Monopoles, confinement and deconfinement in lattice compact QED in (2+1)D with external fields

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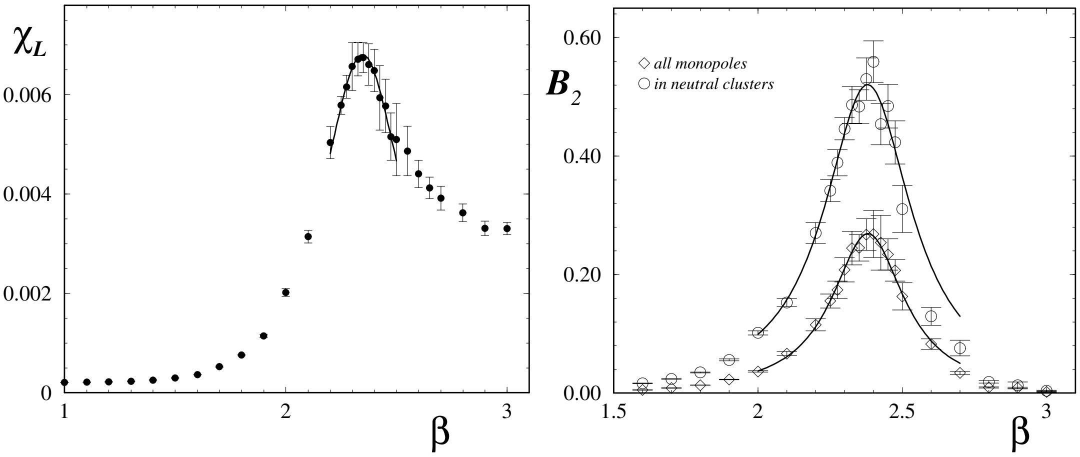
1. Introduction

In 3D cQED confinement is proven and understood [1].

The confinement is due to presence of Abelian monopoles – topological defects which appear due to the compactness of the gauge group.

The confining property is lost at sufficiently high temperature. The confinement-deconfinement phase transition has been studied on the lattice both analytically [2] and numerically [3].

In [4] we have demonstrated that the monopoles are sensitive to the transition. In the confinement phase the monopoles are observed in the plasma state while in the deconfinement phase the monopoles appear in the form of a dilute gas of magnetic dipoles [5].



Susceptibility of the Polyakov loop (l) and Binder second cumulants of monopole densities (r) *vs.* β at zero field.

In an alternative explanation, based on Svetitsky–Yaffe universality arguments [6], the transition is explained in terms of $U(1)$ vortex dynamics of the corresponding $2D$ spin system. Approaching T_c, vortices and anti-vortices start to form bound states, at $T > T_c$ no unbound vortices and anti-vortices are left.

The monopole pairing machanism seems to have counterparts in realistic gauge theories: instanton molecules in $T > 0$ QCD [7], formation of Nambu $M\bar{M}$ pairs at the electroweak theory phase transition [8]...

Here we study the influence of the external electromagnetic field on the confining and monopole properties of the compact Abelian gauge model in 2+1 dimensions [9].

2. Model and restrictions on external fields

The action with an external field is given by [10]

$$S[\theta, \theta^{\text{ext}}] = -\beta \sum_p \cos\Big(\theta_p - \theta_p^{\text{ext}}\Big)\,, \quad \beta = \frac{1}{a\, g_3^2}\,, \quad \frac{T}{g_3^2} = \frac{\beta}{L_t}\,.$$

External electric field E: field directly coupled to plaquettes lying in the 31 plane,

$$E = \theta_{31}^{\text{ext}} = \frac{2\pi n_E}{L_1\, L_3}\,, \quad n_E \in \mathbb{Z}\,, \quad \theta_{12}^{\text{ext}} = \theta_{23}^{\text{ext}} = 0$$

Magnetic field B: field coupled to plaquettes lying in plane 12,

$$B = \theta_{12}^{\text{ext}} = \frac{2\pi n_M}{L_1\, L_2}\,, \quad n_M \in \mathbb{Z}\,, \quad \theta_{31}^{\text{ext}} = \theta_{23}^{\text{ext}} = 0$$

The quantization of external fields is necessary to match with p.b.c. imposed on the lattice [10]:
Wilson loop $W(L_i, L_k) = 1 = \exp\{i\Phi\}$, Φ flux through the ik plane $\longrightarrow$ $\Phi = 2\pi n_{\text{ext}}$, $n_{\text{ext}} \in \mathbb{Z}$

From the form of action the maximal number of the external flux is restricted. We restrict that number to $0 < n_{\text{ext}} < L_i L_j/4$. The largest fluxes might correspond to lattice artefacts.

