One-loop renormalisation of quarks currents with overlap fermions and improved gauge action

A. Schiller

Leipzig University

in collaboration with

R. Horsley (Edinburgh)
H. Perlt (Regensburg, Leipzig)
P. Rakow (Liverpool)
G. Schierholz (DESY)

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Overview

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- Calculational techniques
- Results for Z's
- Tadpole improvement
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- Summary

Introduction

- Currently, two promising improvements are used in new QCD simulations:
  - Fermionic sector: overlap fermions
  - Gauge field sector: improved gauge action

- Relate lattice measurements of operators to physical quantities in continuum by renormalisation
  $\longrightarrow$ need Z-factors to convert lattice results at a into results of a continuum scheme (e.g. $\overline{MS}$) at scale μ

$$S^{\overline{MS}} = Z_\psi^{\overline{MS}}(a,\mu)\, S^{\text{lat}}\,, \quad \Lambda_{\mathcal{O}}^{\overline{MS}} = \frac{Z_{\mathcal{O}}^{\overline{MS}}(a,\mu)}{Z_\psi^{\overline{MS}}(a,\mu)}\, \Lambda_{\mathcal{O}}^{\text{lat}}$$

- Z-factors determined with perturbative and non-perturbative methods
- Some published work for perturbative Z-factors:
  - Overlap fermions and Wilson plaquette action:
    - Quark bilinears (Alexandrou et al., Capitani & Giusti)
    - First moments of DIS structure functions (Capitani)
  - Domain wall fermions and RG improved actions:
    - Quark bilinears (Aoki et al.)
- This talk:
  Combine overlap fermions with RG improved actions to obtain perturbative Z-factors of quark bilinear operators

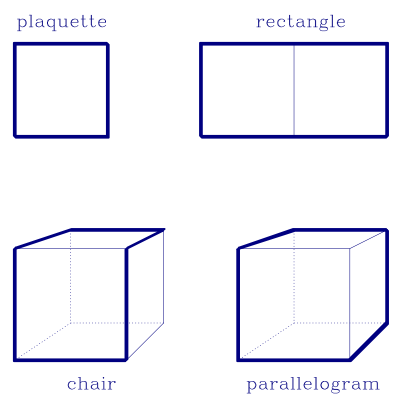
Calculational techniques

RG-improved gauge action including 6 links

$$S_{\text{gluon}} = \frac{6}{g^2}\Bigg\{c_0 \sum_{\text{plaquette}} \frac{1}{3}\mathrm{ReTr}U_{\text{pl}} + c_1 \sum_{\text{rectangle}} \frac{1}{3}\mathrm{ReTr}U_{\text{rect}}$$

$$+c_2 \sum_{\text{chair}} \frac{1}{3}\mathrm{ReTr}U_{\text{chair}} + c_3 \sum_{\text{parallelogram}} \frac{1}{3}\mathrm{ReTr}U_{\text{par}}\Bigg\}$$

with $c_0 + 8c_1 + 16c_2 + 8c_3 = 1$



Various values for parameters suggested

- Tree level Symanzik: $c_1 = -1/12$, $c_2 = c_3 = 0$
- Tadpole improved Lüscher-Weisz (TILW) ($c_2 = 0$) c_1 and c_3 functions of β
- DBW2: $c_1 = -1.4086, c_2 = c_3 = 0$ (determined non-perturbatively at $\beta = 7.986$ using RG, same for all β)
- Iwasaki $c_1 = -0.331, c_2 = c_3 = 0$ (found perturbatively using approximate RG, β independent)

Gluon propagator $D_{\mu\nu}(k)$ from inversion relation:

$$\sum_{\rho}\left(G_{\mu\rho}-\frac{\xi}{\xi-1}\widehat{k}_{\mu}\widehat{k}_{\rho}\right)\times D_{\rho\nu}(k)=\delta_{\mu\nu}$$

with

$$\begin{aligned} G_{\mu\nu}(k) \;&=\; \widehat{k}_{\mu}\widehat{k}_{\nu}+\sum_{\rho}\Big\{\left(\widehat{k}_{\rho}^{2}\delta_{\mu\nu}-\widehat{k}_{\mu}\widehat{k}_{\rho}\delta_{\rho\nu}\right)(1-\delta_{\mu\rho})\times \\ &\quad \left[1-a^{2}\left((c_{2}+c_{3})\widehat{k}^{2}+(c_{1}-c_{2}-c_{3})(\widehat{k}_{\mu}^{2}+\widehat{k}_{\rho}^{2})\right)\right]\Big\} \end{aligned}$$

$$\widehat{k}_{\mu}=\frac{2}{a}\sin\frac{ak_{\mu}}{2}\,,\quad \widehat{k}^{2}=\sum_{\mu}\widehat{k}_{\mu}^{2}$$

