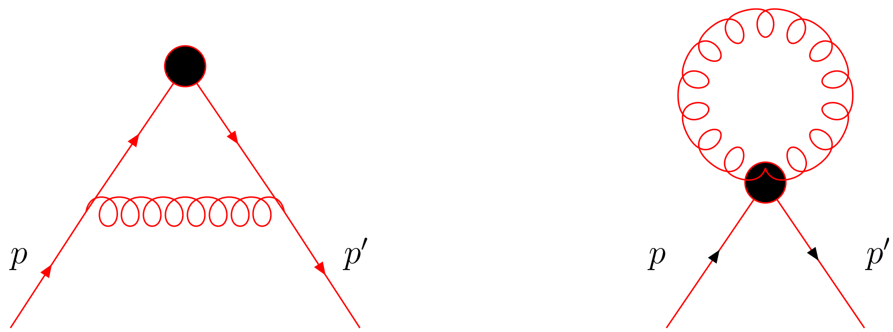
One-loop diagrams

Quark self energy

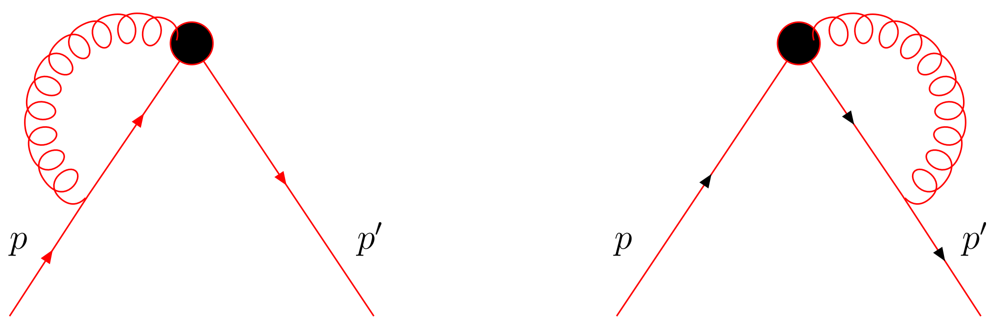


One-loop diagrams contributing to the quark self-energy

Vertex function



One-loop vertex and tadpole diagrams



One-loop cockscomb diagrams

Computation

Mathematica package extended developed for forward matrix elements (Wilson, clover, overlap)

Evaluate a typical lattice integral of the form

$$\mathcal{I}_{\mu_1 \cdots \mu_n}(a, p', p) = \int_{-\pi/a}^{\pi/a} \frac{d^4k}{(2\pi)^4} \, \mathcal{K}_{\mu_1 \cdots \mu_n}(a, p', p, k)$$

Numerator of $\mathcal{K}$: polynomial in sines and cosines of lattice momenta k

Denominator of $\mathcal{K}$ contains the denominators of lattice quark and gluon propagators

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- Calculation is performed following Kawai, Nakayama, Seo (1981) (dimensional regularisation, two external momenta): expand d-dimensional lattice integrals in external momenta and perform a "continuum" calculation in dim. regularisation such that the $1/\epsilon$-poles have to cancel

$\mathcal{I}$ is calculated by rearranging it into two parts

$$\mathcal{I} = \tilde{\mathcal{I}} + (\mathcal{I} - \tilde{\mathcal{I}})$$

$\tilde{\mathcal{I}}$: Taylor expansion of original I in p and p'

$$\begin{aligned}\tilde{\mathcal{I}}(a,p',p) &= \mathcal{I}(a,0,0) + \\ &+ \sum_\alpha \left\{ p'_\alpha \frac{\partial \mathcal{I}(a,p',p)}{\partial p'_\alpha}\Big|_{p'=p=0} + p_\alpha \frac{\partial \mathcal{I}(a,p',p)}{\partial p_\alpha}\Big|_{p'=p=0} \right\} + \ldots\end{aligned}$$

Order of expansion given by the degree of UV divergence of $\mathcal{I}$

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Order of expansion given by the degree of UV divergence of $\mathcal{I}$

$\longrightarrow$ Difference $\mathcal{I} - \tilde{\mathcal{I}}$ UV finite

calculated in the (Euclidean) continuum ($a \to 0$)

Original UV poles appear now as IR poles in the Taylor expansion

and are regularised using dimensional regularisation with $d > 4$

For IR regularization

$$(\mathcal{I} - \tilde{\mathcal{I}})|_{a \to 0} \to \mathcal{I}|_{a \to 0} = \mathcal{I}^{\text{cont}}(p', p)$$

Meaning: one-loop continuum calculation in d dimensions

Some semi-analytic approaches known for $\mathcal{I}^{\text{cont}}$ (loop diagrams with three different propagators): Davydychev, Tarasov, Campbell,...