Monte Carlo algorithm:

combines local Monte Carlo step (Metropolis + microcanonical sweep) with a global refreshment step to improve ergodicity

Global step: attempt to add an additional unit of flux (sign and direction chosen randomly) to the dynamical gauge field $\theta_i \to [\theta_i + \tilde{\theta}_i]_{\mathrm{mod} 2\pi}$ subject to a global Metropolis acceptance check

Lattice: $32^2 \times 8$

3. Polyakov loop operators and screening

Usually, the Polyakov loop $L(\mathbf{x}) = \exp\left\{ i \int_0^{L_t} dt\, A_0(\mathbf{x}, t) \right\}$ is used to probe confinement. In the absence of an external field:

$$\langle L(\mathbf{0}) L^*(\mathbf{R}) \rangle \propto \exp\{-L_t\, V(\mathbf{R})\}$$

$T < T_c$: $V(R) = \sigma\, R$
$T > T_c$: $V(R)$ logarithmic Coulomb type

Due to the Abelian nature of the Polyakov loop the correlator in the presence of field $\vec{F}^{\mathrm{int}} + \vec{F}^{q}$ can be written as product of two contributions:

$$\langle L(\mathbf{0}) L^*(\mathbf{R}) \rangle_{\vec{F}^{\mathrm{ext}}} = \langle \mathrm{e}^{i \oint_{\mathcal{C}} dx_\mu A_\mu(x)} \rangle_{\vec{F}^{\mathrm{ext}}} = \mathrm{e}^{i \Phi_{\mathcal{C}}(\vec{F}^{\mathrm{int}})} \cdot G(\mathbf{R}, \vec{F}^{\mathrm{ext}}) \,,$$

$$G(\mathbf{R}, \vec{F}^{\mathrm{ext}}) = \langle L(\mathbf{0}) L^*(\mathbf{R}) \rangle^{q}_{\vec{F}^{\mathrm{ext}}} = \langle \mathrm{e}^{i \Phi_{\mathcal{C}}(\vec{F}^{q})} \rangle^{q}_{\vec{F}^{\mathrm{ext}}} \,.$$

$\mathcal{C}$ is the contour corresponding to the test particle trajectories, $\Phi_{\mathcal{C}}(\vec{F}) = \oint_{\mathcal{C}} dx_\mu A_\mu(x) = \int_{\Sigma_{\mathcal{C}}} d\sigma_\mu\, F_\mu(x)$ is the flux of the electromagnetic field F which goes through the surface $\Sigma_{\mathcal{C}}$ spanned on the contour $\mathcal{C}$.
The factor G defines the potential

$$V(\mathbf{R}; \vec{F}^{\mathrm{ext}}) \propto -\log G(\mathbf{R}, \vec{F}^{\mathrm{ext}}) / L_t \,.$$

Summarizing:

$$\langle L(\mathbf{0}) L^*(\mathbf{R}) \rangle_{\vec{F}^{\mathrm{ext}}} \propto \exp\left\{ i \Phi_{\mathcal{C}}(\vec{F}^{\mathrm{int}}) - L_t\, V(\mathbf{R}; \vec{F}^{\mathrm{ext}}) \right\}$$

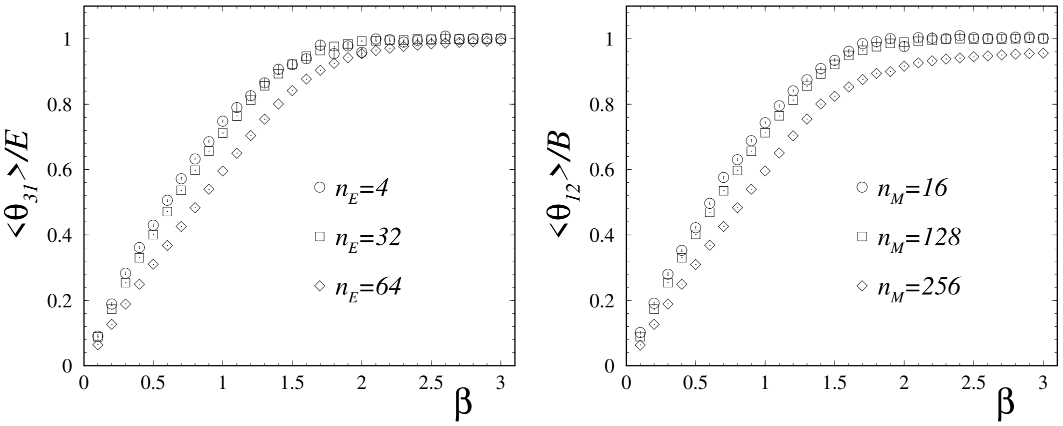
A *non–vanishing* flux $\Phi_{\mathcal{C}}(\vec{F}^{\text{int}})$ pierces the surface $\Sigma_{\mathcal{C}}$.

