Monopoles, confinement and the photon propagator in QED_3

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Abstract

We study the lattice gauge boson propagator of 3D cQED in Landau gauge at zero and non-zero temperature. The non-perturbative effects are taken into account by the generation of a mass, by an anomalous dimension and by the photon wave function renormalisation. All these effects can be attributed to the monopoles: they are absent in the propagator of the singularity-free part of the gauge field.

1. Introduction

3D compact electrodynamics ($cQED_3$) has two essential features in common with QCD, confinement [1] and chiral symmetry breaking [2]. Confinement is caused by a plasma of monopoles which emerge due to the compactness of the gauge field.

Recently we have demonstrated how the deconfinement phase transition in $cQED_{2+1}$ finds its explanation from the monopole point of view [3] and why the deconfinement phase transition is independent on the strength of the external fields [4].

Found: confining monopole plasma turns into dipole plasma at the deconfinement phase transition
Analogously in QCD_4: Studying the dynamics of the monopole current inside gluodynamics, monopole de-condensation at T_c appears as *de-percolation* (decay of the infrared, percolating monopole cluster into short monopole loops)

Here we focus on possible similarities between the gauge boson propagators of both theories [5, 6].

The infrared physics in QCD is governed by non–perturbative effects which should become manifest in the low p^2 behaviour of the propagator.
Numerical lattice results for gluodynamics:
the propagator for all considered gauges in momentum space is less singular than p^{-2} near $p^2 = 0$
Results at zero momentum are ranging from a finite [7] (Laplacian gauge) to a strictly vanishing [8, 9, 10] (Coulomb gauge) value
In the Landau gauge, besides the suppression at $p \to 0$, the propagator is enhanced at intermediate momenta which can be characterized by an anomalous dimension [11]

Our work: the p^2 dependence of the $T = 0$ photon propagator in $cQED_3$ is also described by a Debye mass and by an anomalous dimension
At finite temperature more structure functions are necessary, one of them is sensitive to the deconfinement phase transition where both the generated mass and the anomalous dimension vanish.

This mechanism can be clearly attributed to magnetic monopoles

2. The $3D$ compact U(1) model and the photon propagator

Action:

$$S[\theta] = -\beta \sum_p \cos\theta_p \,, \quad \beta = \frac{1}{a\, g_3^2} \,, \quad \frac{T}{g_3^2} = \frac{\beta}{L_t} \,.$$

$T = 0$ simulations mainly on a 32^3 lattice, at finite temperature on a $32^2 \times 8$ lattice

We do not intend to study finite size scaling aspects

We study the gauge boson propagator (in a specified gauge) in lattice momentum space. It is defined in terms of the Fourier transformed gauge potential ($\vec{x} = \vec{n} + \frac{1}{2}\vec{\mu}$)

$$\tilde{A}_{\vec{k},\mu} = \frac{1}{\sqrt{L_1 L_2 L_3}} \sum_{\vec{n}} \exp\left(2\pi i \sum_{\nu=1}^{3} \frac{k_\nu (n_\nu + \frac{1}{2}\delta_{\nu\mu})}{L_\nu}\right) A_{\vec{n}+\frac{1}{2}\vec{\mu},\mu}$$

by the gauge-fixed ensemble average

$$D_{\mu\nu}(\vec{p}) = \langle \tilde{A}_{\vec{k},\mu} \tilde{A}_{-\vec{k},\nu} \rangle \,.$$

Two identifications of $A_{\vec{x},\mu}$

(a) the angle-definition

$$A_{\vec{n}+\frac{1}{2}\vec{\mu},\mu} = \theta_{\vec{n},\mu}/(g_3 a) = \log\left(U_{\vec{n},\mu}\right)/(i g_3 a)$$

(b) the sinus-definition

$$A_{\vec{n}+\frac{1}{2}\vec{\mu},\mu} = \sin\left(\theta_{\vec{n},\mu}\right)/(g_3 a) = \left(U_{\vec{n},\mu} - U^*_{\vec{n},\mu}\right)/(2 i g_3 a) \,.$$

Relation between lattice and Fourier momenta

$$p_\mu = \frac{2}{a} \sin\frac{\pi k_\mu}{L_\mu} \,, \quad k_\mu = 0, \pm 1, \ldots, \pm\frac{L_\mu}{2} \,, \quad p^2(\vec{k}) = \sum_\mu p_\mu p_\mu$$

The most general tensor structure at $T = 0$ in the continuum

$$D_{\mu\nu}(\vec{p}) = P_{\mu\nu}(\vec{p}) D(p^2) + \frac{p_\mu p_\nu}{p^2} \frac{F(p^2)}{p^2}$$

with the d-dimensional (here $d = 3$) transverse projection operator

$$P_{\mu\nu}(\vec{p}) = \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}$$

In a covariant gauge specified by F gauge invariance requires $p_\mu D_{\mu\nu} p_\nu = F$.
If the Landau gauge would be fulfilled exactly: $F(p^2) \equiv 0$.

