金沢大学 素粒子論研究室

Matter near the Endpoint of the Electroweak Phase Transition

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- Summary

Results presented here complement recent studies of the Oxford group at much lighter and clearly larger Higgs masses

We concentrate on qualitative change of screening spectrum across transition and in crossover region evaluating the wave function in configuration space

Introduction

during last years big efforts to study phase transition of ew theory by lattice methods

due do newest lower bound on M_H very likely that standard model does not pass through true phase transition

BAU generation already ruled out
$\rightarrow$ interest to extensions of standard model

Reasons for interest to standard variant

- laboratory for investigating behavior of hot gauge fields coupled to scalar matter
- characterize possible bounds states and understand physics in strongly coupled high T phase
- understanding of real time topological transitions between different vacua
- influence of embedded topological defects on transition

The Model

$3D$ effective model for the electroweak sector of the standard model from dimensional reduction (Kajantie et al.)

$$S = \int d^3x (\frac{1}{4}F^b_{\mu\nu}F^b_{\mu\nu} + (D_\mu\phi)^+(D_\mu\phi) + {m_3}^2\phi^+\phi + \lambda_3(\phi^+\phi)^2)$$

$3D$ continuum model superrenormalizable: $m_3(\mu_3), g_3^2, \lambda_3$

Lattice formulation ($\rho_x^2 = \frac{1}{2}\mathrm{tr}(\Phi_x^+\Phi_x)$)

$$\begin{aligned} S[U,\phi] &= \beta_G \sum_{\mathcal{P}}(1 - \frac{1}{2}\mathrm{tr}U_{\mathcal{P}}) - \beta_H \sum_{links} \mathrm{tr}(\Phi_x^+ U_{x\mu}\Phi_{x+\hat{\mu}}) \\ &+ \sum_x (\rho_x^2 + \beta_R(\rho_x^2 - 1)^2) \end{aligned}$$

$$\begin{aligned} \beta_G &= \frac{4}{a g_3^2}\ , \quad \beta_H = \frac{2(1-2\beta_R)}{6 + a^2 m_3^2} \\ \beta_R &= \frac{\lambda_3}{g_3^2}\frac{\beta_H^2}{\beta_G} = \frac{1}{8}\left(\frac{M_H^*}{80\text{ GeV}}\right)^2 \frac{\beta_H^2}{\beta_G} \end{aligned}$$

Laine 95

$$m_3^2(\mu_3) = m_3^2 + m_{1-loop}^2 + m_{2-loop}^2$$

choice (arbitrary) $\mu_3 = g_3^2$

$3D$ cont. couplings $g_3^2, \lambda_3, m_3(\mu_3)$ *perturbatively known* functions of $4D$ running coupling constants (Kajantie et al., NP B458)

Cross Correlation Technique

Method

Aim: study simultaneously ground state *and* excited states and possibly their wave functions

- Consider a complete set of operators for given quantum numbers J^{PC} $\mathcal{O}_i$
- According to transfer matrix formalism write connected correlation matrix at time separation t in spectral decomposition form

$$C_{ij}(t) = \sum_{n=1}^{\infty} \Psi_i^{(n)} \Psi_j^{(n)*} e^{-m_n t}$$

$$\Psi_i^{(n)} = \langle \text{vac} | \mathcal{O}_i | \mathbf{\Psi}^{(n)} \rangle$$

  $|\mathbf{\Psi}^{(n)}\rangle$: n-th (zero momentum) energy eigenstate

- by suitable diagonalization find masses *and* wave functions of mass screening states
- Praxis: truncated set of operators $\mathcal{O}_i$, $(i = 1, \ldots, N)$
- Hope: lowest lying states ($k = 1, \ldots, r$, with $r << N$) might not be essentially affected by truncation
  $\rightarrow$ solve direct eigenvalue problem for $C(t)$

$$C(t)\Psi^{(n)} = \lambda^{(n)}(t)\Psi^{(n)}$$

- Experience: solution suffers from big systematic truncation errors

- $\rightarrow$ solve generalized eigenvalue equation

$$C(t)\ \Psi^{(n)} = \lambda^{(n)}(t,t_0)C(t_0)\ \Psi^{(n)}$$

  which minimizes the errors (Lüscher/Wolff, Gattringer/Lang)

- Optimised eigenfunctions $\Psi^{(n)}$ (obtained with t_0) give an information about the overlap of the source operators $\mathcal{O}_i$ with the actual eigenstates $|\boldsymbol{\Psi}^{(n)}\rangle$. Due to truncation the eigenvectors $\Psi^{(n)}$ are not supposed to be orthogonal to each other since $C^{-1}(t_0)C(t)$ is not symmetric