Both external and internal fluxes through the corresponding planes are quantized.
We use

$$\vec{F}^{\text{ext}} = (0, E, 0)\,, \quad \text{or} \quad \vec{F}^{\text{ext}} = (0, 0, B)\,.$$

Only the electric component contributes to the flux $\Phi_{\mathcal{C}}(\vec{F}^{\text{int}})$.
The mean internal electric and magnetic fields are $\langle\theta_{31}\rangle$ and $\langle\theta_{12}\rangle$, respectively:

$$\langle\theta_{ik}\rangle = \langle\ \frac{1}{L_s^2 L_t} \sum_{\vec{x}} [\theta_{\vec{x},ik}]_{\text{mod}2\pi}\ \rangle\,.$$



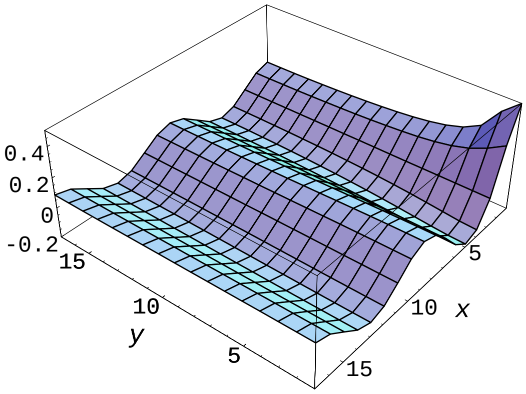
The normalized internal electric (l) and magnetic (r) fields *vs.* β at fixed $n_{E/M}$.

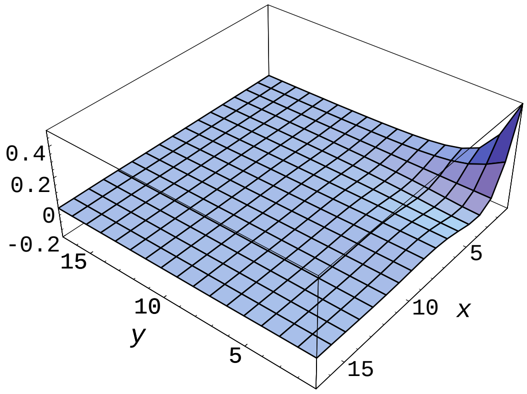
The attenuation of the external fields in the confinement phase resulting in weaker average fields in the medium happens due to the monopole plasma, it is called screening.

Consider the correlator in an external electric field

$$\langle L(0,0)\, L^*(x,y)\rangle_E \propto \exp\Big\{2\pi i\, \frac{x n_{\text{int}}}{L_s} - L_t\, V(x,y;\, E)\Big\}\,,$$

$\Phi_{\mathcal{C}} = 2\pi\, n_{\text{int}}\, x\, / L_s$.
Our numerical results suggest that the internal fluxes can adequately be described by taking into account only the “most probable” flux state $n_{\text{int}} = n_{\text{int}}(n_{\text{ext}}, \beta, L_i)$. The oscillating part of the correlator is defined by the electric component of the internal field. The correlator simultaneously characterizes both the screening of the external field (phase factor) and the potential of the test electric charges (modulus).





Real part of the Polyakov loop correlator in the $x-y$ plane for $n_E = 4$ in deconfinement (l, $\beta = 2.6$) and confinement (r, $\beta = 2.0$) phases. Half of the lattice $32^2 \times 8$ is shown.

The external magnetic field is directed along the Polyakov loop, therefore, the induced internal field does not contribute to $\Phi_{\mathcal{C}}$.

4. String tension and phase structure

String tension σ in external fields:

To evaluate σ, we use two Polyakov "plane" operators which are defined as a sum of the Polyakov loop along and perpendicular to the external field.
As an example, we take the electric field in the y direction

$$L_{||}(x) = \sum_{y=1}^{L_s} L(x,y)\,, \qquad L_{\perp}(y) = \sum_{x=1}^{L_s} L(x,y)\,.$$

It can be shown (using results of [11]) that in the presence of an electric field

$$\langle L_{||}(0) L_{||}^*(x)\rangle_{n_E} = \text{const} \cdot \mathrm{e}^{2\pi i n_{\text{int}}\, x/L_s} \cosh\left[\sigma\, L_t \left(x - \frac{L_s}{2}\right)\right],$$

therefore, the plane–plane Polyakov loop correlator parallel to the electric field oscillates with a decreasing amplitude.