Exact solution only for fixed integer dimensions ($D=2,3,4$) (Weisz, 1983))

$\longrightarrow$ sufficient because improved part is IR finite

Decompose propagator

$$D^{\text{imp}}_{\mu\nu} = D^{\text{Wilson}}_{\mu\nu} + \Delta D_{\mu\nu}\,, \qquad D^{\text{Wilson}}_{\mu\nu} = \frac{1}{\widehat{k}^2}\left(\delta_{\mu\nu} - \xi\,\frac{\widehat{k}_\mu \widehat{k}_\nu}{\widehat{k}^2}\right)$$

$$\Delta D_{\mu\nu} = D^{\text{imp}}_{\mu\nu} - D^{\text{Wilson}}_{\mu\nu} \quad \text{at} \quad D = 4$$

Found analytic $4D$ representation (20 terms) which explicitly depends on external Lorentz indices only

$\longrightarrow$ manipulate contributions symbolically

All divergences with Wilson part of propagator !

Overlap fermions

- Overlap Dirac operator, Neuberger,...
- Feynman rules for overlap fermions, see Kikukawa, Yamada) and Ishibashi et al.
- Feynman rules for lattice operators needed for moments of structure functions, see Göckeler et al.

About computation

- Calculation performed in Kawai scheme
- Complete computation of diagrams with symbolic program (*Mathematica*)

(+) A number of analytic and numeric checks:

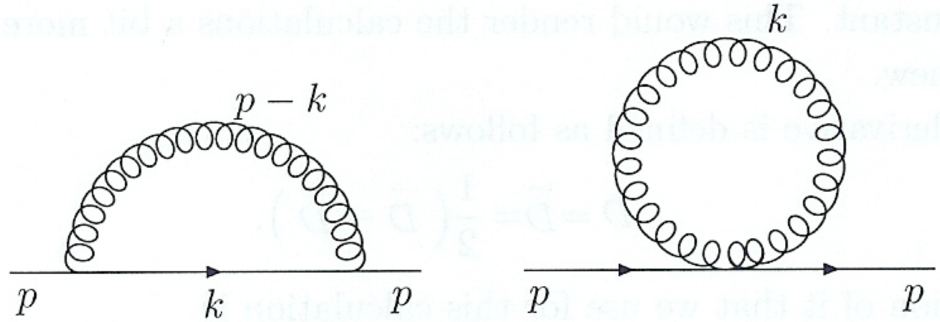
- Computations in general covariant gauge
- Analytic cancellation of pole terms
- No $1/a$ terms for overlap fermions because of chirality property

(+) Decoupling between symbolic computation of diagrams and numeric computations of lattice integrals

(−) Expansive in CPU time and memory

Results for Z's

Quark self energy



One loop correction from

$$S_O^{-1} = \mathrm{i}\,\not{p}\left[1 - \frac{g^2\,C_F}{16\pi^2}\,\Sigma_1 + \cdots\right],$$

$$\Sigma_1(a,p) = (1-\xi)\,\log(a^2p^2) + 4.79201\,\xi + b_\Sigma$$

Example:

$r = 1$, $\rho = 1.4$, TILW at $\beta = 8.45$: $\quad b_\Sigma = -16.179$

Z-factor relating lattice to $\overline{MS}$

$$Z_\psi^{\overline{MS}} \;=\; 1 - \frac{g^2 C_F}{16\pi^2}\Big(2(1-\xi)\log(a\mu) + 3.79201\,\xi + B_\psi\Big)$$

$$B_\psi^{\text{over}} = -15.179,\;\; B_\psi^{\text{Wilson}} = 12.8524$$

For plaquette gauge action results coincide with

Alexandrou et al., Capitani and Giusti

Bilinear quark operators

$$\mathcal{O}_F = \bar{\psi}(x)\Gamma^F\psi(x)$$

Γ^F generic Dirac matrices

$$\Gamma^S = 1\,,\ \Gamma^P = \gamma_5\,,\ \Gamma^V = \gamma_\mu\,,\ \Gamma^A = \gamma_\mu\gamma_5\,,\ \Gamma^T = \sigma_{\mu\nu}\gamma_5$$