We use our own parametrisation

The first Taylor expanded part $\tilde{I}(p', p, a)$ at finite a is calculated in d dimensions as well

Poles in ϵ (with $d = 4 - 2\epsilon$) analytically cancel those of $\mathcal{I}^{\text{cont}}$

- Complete computation of diagrams in symbolic terms
  - Free Lorentz index structure $\longrightarrow$ construct all possible representations
  - Number of analytic and numeric checks:

    Analytic cancellation of pole terms

    Recover results for forward case
  - Decoupling between symbolic computation of diagrams and numeric computations of lattice integrals
  - Expensive in CPU time and memory

Examples for renormalisation factors

Define index combinations:

$$\begin{aligned}
\mathcal{O}_{\{\nu_1\nu_2\nu_3\}} &= \frac{1}{6}\big(\mathcal{O}_{\nu_1\nu_2\nu_3} + \mathcal{O}_{\nu_1\nu_3\nu_2} \\
&\quad + \mathcal{O}_{\nu_2\nu_1\nu_3} + \mathcal{O}_{\nu_2\nu_3\nu_1} + \mathcal{O}_{\nu_3\nu_1\nu_2} + \mathcal{O}_{\nu_3\nu_2\nu_1}\big) \\
\mathcal{O}_{\|\nu_1\nu_2\nu_3\|} &= \mathcal{O}_{\nu_1\nu_2\nu_3} - \mathcal{O}_{\nu_1\nu_3\nu_2} \\
&\quad + \mathcal{O}_{\nu_3\nu_1\nu_2} - \mathcal{O}_{\nu_3\nu_2\nu_1} - 2\,\mathcal{O}_{\nu_2\nu_3\nu_1} + 2\,\mathcal{O}_{\nu_2\nu_1\nu_3} \\
\mathcal{O}_{\langle\langle\nu_1\nu_2\nu_3\rangle\rangle} &= \mathcal{O}_{\nu_1\nu_2\nu_3} + \mathcal{O}_{\nu_1\nu_3\nu_2} - \mathcal{O}_{\nu_3\nu_1\nu_2} - \mathcal{O}_{\nu_3\nu_2\nu_1}
\end{aligned}$$

Present renormalisation matrix in the form:

$$Z_{jk} = \delta_{jk} - \bar{g}_R^2\left(\gamma_{jk}\ln(a^2\mu^2) + c_{jk}\right)$$

Consider the following irreducible epresentations:

$\tau_2^{(4)}$, $C = -1$

$$\mathcal{O}^{DD}_{\{124\}} \qquad \mathcal{O}^{\partial\partial}_{\{124\}}$$

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$$\gamma_{jk} = \begin{pmatrix} \frac{25}{6} & -\frac{5}{6} \\ 0 & 0 \end{pmatrix}$$

$$c_{jk} = \begin{pmatrix} -11.563 + 2.898\, c_{sw} - 0.984\, c_{sw}^2 & 0.024 - 0.255\, c_{sw} - 0.016\, c_{sw}^2 \\ 0 & 20.618 + 4.746\, c_{sw} - 0.543\, c_{sw}^2 \end{pmatrix}$$

Numbers in red agree with previously calculated forward matrix elements (action improvement only)

$\tau_1^{(8)}$, $C = -1$

$$
\begin{aligned}
\mathcal{O}_1 &= \mathcal{O}^{DD}_{\{114\}} - \frac{1}{2}\left(\mathcal{O}^{DD}_{\{224\}} + \mathcal{O}^{DD}_{\{334\}}\right) \\
\mathcal{O}_2 &= \mathcal{O}^{\partial\partial}_{\{114\}} - \frac{1}{2}\left(\mathcal{O}^{\partial\partial}_{\{224\}} + \mathcal{O}^{\partial\partial}_{\{334\}}\right) \\
\mathcal{O}_3 &= \mathcal{O}^{DD}_{\langle\langle 114\rangle\rangle} - \frac{1}{2}\left(\mathcal{O}^{DD}_{\langle\langle 224\rangle\rangle} + \mathcal{O}^{DD}_{\langle\langle 334\rangle\rangle}\right) \\
\mathcal{O}_4 &= \mathcal{O}^{\partial\partial}_{\langle\langle 114\rangle\rangle} - \frac{1}{2}\left(\mathcal{O}^{\partial\partial}_{\langle\langle 224\rangle\rangle} + \mathcal{O}^{\partial\partial}_{\langle\langle 334\rangle\rangle}\right) \\
\mathcal{O}_5 &= \mathcal{O}^{\partial D,5}_{||213||} \\
\mathcal{O}_6 &= \mathcal{O}^{\partial D,5}_{\langle\langle 213\rangle\rangle}
\end{aligned}
$$