The plane–plane correlator perpendicular to the field decreases exponentially (without oscillations) as function of the distance between the planes:

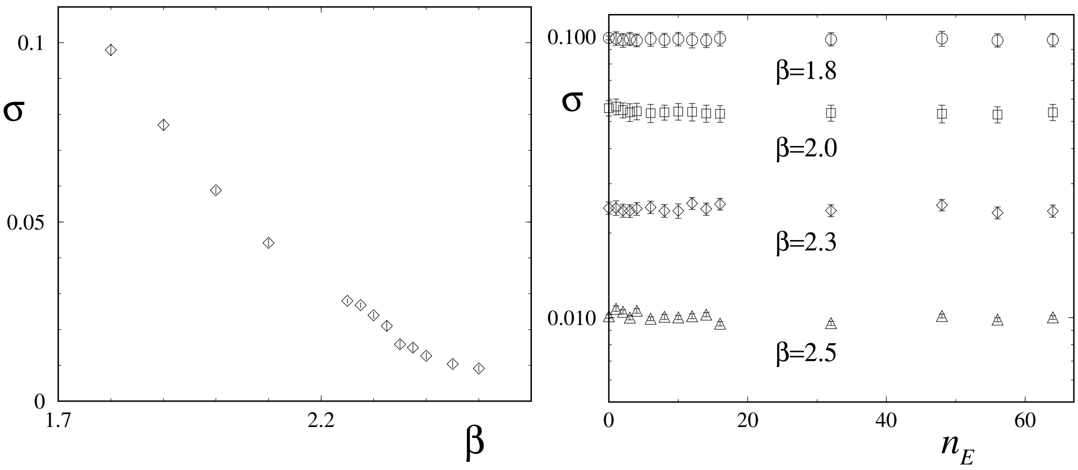
$$\langle L_{\perp}(0) L_{\perp}^*(y)\rangle_{n_E} = \text{const} \cdot \cosh\left[\sigma_{\text{eff}}(\sigma, n_{\text{int}})\, L_t \left(y - \frac{L_s}{2}\right)\right]$$

where

$$\sigma_{\text{eff}}(\sigma, n_{\text{int}}) = \frac{1}{L_t} \operatorname{arccosh}\Big[\cosh(\sigma\, L_t) - \cos(2\pi n_{\text{int}}\,/L_s) + 1\Big]\,.$$

The essential message is that the effective string tension σ_{eff} does not tell anything about confinement properties described by σ.

Note that the effect of the external electric field is absent if the internal flux is equal to quantized values $n_{\text{int}} = N \cdot L_s\,, \in \mathbb{Z}$.

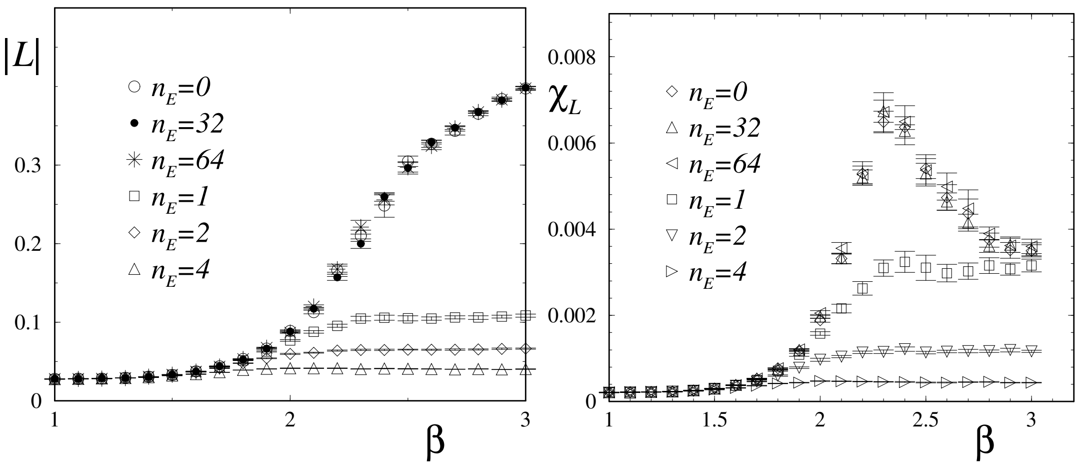


The string tension *vs.* β without external field (l) [4]. Fitted string tension for various β values as function n_E (r).

Summarizing, both magnetic and electric fields do not influence the string tension.

Polyakov loop in external electric field:

The plot of the Polyakov loop at zero field agrees with general expectations. However, as the electric field is turned on, the vacuum expectation value of the Polyakov loop is decreasing.