The two structure functions D and F can be extracted by projection also on the lattice

$$F(p^2) = \sum_{\mu,\nu=1}^{3} p_\mu D_{\mu\nu}(\vec{p}) p_\nu \,, \quad p^2 D(p^2) = \frac{1}{2} \sum_{\mu,\nu=1}^{3} P_{\mu\nu}(\vec{p}) D_{\mu\nu}(\vec{p})$$

and are found, in the best case, only approximately rotationally invariant.

$T > 0$: the propagator lacks $O(d)$ rotational symmetry (the d-th dimension defines T)
Now we have to consider two scalar functions:
D_T for the $(d-1)$-dimensional transverse operator P^T
D_L for the $(d-1)$-dimensional longitudinal operator P^L :

$$D_{\mu\nu}(\vec{p}) = P^T_{\mu\nu} D_T(|\mathbf{p}|, p_d) + P^L_{\mu\nu} D_L(|\mathbf{p}|, p_d) + \frac{p_\mu p_\nu}{p^2} \frac{F(|\mathbf{p}|, p_d)}{p^2}$$

The projection operators (transverse in d dimensions) are given as follows ($i, j = 1, \ldots, d-1$, $\mathbf{p}^2 = p_i p_i$):

$$P^T_{ij} = \delta_{ij} - \frac{p_i p_j}{\mathbf{p}^2} \,, \quad P^T_{dd} = P^T_{di} = P^T_{id} = 0 \,, \quad P^L_{\mu\nu} = P_{\mu\nu} - P^T_{\mu\nu}$$

The scalar functions D_T and D_L are extracted on the lattice from $D_{\mu\nu}$ ($d = 2 + 1$)

$$D_T(|\mathbf{p}|, p_d) = \sum_{\mu,\nu=1}^{3} P^T_{\mu\nu} D_{\mu\nu} \,, \quad D_L(|\mathbf{p}|, p_d) = \sum_{\mu,\nu=1}^{3} P^L_{\mu\nu} D_{\mu\nu} \,.$$

In the static limit $p_3 = 0$

$$D_L(|\mathbf{p}|, 0) \equiv D_{33}(|\mathbf{p}|, 0) \,.$$

This propagator for the angle definition, its fit in terms of mass and anomalous dimension, and the change at the deconfinement transition have been discussed in [5].

In order to discuss the functional form of the propagator from the point of view of confinement induced by the monopole plasma decompose the gauge fields into singular (monopole) and regular (photon) contributions on level of links

In notation of lattice forms:

$$\theta = \theta^{\text{phot}} + \theta^{\text{mon}}\,, \quad \theta^{\text{mon}} = 2\pi\Delta^{-1}\delta p[j]$$

0-form ${}^*j \in \mathbb{Z}$ is nonvanishing on sites of the dual lattice occupied by monopoles
1-form ${}^*p[j]$ corresponds to the Dirac strings (living on links of the dual lattice) which connect monopoles with anti–monopoles: $\delta^* p[j] = {}^*j$
This reconstruction is unique in the Landau gauge.

For a Monte Carlo configuration $\{\theta_l\}$:
1) fix the gauge
2) locate the Dirac strings, $p[j] \neq 0$
3) construct θ^{mon}
4) define $\theta^{\text{phot}} = \theta - \theta^{\text{mon}}$

This additive (de)composition is reflected in the corresponding exact additive decomposition of the transverse and longitudinal propagators $D^{\text{ang}}(p^2)$ and $F^{\text{ang}}(p^2)$ (angle-definition):

$$D^{\text{ang}} = D^{\text{ang,phot}} + D^{\text{ang,mon}} + 2D^{\text{ang,mixed}}$$

$$F^{\text{ang}} = F^{\text{ang,phot}} + F^{\text{ang,mon}} + 2F^{\text{ang,mixed}}$$

Having the decomposition of the link angles one is free to define the corresponding sinus-propagators in position space, proceed to momentum space, define the decomposition of D^{sin} and F^{sin}.

Analogously: decomposition for the structure functions $D_{L/T}$

3. Landau gauge fixing and Gribov copies

In Landau gauge the gauge propagator is expected to fulfill the transversality condition

$$F(p^2) = p_\mu D_{\mu\nu}(\vec{p}) p_\nu \equiv 0 \ ,$$

which allows to describe the propagator by a single function $D(p^2)$ alone.
For any practical implementation that condition is slightly violated.

Depending on the definition of the vector potential, there is need to select the transversal part by projection
of interest to see the origin violation of transversality, its control by the precision of the Landau gauge fixing procedure

Technically, the Landau gauge fixing is defined by maximizing the functional $\sum_l \cos\theta_l^g$ over all gauge transformations g (starting from a MC configuration $\{\theta_l\}$).