- Praxis: decompose matrix $C(t_0)$ using Cholesky decomposition $C(t_0) = LL^T$
  Remaining problem is that of a symmetric matrix
  $\tilde{C}(t,t_0) = L^{-1}C(t)L^{-T}$ with rotated eigenvectors $\tilde{\Psi}^{(n)} = L^T\Psi^{(n)}$

$$\tilde{C}(t,t_0)\tilde{\Psi}^{(n)} = \lambda^{(n)}(t,t_0)\tilde{\Psi}^{(n)}$$

- Extract the masses by fitting the diagonal elements $\mu^{(n)}$ to a cosh forms with t in some plateau region of local effective mass

$$\mu^{(n)}(t,t_0) = \sum_{ij} \tilde{\Psi}_i{}^{(n)}\tilde{C}_{ij}(t)\tilde{\Psi}_j{}^{(n)}.$$

- Wave function $\Psi_j^{(n)}$ shows projection properties of operator $\mathcal{O}_j$

$$\mathcal{O}^{(n)} = \sum_{i=1}^{N} \Psi_i^{(n)}\mathcal{O}_i$$

the coefficients $\Psi_i^{(n)}$ are related to normalized solutions $\tilde{\Psi}_i^{(n)}$

$$\Psi_i^{(n)} = C^{-1/2}(t_0)_{ij}\tilde{\Psi}_j^{(n)}/(\mu^{(n)}(t_0,t_0))^{1/2}$$

- The $\Psi_i^{(n)}$ define weight of chosen operators to optimal operator $\mathcal{O}^{(n)}$ and express the spatial extendedness of eigenstate $|\Psi^{(n)}\rangle$

Used operator set in 2+1

- Only gauge invariant operators used
  classify them with respect to P, C and J
- eigenvalues of C and P have same sign for operators living in same $x-y$-plane in $2+1$ dimensions
- Classification with respect to J requires some care due to discreteness of rotation group
  $SO(2)$ + reflections $\to$ D_4 (dihedral group)
- Outcome: $J(\text{mod } 4)$, $\mu = 3$ correlation direction

0^{++}:	ρ_x^2, $S_{x,1}(l) + S_{x,2}(l)$, $W_{x,1,2}(l) + W_{y,1,2}(l)$
1^{--}:	$V_{x,1}^b(l) + V_{x,2}^b(l)$
2^{++}:	$S_{x,1}(l) - S_{x,2}(l)$, $W_{x,1,2}(l) - W_{y,1,2}(l)$

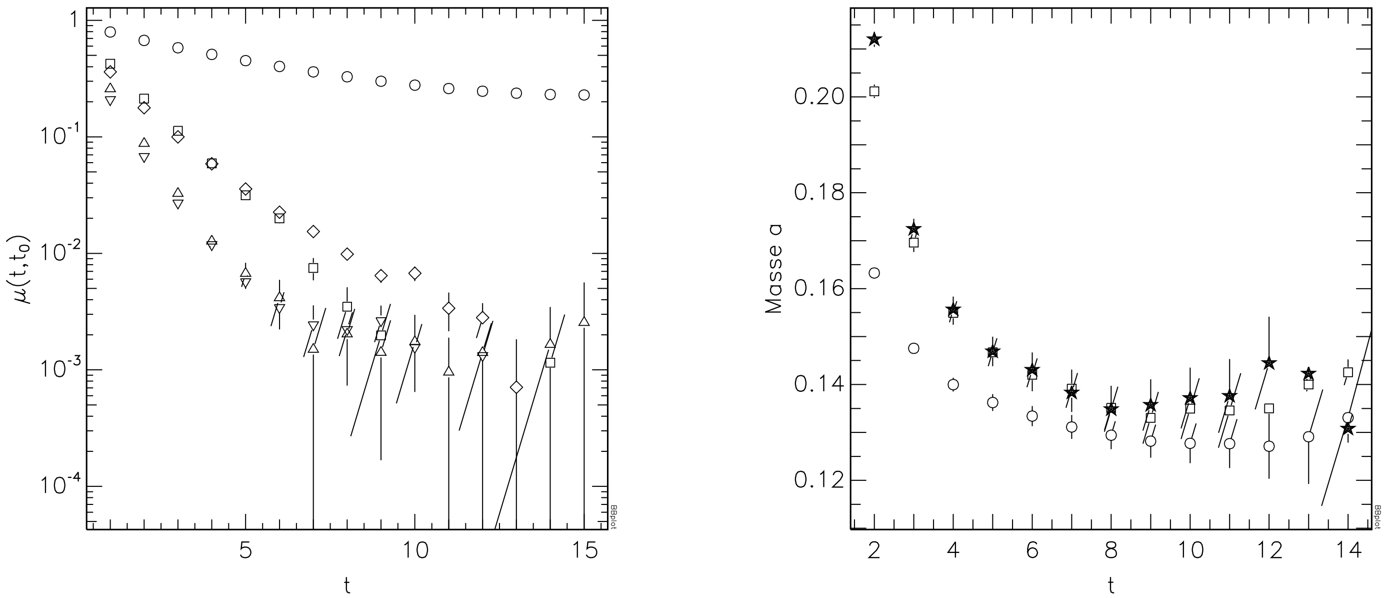
- Notation:

  $W_{x,\mu,\nu}(l)$ quadratic Wilson loop of size $l \times l$

$$S_{x,\mu}(l) = \frac{1}{2}\mathrm{tr}(\Phi_x^+ U_{x,\mu} \dots U_{x+(l-1)\hat{\mu},\mu} \Phi_{x+l\hat{\mu}}) \,,$$
$$V^b_{x,\mu}(l) = \frac{1}{2}\mathrm{tr}(\tau^b \Phi_x^+ U_{x,\mu} \dots U_{x+(l-1)\hat{\mu},\mu} \Phi_{x+l\hat{\mu}}) \,.$$

- restrict $l \leq L/2$
- index i of $\Psi_i^{(n)}$ labels size l in configuration space in 1^{--} and 2^{++} (no Wilson loops included)
  0^{++}: $l = 0$ refers to ρ_x^2; for $l \geq 1$ we have to discriminate between S and W.

Example of such an analysis:
Exponential decay of $\mu^{(n)}(t, t_0)$ for 5 lowest states
0^{++} ground state effective mass obtained from cross correlations ($\bigcirc$), $S_{x,1} + S_{x,2}$ (open box) and ρ_x^2 ($\star$)



Some simulation details

$M_H^* = 70$ GeV ($\lambda_3/g_3^2 \approx 0.09570$) below but near to the endpoint of the phase transition

In symmetric and in Higgs phase, we used lines of constant physics

$$m_3^2(g_3^2)/g_4^2 \approx 0.00023 \quad T \approx 1.02 T_c$$

$$m_3^2(g_3^2)/g_4^2 \approx -0.043 \quad T \approx 0.98 T_c$$

β_G: 8, 12, 16 $\rightarrow$ extrapolate to continuum

Choice of parameters ?: avoid tunnelling between the phases

Lattice size: 30^3, partly 50^3 to increase the range of l in the operator basis

$M_H^* = 100$ GeV ($\lambda_3/g_3^2 \approx 0.1953$) above the endpoint of the phase transition

We studied spectral change with decreasing temperature (increasing β_H) from "symmetric" to "Higgs" side of the crossover line ($\beta_G = 12$ fixed)

From autocorrelation checks for operators of interest frequency of measurements was determined to obtain independent configurations

Error analysis: jackknife

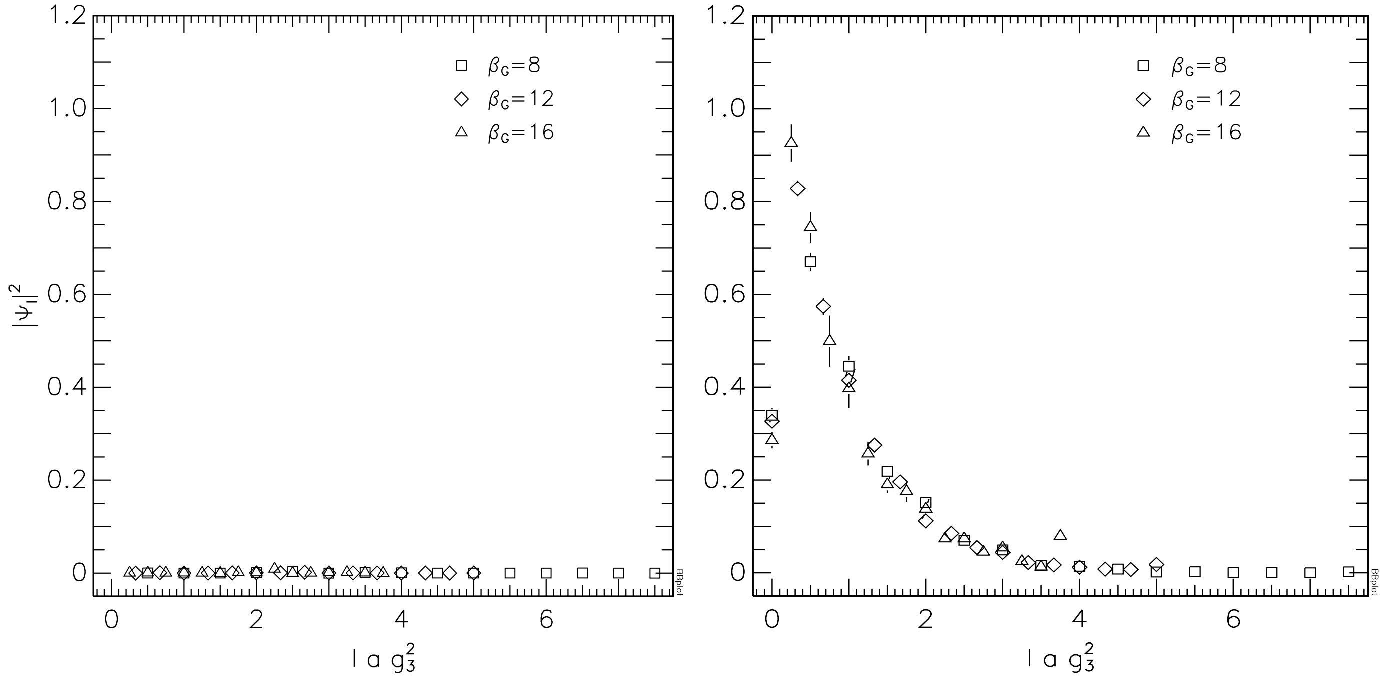
Oxford group results at $M_H^* = 35$ GeV and $M_H^* \approx 120$ GeV