An additional operator is zero in one-loop

$\{\mathcal{O}_1, ..., \mathcal{O}_6\}$, same dimension

$$\gamma_{jk} \quad = \begin{pmatrix} \frac{25}{6} & -\frac{5}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{7}{6} & -\frac{5}{6} & 1 & -\frac{3}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 \\ 0 & 0 & 0 & 0 & -\frac{2}{3} & \frac{2}{3} \end{pmatrix}$$

Wilson fermion case $O(c_{sw}^0)$

$$c_{jk}^{(I,II)} = \begin{pmatrix} -12.1274 & -2.7367/1.4913 & 0.3685 & 0.9934/-0.4160 & 0.0156 & 0.1498 \\ 0 & 20.6178 & 0 & 0 & 0 & 0 \\ 3.3060 & 18.1841/-8.0156 & -14.8516 & \mp 4.3023 & -0.9285 & 0.7380 \\ 0 & 0 & 0 & 20.6178 & 0 & 0 \\ 0 & 3.2644 & 0 & 0 & 0.3501 & 0.0149 \\ 0 & 3.2644 & 0 & 0 & 0.0050 & 0.3600 \end{pmatrix}$$

Extra clover contributions (preliminary, without operator improvement)

$$c_{jk}^{(I)}(c_{sw}) = \begin{pmatrix} 2.9217 & -0.6864 & -0.0328 & 0.1728 & -0.0188 & 0.0570 \\ 0 & 4.7456 & 0 & 0 & 0 & 0 \\ 0.3333 & -0.0551 & 2.1523 & 0.9696 & -1.7581 & 2.2984 \\ 0 & 0 & 0 & 4.7456 & 0 & 0 \\ 0 & -1.4411 & 0 & 0 & 1.6479 & 0.8658 \\ 0 & -1.4411 & 0 & 0 & 0.2886 & 2.2251 \end{pmatrix}$$

$$c_{jk}^{(I)}(c_{sw}^2) = \begin{pmatrix} -0.9817 & -0.1012 & -0.0291 & 0.0424 & -0.0100 & 0.0069 \\ 0 & -0.5432 & 0 & 0 & 0 & 0 \\ 0.3705 & 0.2154 & -1.7074 & 0.1159 & -0.4429 & 0.1033 \\ 0 & 0 & 0 & -0.5432 & 0 & 0 \\ 0 & 1.4157 & 0 & 0 & -1.7033 & 0.5676 \\ 0 & 1.4157 & 0 & 0 & 0.1892 & -1.3249 \end{pmatrix}$$

$\mathcal{O}_1$ - $\frac{1}{a}$ part

In one-loop $\frac{1}{a}$ contributions to the matrix element of operator $\mathcal{O}_{\mu\nu\omega}$

Group theory and charge conjugation:

construct a possible candidate from the lower dimensional operator

$$\mathcal{O}^{\partial}_{\mu\nu\omega} = -\frac{i}{2}\partial_\nu \left(\overline{\psi}\left[\gamma_\mu, \gamma_\omega\right]\psi\right)$$

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Operator in the same representation as $\mathcal{O}_1$:

$$\mathcal{O}_8 \;=\; \mathcal{O}^{\partial}_{114} - \frac{1}{2}\left(\mathcal{O}^{\partial}_{224} + \mathcal{O}^{\partial}_{334}\right)$$

Get multiplicative mixing

$$\mathcal{O}_1\Big|_{1/a-\text{part}} \;=\; \bar{g}_R^2(-0.518 + 0.0832\, c_{sw} - 0.00983\, c_{sw}^2)\frac{1}{a}\mathcal{O}_8^{\text{Born}}$$

Subtract nonperturbatively from matrix element of $\mathcal{O}_1$

difficult task in simulations

Summary & Outlook

- Found perturbative Z-factors for 2nd moments of GPDs - Wilson fermion and clover case
  Operators with γ_μ, $\gamma_\mu\gamma_5$ and transversity operators

- Mixing more complicated than for forward matrix elements

- Small mixing for $\tau_2^{(4)}$ (three different indices)
  Mixing sizeable for $\tau_1^{(8)}$ (two indices equal)
  Additional mixing with lower dimensional operator

- Results concerning mixing are valid in general
  Applicability for numerical results using clover fermions
  Add tadpole or mean field improvement implemented as renormalisation of the link matrices

Perturbative one-loop Z-factors for moments of structure functions and GPD's

	Wilson	Clover	Overlap Wilson gauge	Overlap improved gauge	Domain wall/ improved gauge
SF					
Quark bilinears	x	x	x	x	x/x
1st moment	x	x	x	x	(S. Aoki et al.)
2nd moment	x	x	x		
3rd moment	x	x	x		
GPD's					
2nd moment	x	x			
3rd moment					

Feynman rules for overlap fermions: Ishibashi, Kikukawa, Noguchi, Yamada

- Future:

  Improvement of operators in the nonforward case ?

  Calculate Z-factors for 2nd moments of GPD with overlap fermions

  Inclusion of improved gauge actions