A gauge transformation in U(1) is specified by angles $g_{\vec{x}}$ such that $\{\theta_l\} \to \{\theta_l^g\}$ is given for all links $l = \{\vec{n} + \frac{1}{2}\vec{\mu}, \mu\}$ by

$$\theta_l^g = -g_{\vec{n}} + \theta_l + g_{\vec{n}+\vec{\mu}} + 2\pi k \quad \theta_l^g \in (-\pi, \pi] \, .$$

Our procedure: iterative mixture of overrelaxation and non-periodic gauge transformations, both applied in alternating order
Stop criterion: change of gauge functional/per link $< 10^{-6}$

The overrelaxation is done in a checkerboard fashion:
find for the even and odd sublattice a suitable set of $g_{\vec{n}}$ ($\vec{n} \in \Lambda_{\text{even}}$ or Λ_{odd}) to maximize the gauge functional
The angle $g_{\vec{n}}$ is easily found:

$$\tan g_{\vec{n}} = \frac{\sum_\mu \left(\sin\theta_{\vec{n},\mu} - \sin\theta_{\vec{n}-\vec{\mu},\mu}\right)}{\sum_\mu \left(\cos\theta_{\vec{n},\mu} + \cos\theta_{\vec{n}-\vec{\mu},\mu}\right)}$$

These gauge angles are multiplied by the overrelaxation factor, $g_{\vec{n}} \to \omega g_{\vec{n}}$ and bounded by $|g_{\vec{n}}| < \pi$ before the gauge transformation is performed on all links. One even/odd pair of gauge updates constitutes a single overrelaxation step (we use $\omega = 1.8$).

At a (local) extremum of the gauge functional the following condition is satisfied everywhere

$$\partial_\mu \theta_{\vec{n},\mu} = \sum_\mu \left(\sin\theta_{\vec{n},\mu} - \sin\theta_{\vec{n}-\vec{\mu},\mu} \right) = 0\,.$$

Consequence: $F(p^2) = 0$ for total propagator with sin-definition.

Non-periodic gauge transformations are used to immediately eliminate zero modes of the gauge field:
Subtract the volume average $\Theta_\mu = \sum_{\vec{n}} \theta_{\vec{n},\mu}/\Lambda$ from each link θ_l in the given direction
The subtraction

$$\theta_l \to \theta_l^g = \theta_l - \Theta_\mu + 2\pi k \qquad \theta_l^g \in (-\pi, \pi]$$

results from a gauge function $g_{\vec{x}} = -\Theta_\mu x_\mu$ (no summation!) which is in general non-periodic.

This algorithm generically will not lead to the absolute maximum of the gauge functional – Gribov problem.
The problem partially cured by repeating the same gauge fixing applying it to random gauge copies of the original MC configuration $\{\theta_l\}$

Dirac Lines
decompose plaquette into gauge invariant flux and discrete gauge-dependent contribution representing Dirac strings.
Set of Dirac plaquettes providing a world line of Dirac strings on dual lattice is called Dirac line
DDL: two lines with opposite flux orientation wrapping around the lattice (no monopoles at the end)
Observation: for larger maximum of gauge functional less hints for DL's and DDL's

Example for dependence of the propagator on number of Gribov copies:

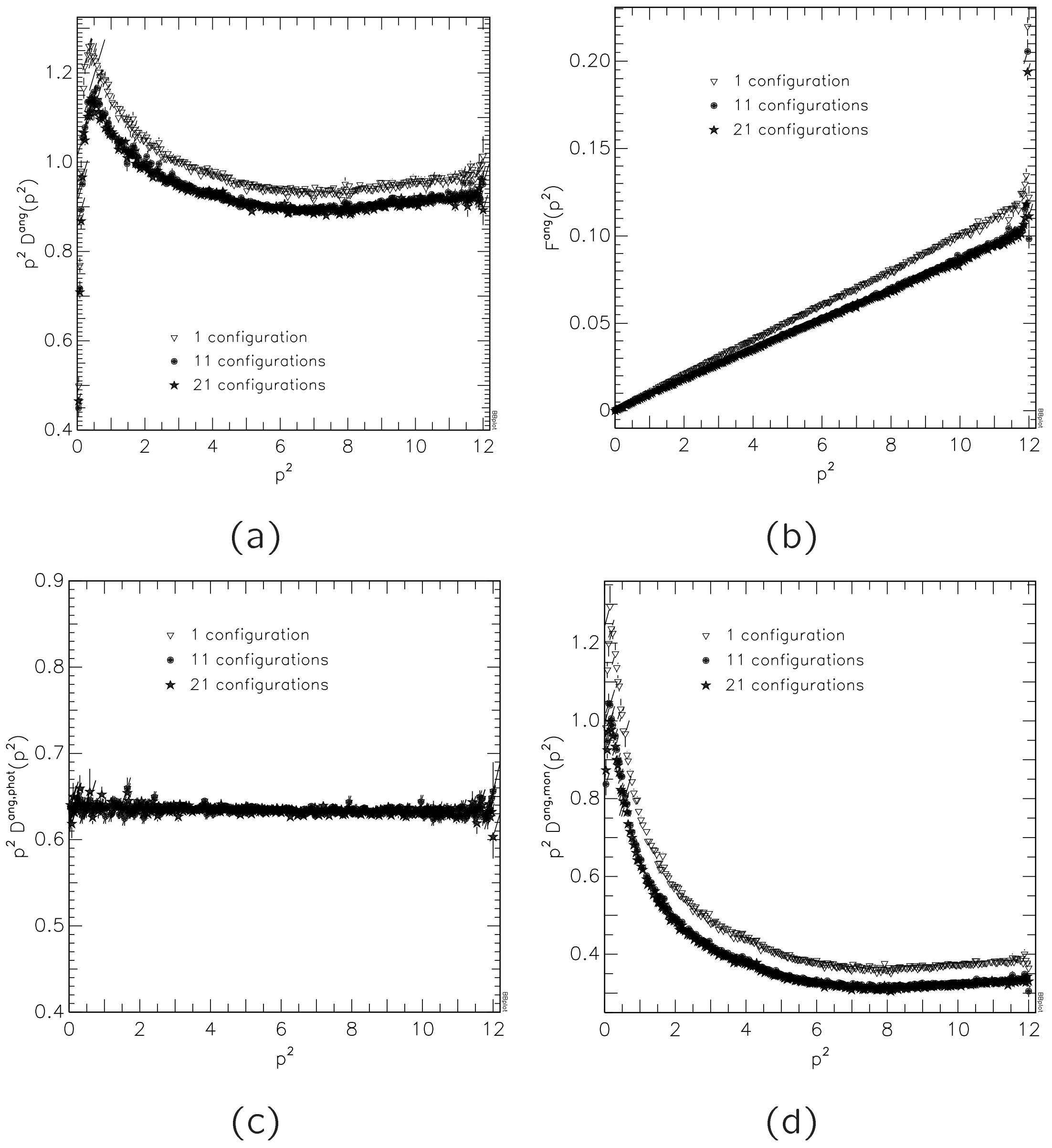

Figure 1: The dependence of the *angle*-propagator on N for a 32^3 lattice ($\beta = 2.0$): (a) the full transversal propagator for $N = 0, 10, 20$; (b) the full longitudinal propagator; (c) the regular part of the transversal propagator; (d) the singular part of the transversal propagator.

4. Results for $T = 0$ propagator in Landau gauge

Example for sinus-definition

All contributions together do *not* give a simple massive Yukawa propagator

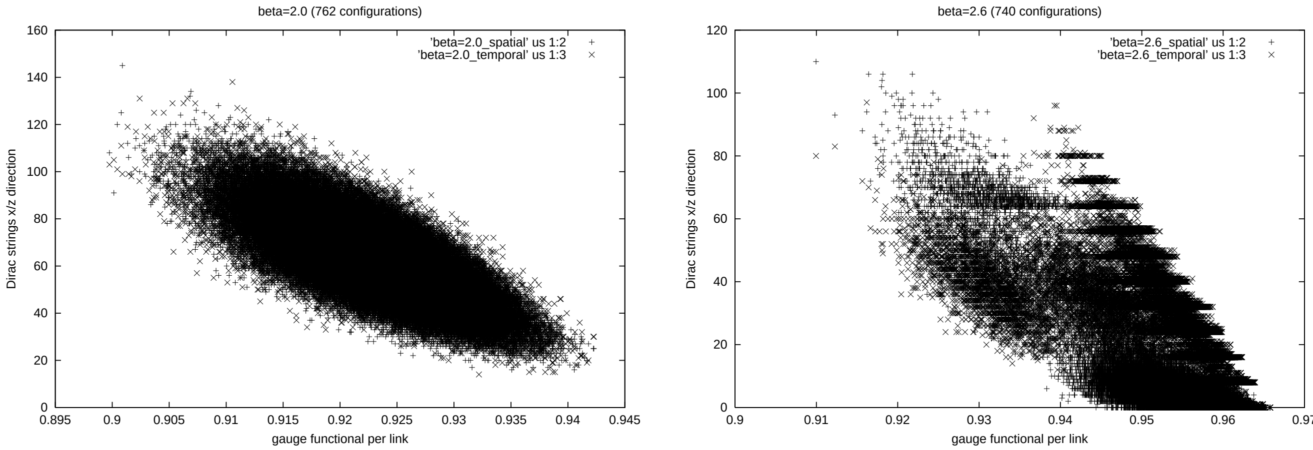

Figure 2: Number of Dirac strings in spatial and temporal directions vs. value of gauge functional for $32^2 \times 8$ in confinement and deconfinement (all 101 Gribov copies)