Wave functions and screening masses at $M_H^* = 70$ GeV

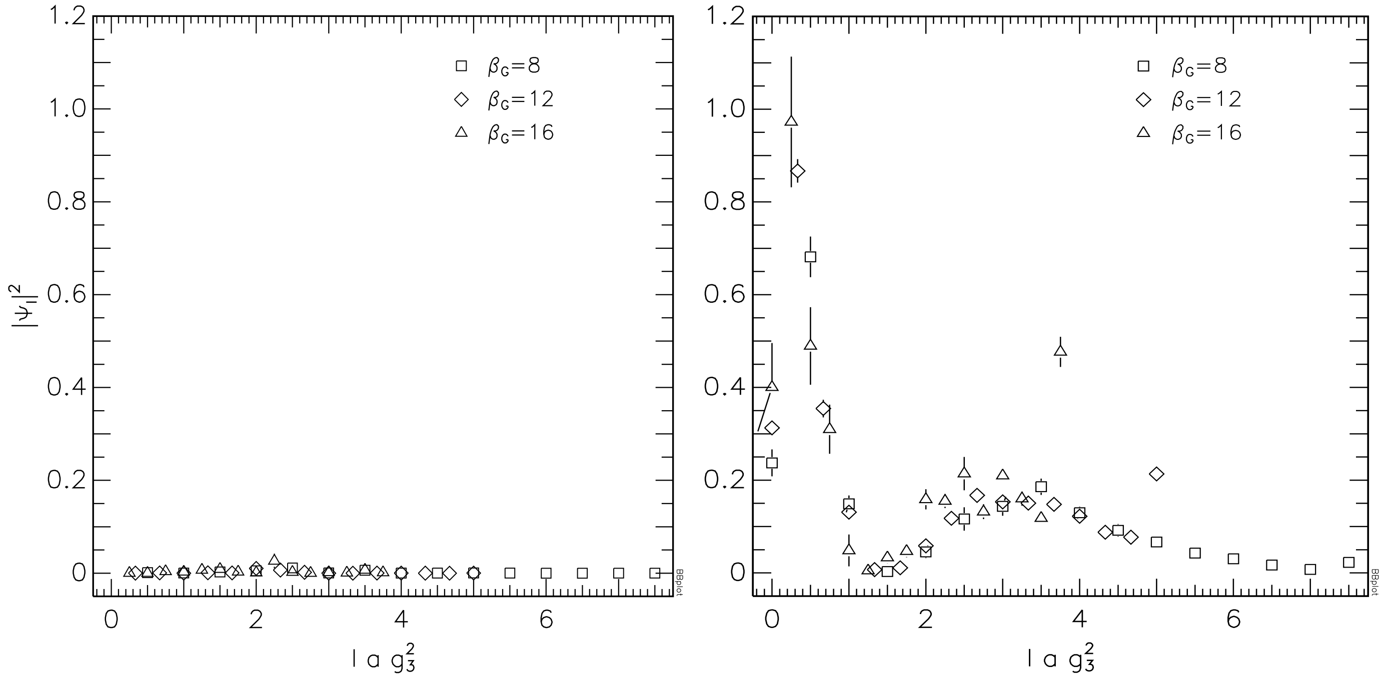
Symmetric phase

- Symmetric phase contains rich spectrum
- 0^{++} channel: Higgs bound states in the ground state and first excitation, followed by a state with pure gauge degree of freedom $\rightarrow$ W-ball in analogy to glueballs in $3D$ $SU(2)$
- 2^{++} channel: W-balls with J=2 are not observed (see Oxford results) since our operator basis does not contain Wilson loops in this channel
- Discussion β_G dependence: 1^{--} wave function for contributions of very large operators vanishes at $\beta_G = 8$
  for higher β_G (smaller volumes) along l.c.p. operator set with $l_{max} = L/2$ does not describe anymore whole wave function
  still fake contributions accumulating in the contribution attributed to longest operator