Amputated 1-loop contribution (without self energies)

Example:

$$\Lambda^{S,P} = \frac{g^2}{16C_F\pi^2}\left[-(4-\xi)\log(a^2p^2) - 5.79201\xi + b_{S,P}\right]\{1,\gamma_5\}$$

Result: $b_{S,P} = 10.512$

Z-factor:

$$Z_{S,P}^{\overline{MS},\text{over}} = 1 - \frac{g^2C_F}{16\pi^2}\left(-6\log(a\mu) - 10.667\right)$$

$$Z_{S,P}^{\overline{MS},\text{Wilson}} = 1 - \frac{g^2C_F}{16\pi^2}\left(-6\log(a\mu) + \{12.95241, 22.59544\}\right)$$

Tadpole improvement

Form of Z-factor in 1-loop :

$$Z_{\mathcal{O}}^{\text{pert}} = 1 - \frac{C_F\, g^2}{16\pi^2} \left[\gamma_{\mathcal{O}} \log a\mu + B_{\mathcal{O}}\right] + O(g^4)$$

g^2 bad expansion parameter (tadpole diagrams)

Express Z-factors in framework of tadpole improved (TI)

perturbation theory (Lepage):

replace coupling by: $g_{\text{TI}}^2 = \frac{g^2}{u_0^4}$

Idea behind: mean-field approximation $U_{x,\mu} \to u_0$ gives good estimate of all Green functions

Use ($g^2 \to g_{\text{TI}}^2$)

$$Z_{\mathcal{O}}^{\text{TI}} = \frac{Z_{\mathcal{O}}^{\text{MF}}}{Z_{\mathcal{O}}^{\text{MF,pert}}} Z_{\mathcal{O}}^{\text{pert}} = Z_{\mathcal{O}}^{\text{MF}} \left(1 - \frac{C_F\, g_{\text{TI}}^2}{16\pi^2} \left[\gamma_{\mathcal{O}} \log a\mu + B_{\mathcal{O}}^{\text{TI}}\right] + \cdots \right)$$

Hope to find smaller $B_{\mathcal{O}}^{\text{TI}} = B_{\mathcal{O}} - B_{\mathcal{O}}^{\text{MF,pert}}$

How to obtain $Z_{\mathcal{O}}^{\mathrm{MF}}$? From: $Z_{\mathcal{O}} = Z_{\Lambda} Z_{\Psi}$

MF result for amputated Green functions

$$\Lambda_{\mathcal{O}}^{\mathrm{MF}} = u_0^{n_D} \Lambda_{\mathcal{O}}^{\mathrm{tree}} \to Z_{\Lambda}^{MF} = u_0^{-n_D}$$

n_D number of covariant derivatives in operator $\mathcal{O}$

Z_{Ψ}^{MF} different for Wilson and overlap fermions

$$\text{Wilson}: \; u_0 \qquad \text{Overlap}: \; \frac{u_0}{1 - \frac{4r}{\rho}(1 - u_0)}$$

Several ways to choose u_0 from measurements:

$$u_0 = \left(\frac{1}{3} \mathrm{Re}\ \mathrm{Tr} \langle U_{\mathrm{pl}} \rangle \right)^{1/4}$$

Perturbative expansion: $u_0^{\mathrm{pert}} = 1 - \frac{g^2 C_F}{16\pi^2} k_u + O(g^4)$

k_u from improved gluon propagator ($k_u^{\text{Wilson}} = \pi^2$)

$$k_u = 4\pi^2 a^4 \int_{-\pi/a}^{\pi/a} \frac{d^4k}{(2\pi)^4} \left(\hat{k}_4^2 D_{11}^{\text{imp}} - \hat{k}_1 \hat{k}_4 D_{14}^{\text{imp}} \right)$$

Operator $\mathcal{O}$	$B_{\mathcal{O}}^{\overline{MS}}$	$B_{\mathcal{O}}^{\text{TI}}$
S,P	-13.051	0.376
V,A	-12.663	0.764
T	-13.867	-0.440

Table 1: Finite one-loop contributions $B_{\mathcal{O}}$ for TILW at $\beta = 8.45$.