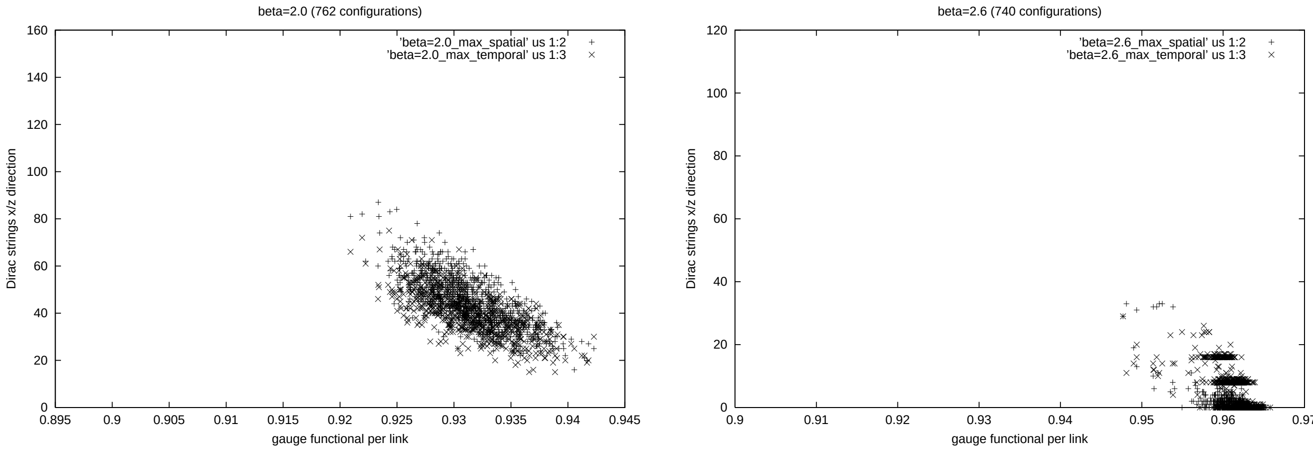

Figure 3: Same for the best out of 101 Gribov copies (configuration used to measure the propagator)

We are describing the full transversal propagator D using a function of the form

$$D(p^2) = \frac{Z}{\beta} \frac{m^{2\alpha}}{p^{2(1+\alpha)} + m^{2(1+\alpha)}} + C\,,$$

Z, α, m and C are fitting parameters

Ansatz implies:

Photon Debye mass m (due to screening) together with an anomalous dimension α

Z renormalisation of photon wave function due to quantum corrections

C δ–function–like interaction in coordinate space

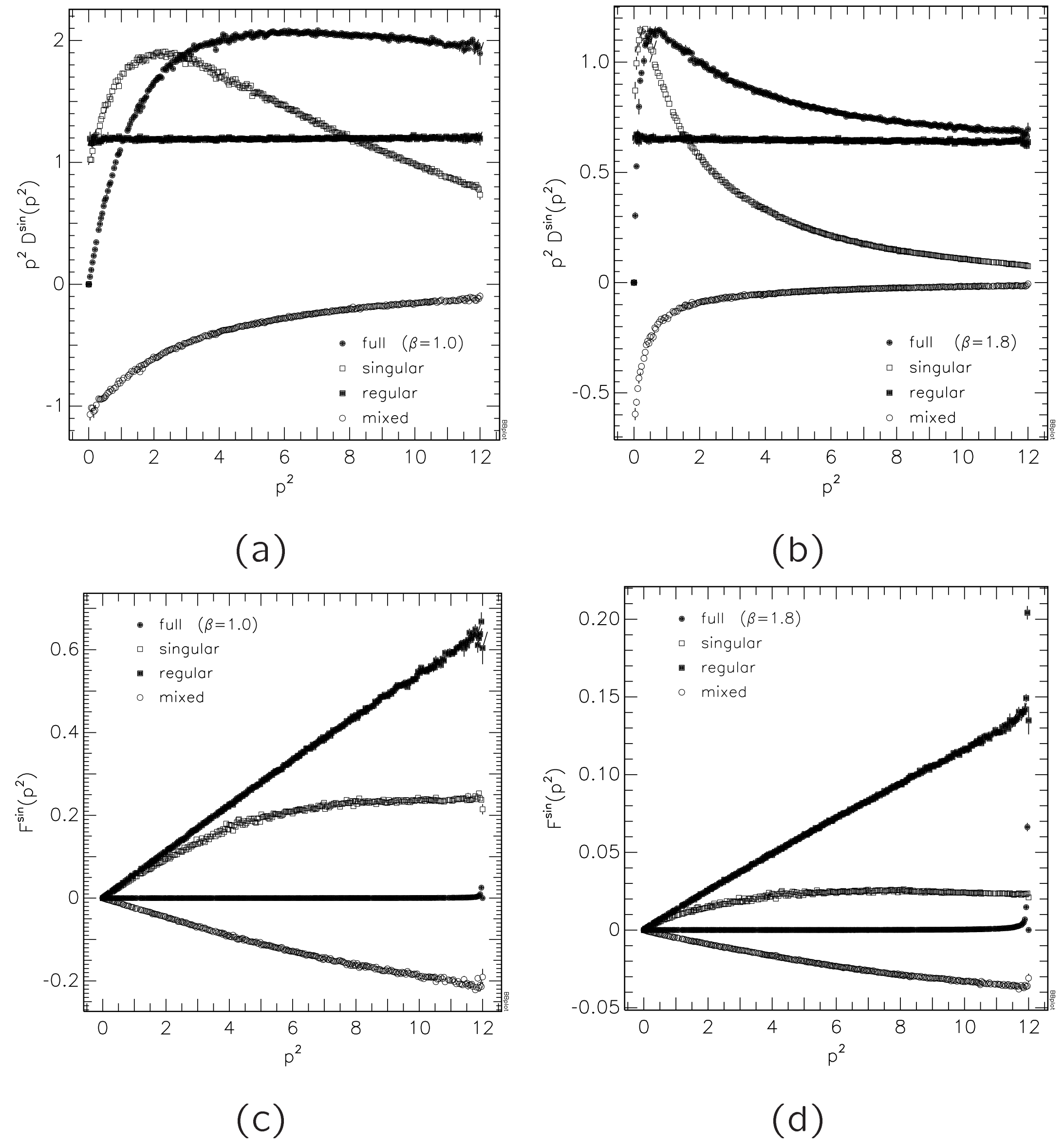

Figure 4: The *sinus*-propagator in the Landau gauge measured on a 32^3 lattice : for the transverse part, D^{sin} ((a) at $\beta = 1.0$ and (b) at $\beta = 1.8$) and for the longitudinal part F^{sin} ((c) at $\beta = 1.0$ and (d) at $\beta = 1.8$) the full, the singular, the regular and the mixed contribution are shown for comparison.

Model similar to some of Refs. [11, 12] where the propagator in gluodynamics has been studied

The regular part of the transversal propagator is expected to be described by a massless propagator with the canonical dimension

$$D^{phot}(p^2) = \frac{Z^{phot}}{\beta}\frac{1}{p^2} + C^{phot}\,.$$

The longitudinal part, if not vanishing, can be fitted by a contact term alone,

$$\frac{F(p^2)}{p^2} = P\,, \qquad \frac{F^{phot}(p^2)}{p^2} = P^{phot}\,,$$

where the fitting parameters P and P^{phot} describe the strength of the δ–function terms. Note that for the full propagator D^{sin} the parameter $P \equiv 0$.

Results of the fits

In the fits we used standard Mathematica packages combined with search for global minimum in $\chi^2/d.o.f.$

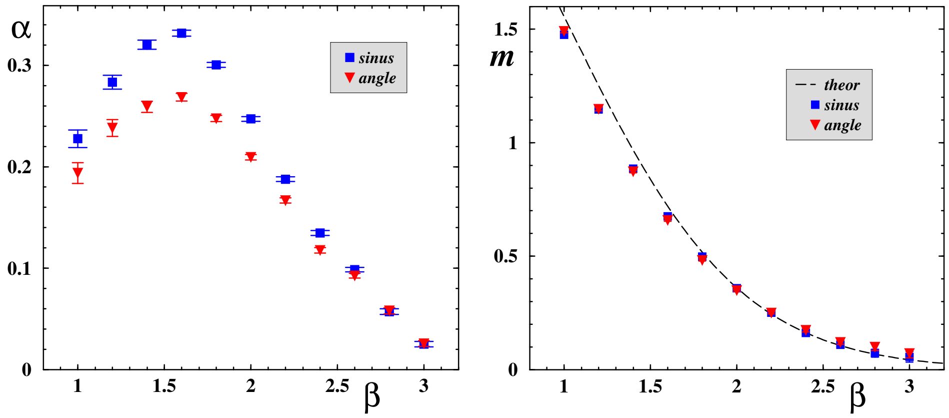

Figure 5: The anomalous dimension and the mass (compared to a theoretical prediction [6]) at zero temperature as function of β.

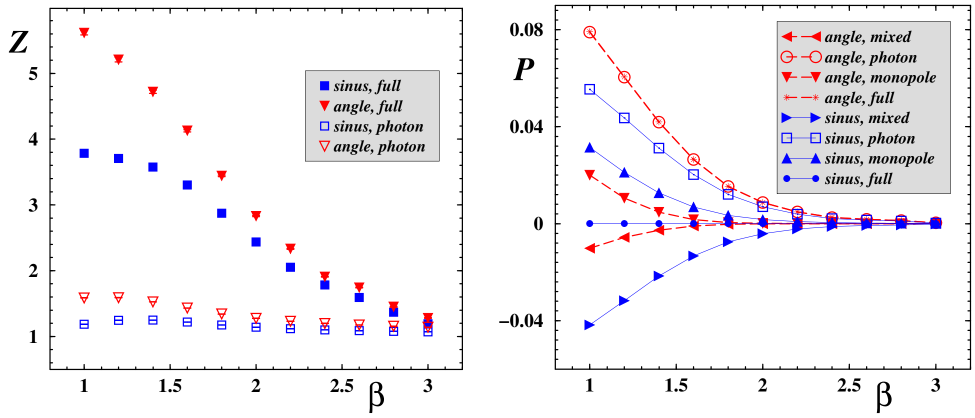

Figure 6: The same for Z and P.

5. Fits for the $T > 0$ propagators

We analyse here the structure functions at $p_3 = 0$. The fit forms are used as in the zero temperature case.