0^{++}

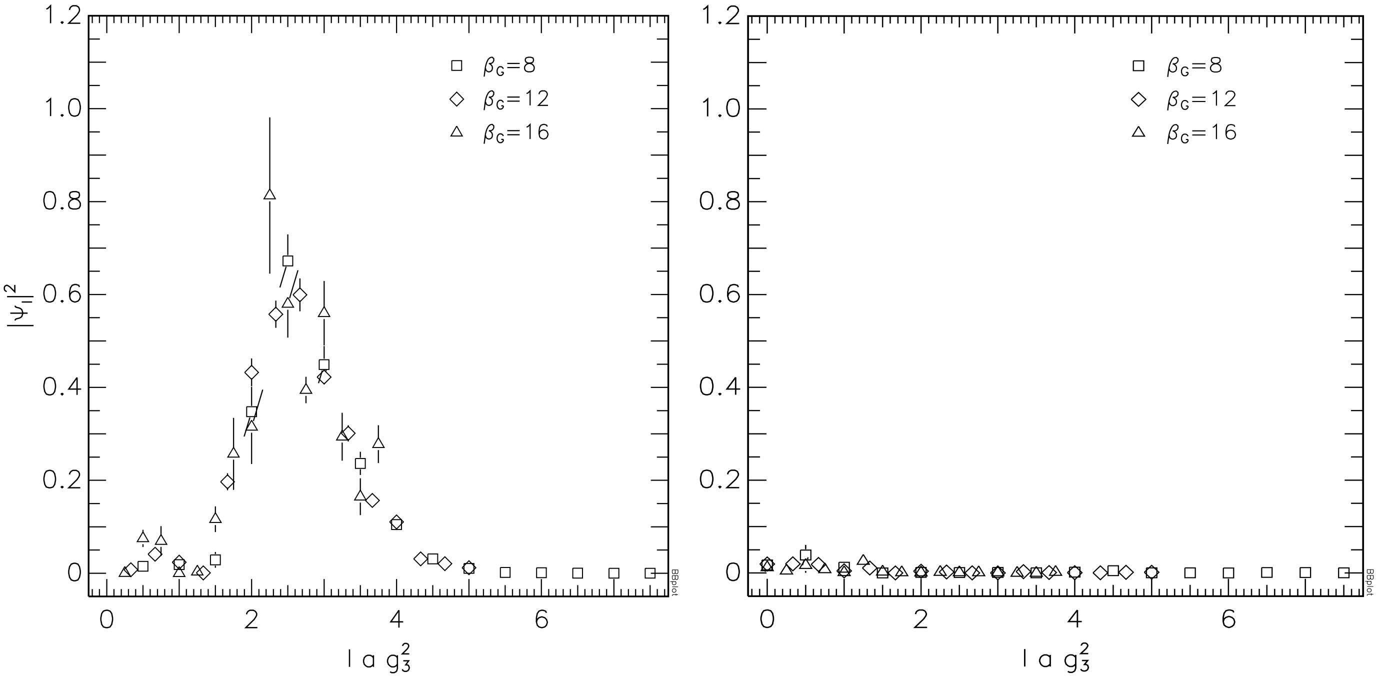


0^{++} ground state: left Wilson loops, right extended Higgs operators $S_{x,\mu}(l)$