Reliability of using boosted coupling depends on the action used

Action fixes relation between Λ parameters of two schemes to 1-loop

$$\frac{1}{g^2_{\overline{MS}}(\mu)} - \frac{1}{g^2(a)} = b_0 \left(\log \frac{\mu^2}{\Lambda^2_{\overline{MS}}} - \log \frac{1}{a^2 \Lambda^2_{\mathrm{latt}}} \right) = 2b_0 \log a\mu + d_g + d_f$$

$b_0 = (11 - 2/3 N_f)/(4\pi)^2$

b_g and b_f calculable/calculated in pert. theory

Gauge action	$\Lambda_{\mathrm{latt}}/\Lambda_{\overline{MS}}$		$\Lambda^{\mathrm{TI}}_{\mathrm{latt}}/\Lambda_{\overline{MS}}$	
	$N_f = 0$	$N_f = 2$	$N_f = 0$	$N_f = 2$
Wilson	0.0347	0.0172	0.380	0.262
Symanzik	0.184	0.115	1.06	0.843
Iwasaki	2.13	1.86	5.82	5.86
DBW2	45.4	60.7	65.6	92.2

Table 2: Λ-ratios using bare g^2 and g^2_{TI}.

Boosting the coupling reasonable for Wilson, Symanzik, TILW actions:

$\Lambda_{\mathrm{latt}} \longrightarrow \Lambda_{\overline{MS}}$

desirable because $g^2_{\overline{MS}}(\mu = 1/a)$ is usually a good expansion parameter

$\longrightarrow$ Do not use TI for Iwasaki and DBW2 actions

On gauge dependence of Green functions

ξ-dependent terms in Green functions independent of

— gauge action (no dependence of c_i) – obvious –

— concrete form of the lattice fermion action

Explain remarkable independence of fermion action:

At one-loop level:

split functional integral over gauge fields into integral over transverse and (unphysical) longitudinal modes

Contribution of longitudinal modes in configuration space:

universal factor multiplying Landau gauge ($\xi = 1$) Green function (Göckeler et al., 1992)

$$\begin{aligned} G(x; g^2, \xi) &= G(x; g^2, \xi = 1) \times \left[1 - (1-\xi) g^2 C_F I(x,a) + \cdots \right] \\ I(x,a) &= \int \frac{d^4k}{(2\pi)^4} \frac{1 - \cos k \cdot x}{(\widehat{k}^2)^2} \end{aligned}$$

$\rightarrow$ exact expression for $O(g^2)$ gauge-dependent term in configuration space

$$G^{\text{gauge}}(x) = g^2 C_F \xi \, G^{\text{tree}}(x) I(x,a)$$

Find gauge term in momentum space:

Fourier transform dominated by $x^2 \sim 1/p^2$

We are interested in $a^2 p^2 \ll 1 \rightarrow x^2 \gg a^2$

$G^{\text{tree}}(x)$ do depend on the fermion action when $x^2 \sim a^2$

But: $G^{\text{tree}}(x) \rightarrow G^{\text{tree}}_{\text{cont}}(x)$ for $x^2 \gg a^2$

$\rightarrow$ Gauge terms universal for all fermion actions

Summary

- Computations performed in general covariant gauge using symbolic language MATHEMATICA
- Z factors found relating lattice and continuum operators of local fermionic currents using overlap fermions and 6-link improved gauge actions
- Exact chiral symmetry maintained in all results
- Comparing with the standard Wilson case the one-loop corrections to the Z-factors for overlap fermions have opposite sign. This is mainly due to the quark self energy
- We have tabulated Z results for a range of parameters in both the gauge and fermion actions
- The Z-factors obtained in the new framework give strong hope for better comparison with lattice measurements, especially when TI is added

TI improvement for Iwasaki and DBW2 actions should be taken with reservation

- Analytic proof for independence of gauge dependent Green functions on lattice fermion action
- for first applications of results see Latt03:
  QCDSF-UKQCD collaboration, hep-lat/0311017