The absolute value of the bulk Polyakov loop (l) and the Polyakov loop susceptibility (r) *vs.* β for various values of the external electric flux n_E.

One might conclude that the external electric field restores the confinement phase which is in clear contradiction with the results for the string tension.

However, we have seen above that the external electric field modifies the properties of the Polyakov loop classically. In particular, for not too large n_E it enhances what we called the *effective* string tension.

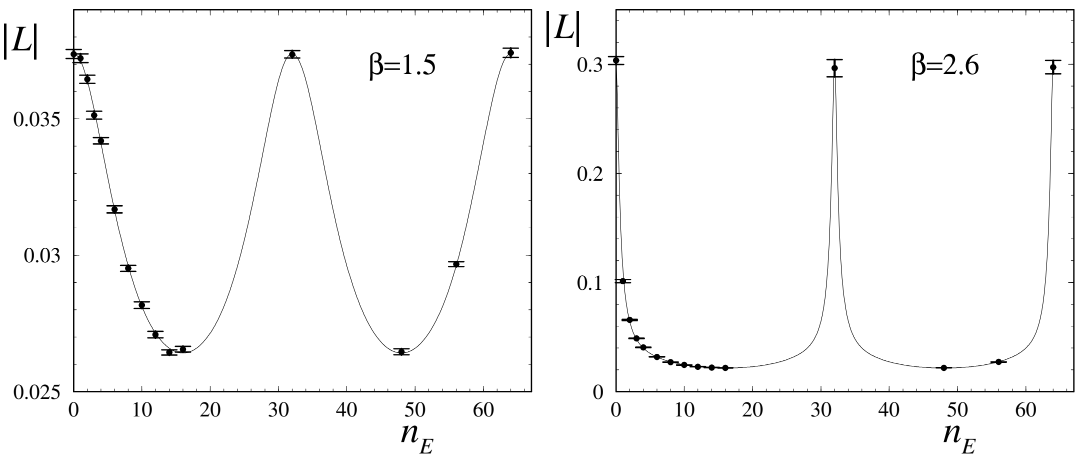
Let us estimate the classical correction to the Polyakov loop expectation value due to the external electric field. First we consider the squared modulus of the Polyakov line:

$$\langle |L|^2 \rangle_E = \mathrm{const} \cdot \sum_{x,y} \mathrm{e}^{2\pi i \, n_{\mathrm{int}} \, x / L_s} \, \langle L(0,0) L^*(x,y) \rangle \, .$$

It can be shown that:

$$\langle |L|^2 \rangle_E = \sum_{m \geq 0} \frac{C_m}{\cos(2\pi n_{\mathrm{int}}/L_s) - \cosh(\sigma_m L_t)} \, .$$

The lowest state in the expansion corresponds to $\sigma_0 = \sigma$. The square root of $\langle |L|^2 \rangle_E$ can serve as a good estimator for the Polyakov loop quantum average $\langle |L| \rangle$.
We use only the first two terms of the expansion to fit the behavior of the *measured* Polyakov loop *vs.* the external electric flux number.



The absolute value of the bulk Polyakov loop *vs.* the external electric flux n_E and its fit in (l) the confinement phase ($\beta = 1.6$) and (r) the deconfinement ($\beta = 2.5$) phase.

The best fits for σ_i are:
$\beta = 1.6$: $\sigma_0 = 0.167(13)$, $\sigma_1 = 0.85(6)$ [$\sigma = 0.170(4)$]
$\beta = 2.5$: $\sigma_0 = 0.0096(5)$, $\sigma_1 = 0.18(1)$ [$\sigma = 0.0101(1)$].

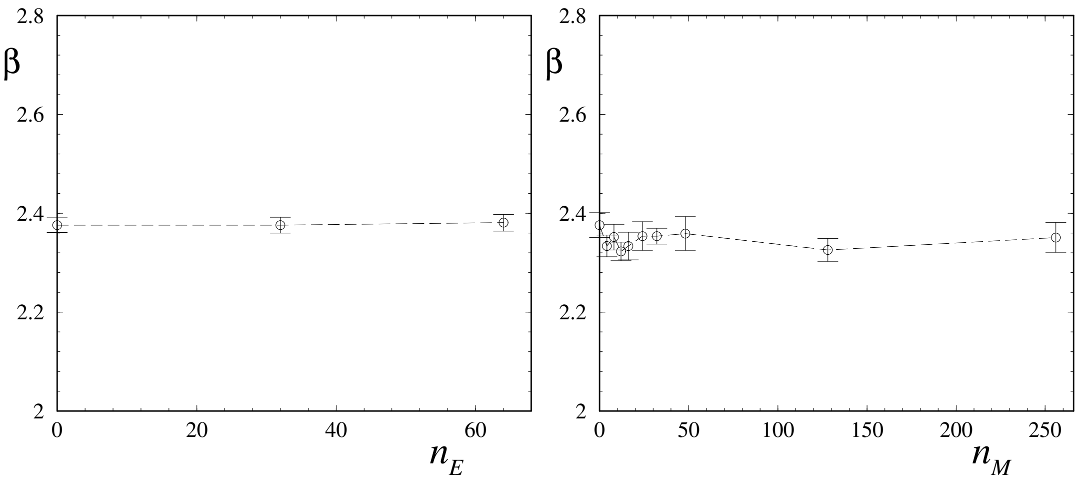
Therefore, the behavior of the Polyakov loop approximated via $\langle |L|^2 \rangle_E$ in the external electric field is consistent with the string tension measurements at zero field.