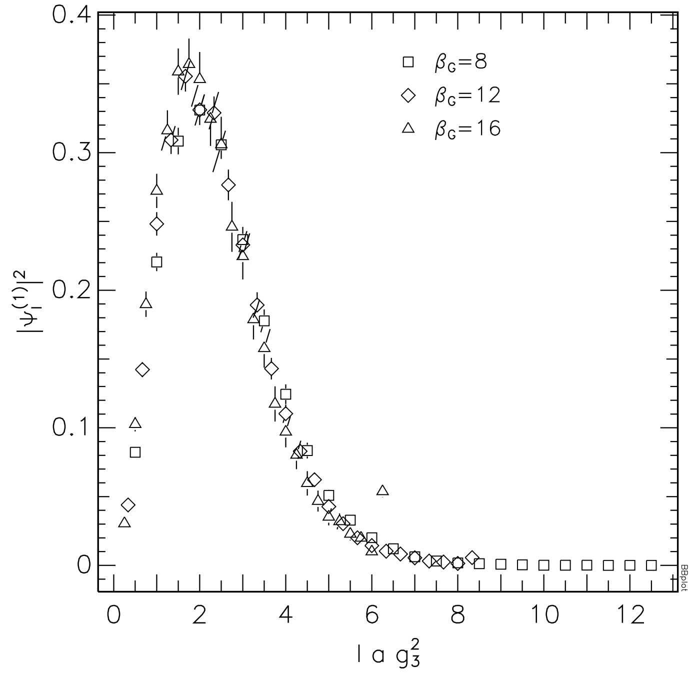
0^{++}: Same for 1st excited states

0^{++}

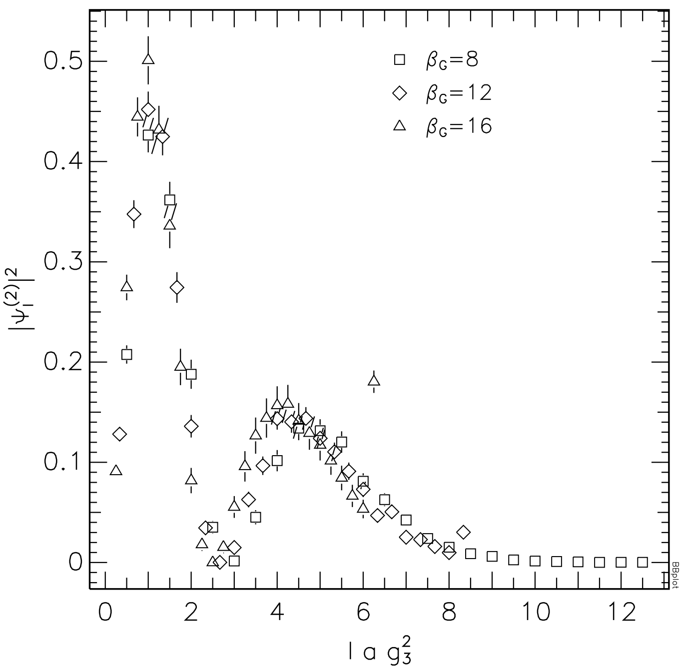


0^{++}: *Same for 2nd excited state*