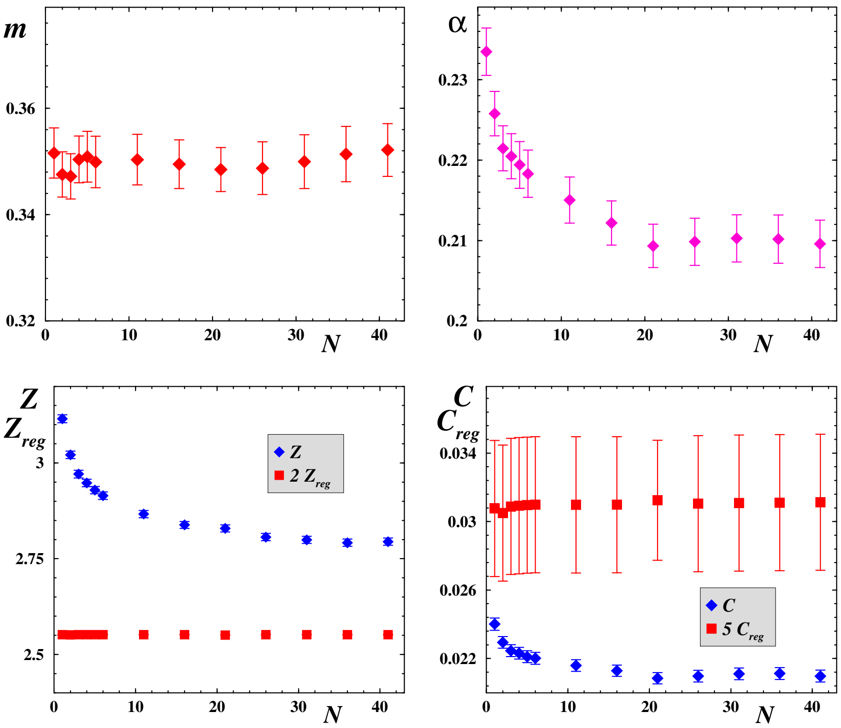

Figure 7: Study of the Gribov copy dependence of the fit parameters (D^{ang} at $T = 0$, $\beta = 2.0$)

The D_L structure function

Z and Z^{phot}:

$Z \approx Z^{\text{phot}}$ in the deconfined phase while it is much larger in the confined phase: photon wave function strongly renormalized by monopole plasma

In contrast, Z^{phot} smoothly changes crossing transition at $\beta_c \approx 2.35$.

The mass scale generated is non–vanishing in the confinement phase due to presence of the monopole plasma

it "vanishes" at T_c when the very dilute remaining monopoles and anti–monopoles form dipoles

In the confinement phase the monopole plasma causes the anomalous dimension growing to $\alpha \approx 0.25$

To characterize the properties of Z and α approaching the phase transition we fitted in [5]

$$R_Z(\beta) = \frac{Z(\beta)}{Z^{\text{phot}}(\beta)} - 1 \,,$$

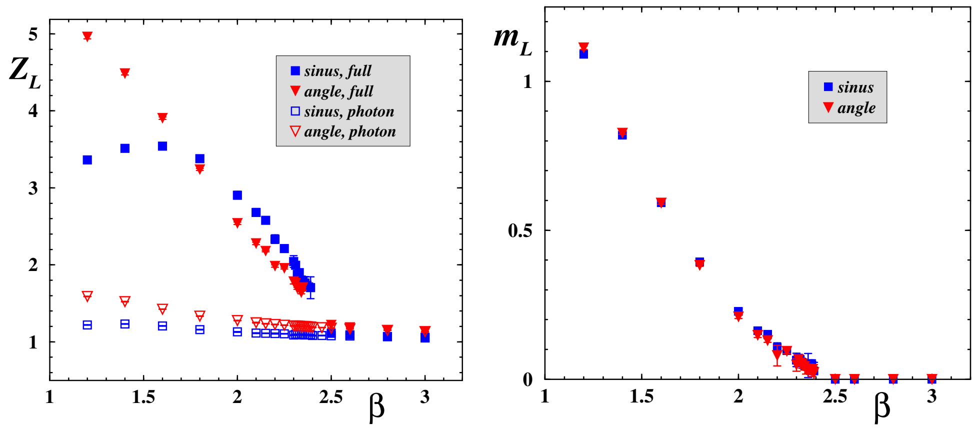

Figure 8: Coefficients Z_L and the Debye mass m_L for D_L vs β for $32^2 \times 8$

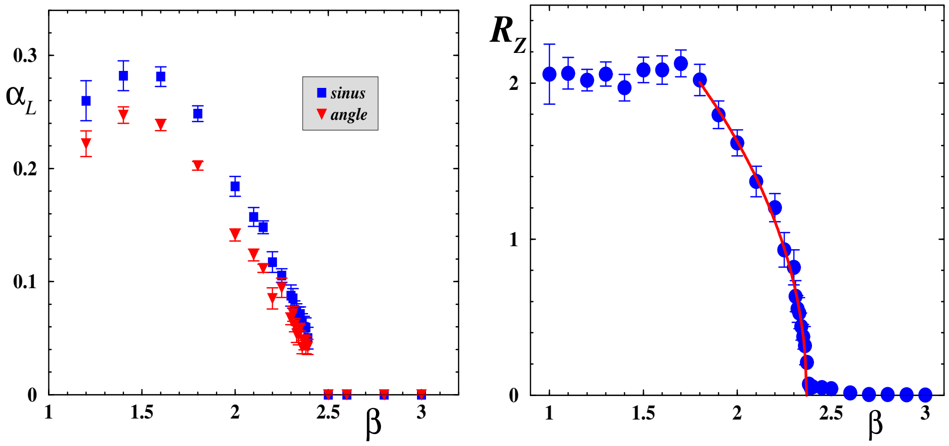

Figure 9: Anomalous dimension α_L for D_L and fit to R_Z (from [5] near β_c.