In the case of non-vanishing external electric field, the Polyakov loop expectation value may vanish regardless of the value of the actual string tension.

The influence of the external field on the Polyakov loop correlations is absent at $n_E = n_{\text{int}} = 32, 64$ and coincides with $n_E = 0$ data (in agreement with the earlier mentioned condition).

The tree level contribution to the Polyakov loop observables is absent in the case of the external magnetic field.

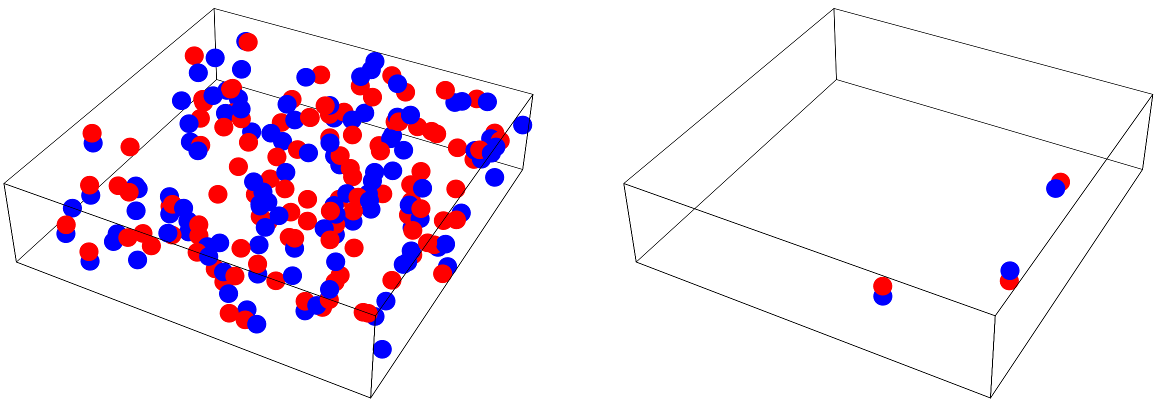
Phase diagram:



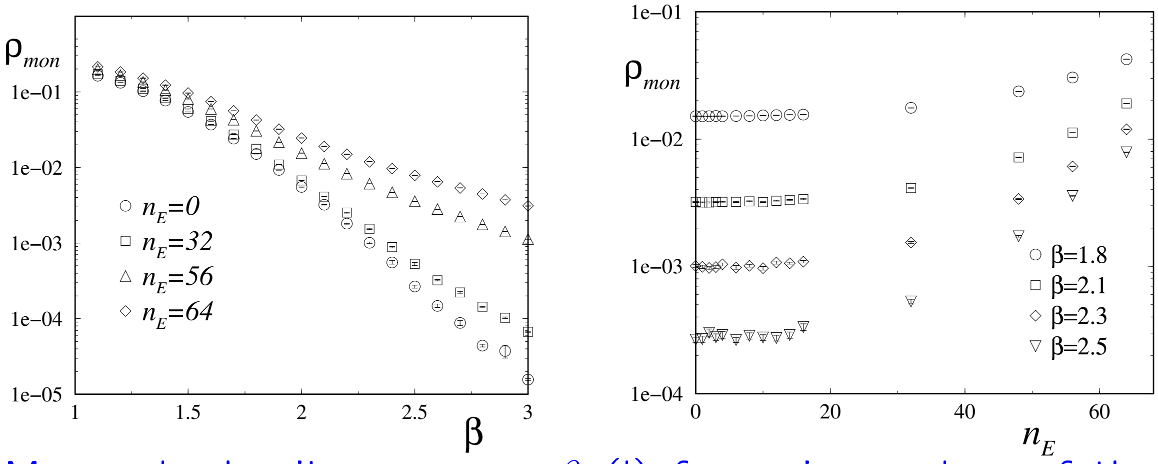
Phase diagrams in the $(n_{E/M}, \beta)$ plane derived from the module of the Polyakov loop for electric (l) and magnetic (r) fields, respectively.