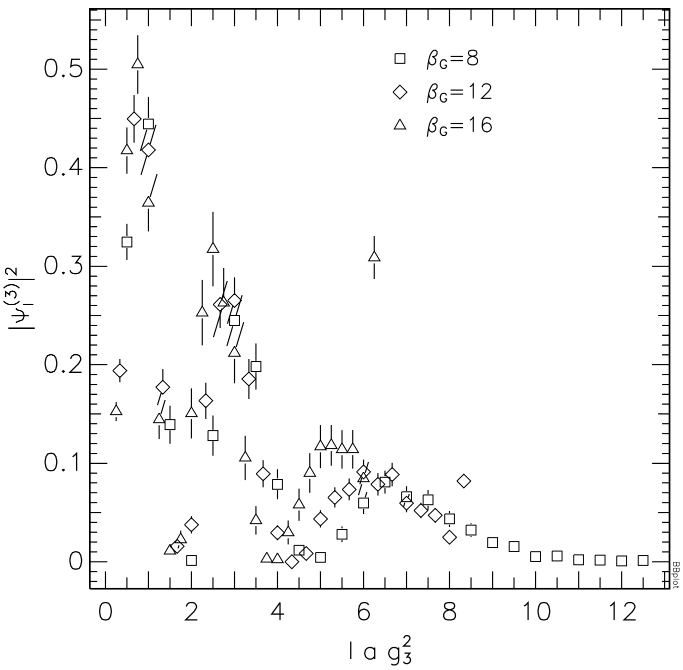
1^{--}



1^{--} ground state

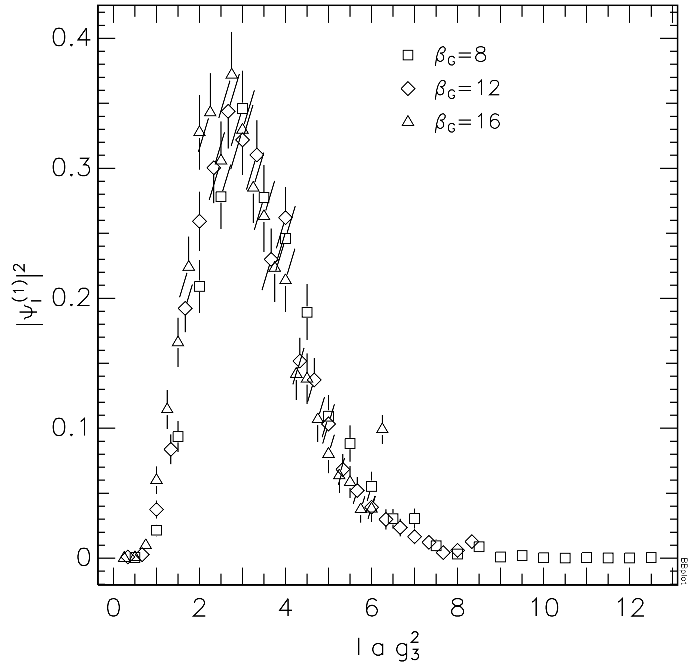


1^{--} 1st excited state

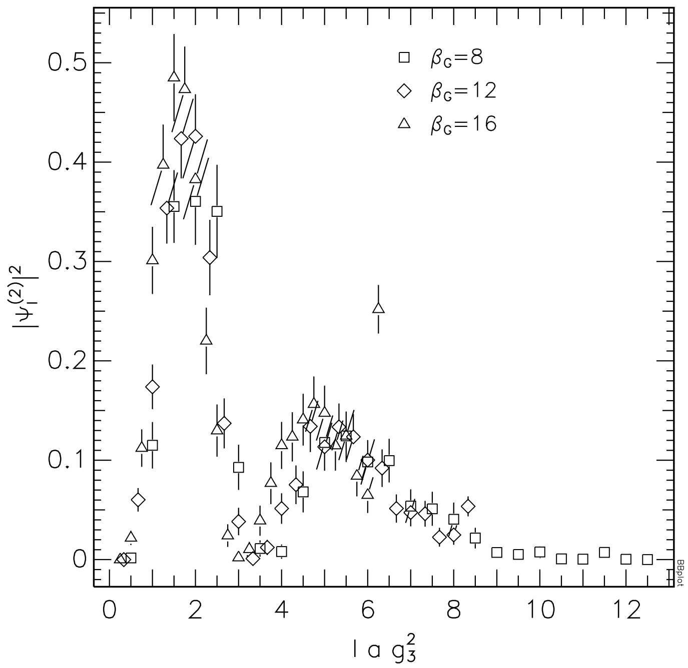


2nd excited state