and the anomalous dimension α in the following form:

$$f_i(\beta) = h_i\,(\beta_c^{(i)} - \beta)^{\gamma_i}\,, \quad \beta < \beta_c^{(i)}\,, \quad (i = \alpha, Z)$$

$\beta_c^{(\alpha,Z)}$ are the pseudo–critical couplings
Results are in agreement with previous studies

The D_T structure function

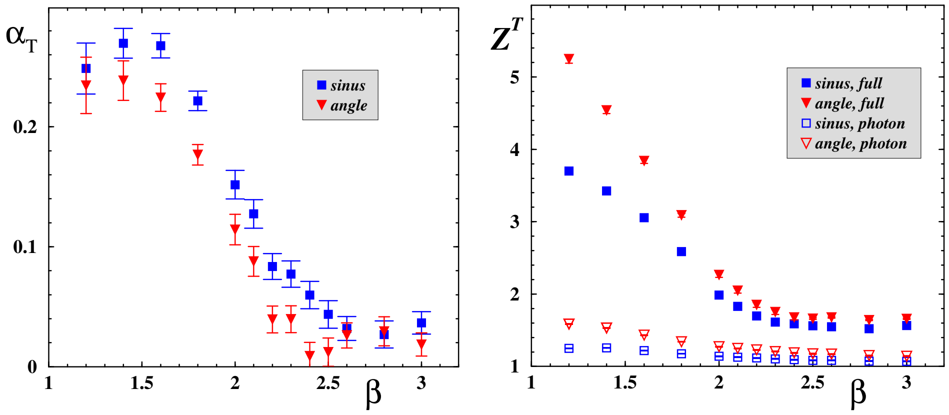

Figure 10: Anomalous dimension and wave function renormalisation for D_T at $32^2 \times 8$ vs β

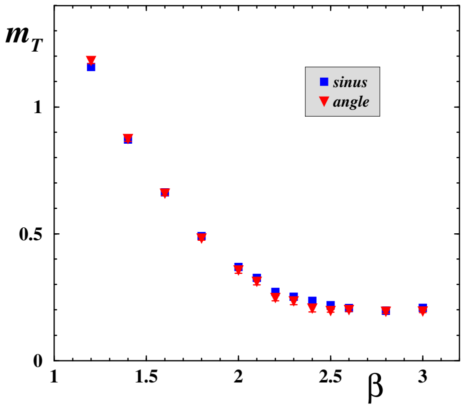

Figure 11: The same for mass m_T

6. Conclusions

The presence of the monopole plasma leads to the appearance of an $\alpha > 0$ in the boson propagator of cQED$_3$ in the confinement phase and the generation of a mass

The effects are absent in the propagator of the singularity-free part of the gauge field

$\rightarrow$ stimulate an analytical explanation for the anomalous dimension for the gauge field

All propagator data at zero and finite temperature in the confinement can be described by the same type of fitting function.

We conjecture that in the case of QCD the Abelian monopoles defined within the Abelian projection may be responsible for the anomalous dimension of the gluon propagator observed (Martinelli group)

Found that α and the ratio of the photon wave function renormalisation constants R_Z obtained from the structure function D_T represent alternative, also non–local order parameters characterizing the confinement phase

The spatial structure function D_T is found not to change suddenly at the deconfinement temperature

Possible work in future:
Influence of external fields and coupling to matter in $cQED_3$
Role of monopoles on gauge boson propagator in SU(2) and external fields
Monopoles and gluon propagator in full QCD including dynamical fermions

References

[1] A. M. Polyakov, Nucl. Phys. B **120**, 429 (1977).

[2] H. R. Fiebig and R. M. Woloshyn, Phys. Rev. D **42**, 3520 (1990).

[3] M. N. Chernodub, E. M. Ilgenfritz and A. Schiller, Phys. Rev. D **64**, 054507 (2001).

[4] M. N. Chernodub, E. M. Ilgenfritz and A. Schiller, Phys. Rev. D **64**, 114502 (2001).

[5] M. N. Chernodub, E. M. Ilgenfritz and A. Schiller, Phys. Rev. Lett. **88**, 231601 (2002).

[6] M. N. Chernodub, E. M. Ilgenfritz and A. Schiller, in preparation.

[7] C. Alexandrou, P. de Forcrand and E. Follana, Phys. Rev. D **63** (2001) 094504; hep-lat/0112043.

[8] V. N. Gribov, Nucl. Phys. B **139**, 1 (1978).

[9] A. Cucchieri and D. Zwanziger, Phys. Lett. B **524**, 123 (2002).

[10] D. Zwanziger, Nucl. Phys. B **364**, 127 (1991).

[11] P. Marenzoni, G. Martinelli, N. Stella and M. Testa, Phys. Lett. B **318**, 511 (1993);

[12] J. P. Ma, Mod. Phys. Lett. A **15**, 229 (2000).