5. Monopole properties in presence of fields

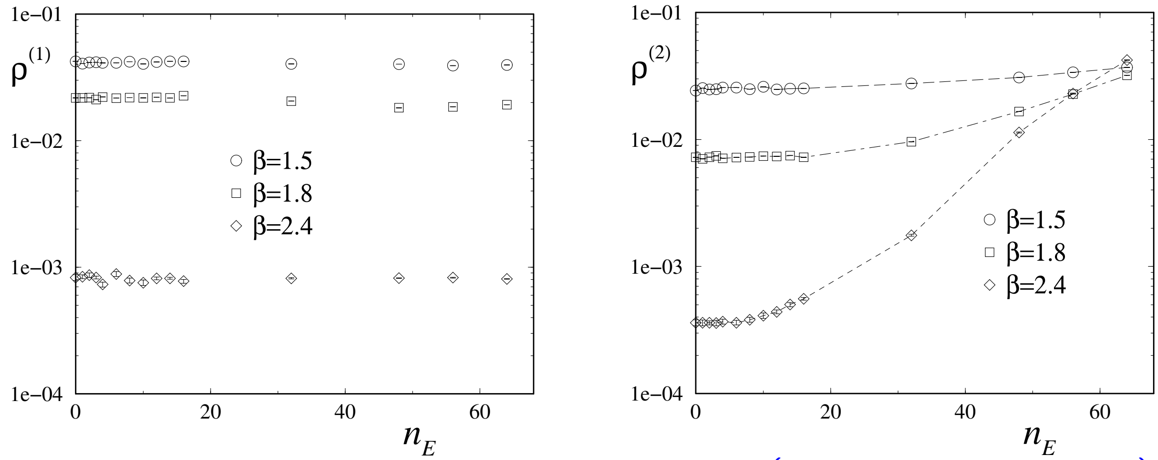
Here we present several results based on a cluster analysis of the monopole configurations [4]. The monopoles are defined as usual [12].



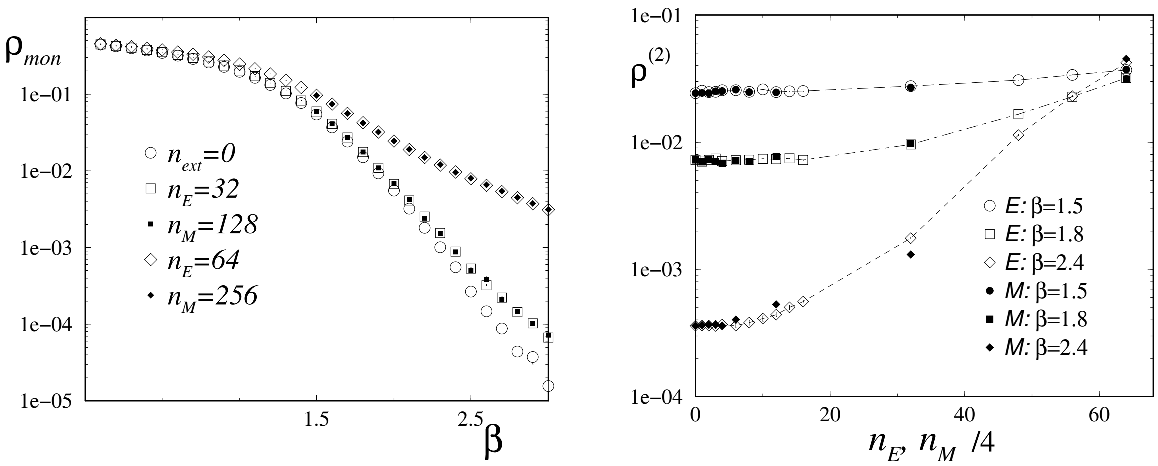
Typical monopole configurations in the confinement (l, $\beta = 1.6$) and deconfinement (r, $\beta = 2.5$) phase at zero field.



Monopole density $\rho_{\rm mon}$ *vs.* β (l) for various values of the external electric field, n_E; *vs.* n_E (r) for various β values.



The density of the clusters of size $N = 1$ (l, single monopoles) and $N = 2$ (r, dipoles) *vs.* external electric field of the flux n_E at various values of the coupling constant β.

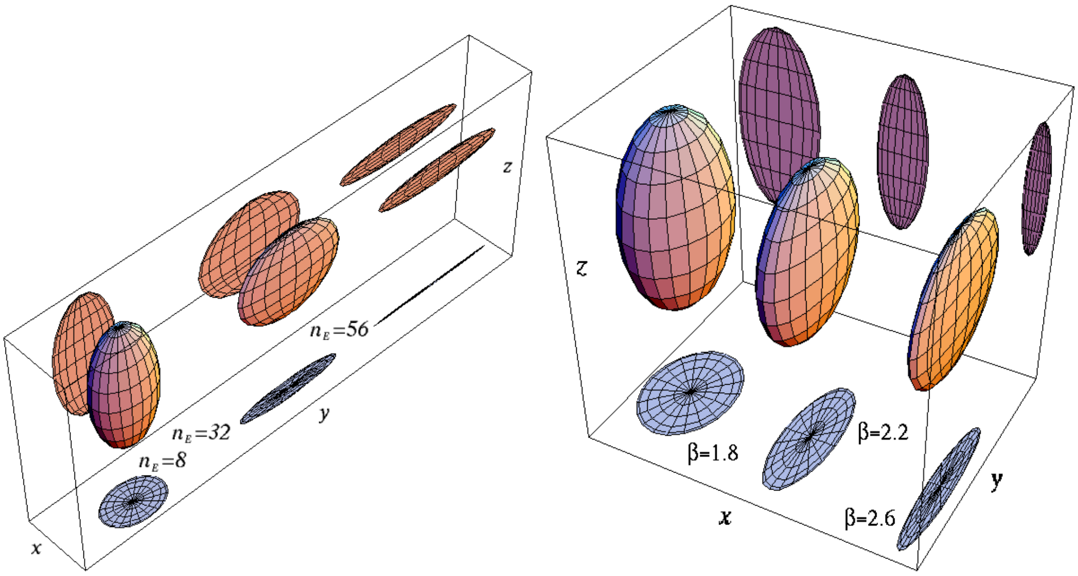


Monopole density $\rho_{\rm mon}$ *vs.* β (l) for various values of the external fields $n_{E/M}$. Clusters of two monopoles (dipoles) *vs.* $n_{E/M}$ (r) for some β values.

We observe that the plasma component of the single monopole ensemble does not feel the external field. Thus the confining

properties of the system should not depend on the external field in agreement with the conclusions made before.
The dipole density changes drastically as the external field increases: the field creates the magnetic dipoles from the vacuum. The larger the temperature (or β) is the larger the increase of the dipole density turning on the external field.

With a non–zero electric field, the system is anisotropic in all directions. The electric field is directed along the y direction, the "temperature" direction, z, is influenced by compactification. As the external field increases, the dipoles become elongated, increasingly with the external field, along the direction of the applied field.



The mean dipole anisotropy as the function of the external electric field (l) at fixed $\beta = 1.8$; *vs.* β (r) at fixed external electric flux n_E.

In the external magnetic field the system is symmetric in the x and y directions. The dipoles become more elongated in in the (temporal) z–directions as the strength of the external field increases.

6. Summary

We have investigated the properties of the $(2+1)D$ compact electrodynamics in external constant electric and magnetic fields. The main result is that the deconfinement temperature is insensitive to both external fields. We have found the reason for this behavior in terms of the monopole degrees of freedom: the external fields create magnetic dipole pairs (monopole–anti-monopole bound states) from the vacuum while the density of unpaired monopoles (which are responsible for the confinement of electric charges) stays unchanged. This result is not obvious from the beginning since another option is possible: the external fields could destroy the monopole bound states enforcing confining properties of the medium.

At zero external fields the magnetic dipole states are more elongated in the temperature direction. The external magnetic field which is parallel to the temperature direction makes this elongation stronger. An external electric field turns the dipoles to the direction of the external field. The effects of the external field on the medium are stronger in the deconfinement phase in which both electric and magnetic external fields are not screened.

The external electric field influences the Polyakov loop classically (or, in other words, on tree level). This might lead to a vanishing Polyakov loop and, in a certain case, to the observation of a non-vanishing *effective string tension* depending on the external field. However, this behavior of the Polyakov loop does not indicate a restoration of confinement for certain (not too large) external field fluxes.

The string tension (correctly defined from the correlation function of Polyakov plane–plane correlators parallel to the external electric field in spatial direction) is not influenced by the external electric field and coincides with the zero–field value.

The tree level effects on Polyakov loops and Polyakov loop correlation functions are absent for external magnetic fields pointing in the time–like direction.

The dynamics of the Abelian system is different from the behavior of the $(3+1)D$ non–Abelian gauge theory [13] where the external magnetic field increases the deconfinement temperature contrary to our results in $(2+1)D$ cQED. The reason of this difference may lie in the different behavior of the monopoles in the Abelian and non-Abelian gauge theories. We plan to investigate the monopole properties in the non-Abelian gauge theory [14].

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