2^{++}

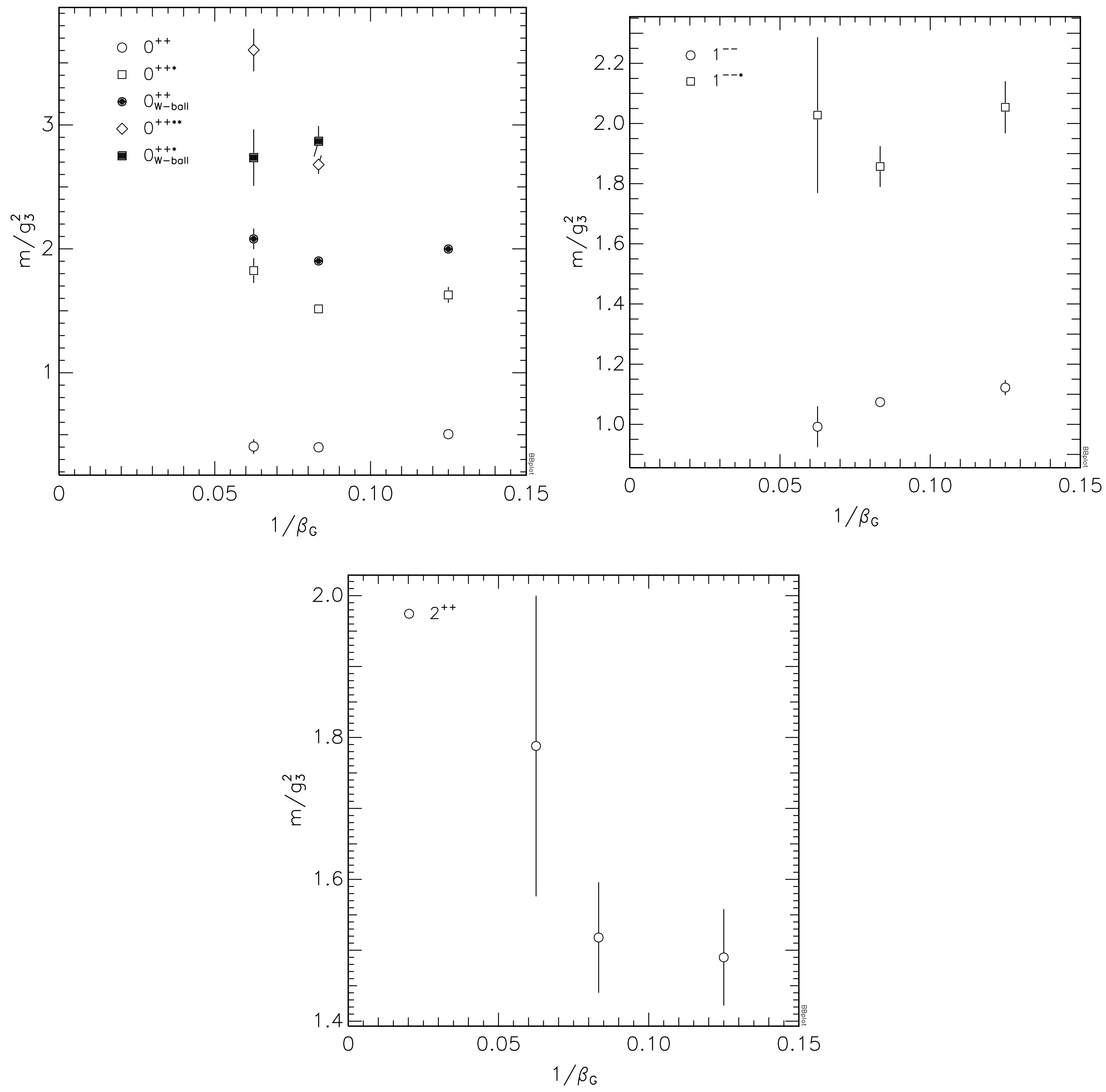


2^{++} *ground state*



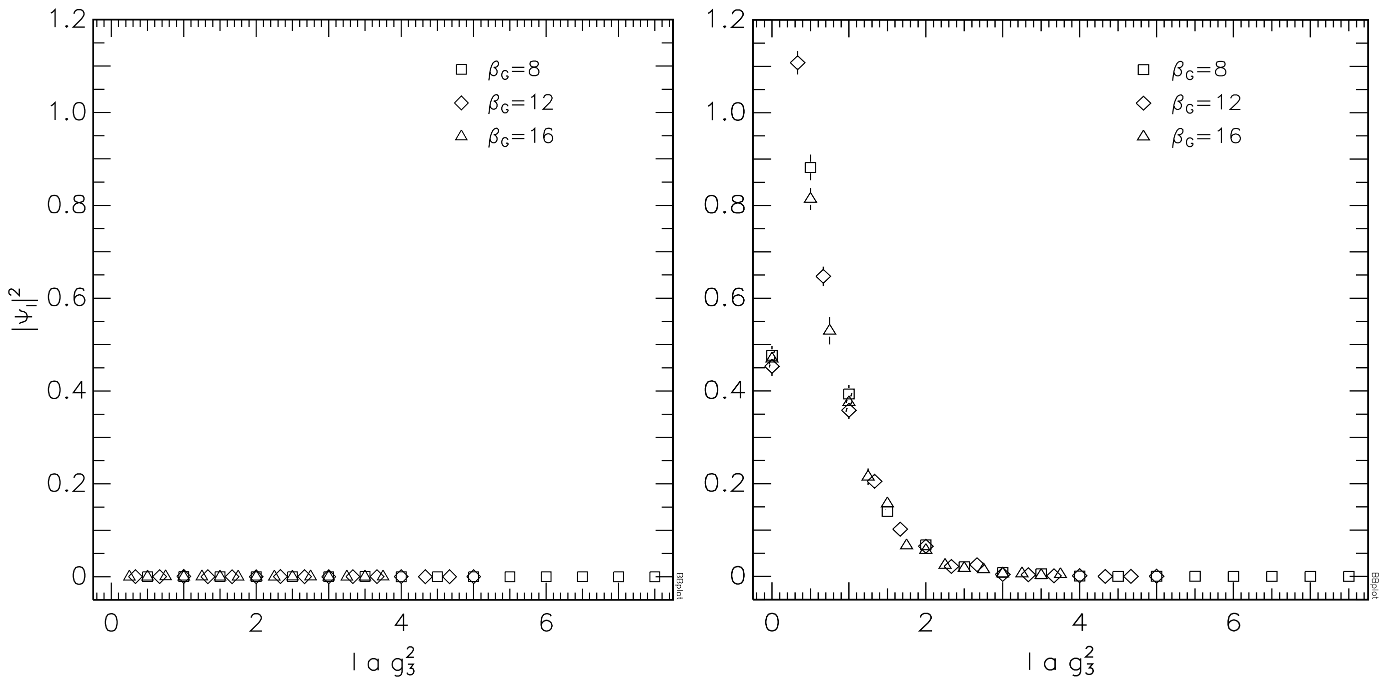
2^{++} *1st excited state*

Masses vs. β_G at l.c.p.

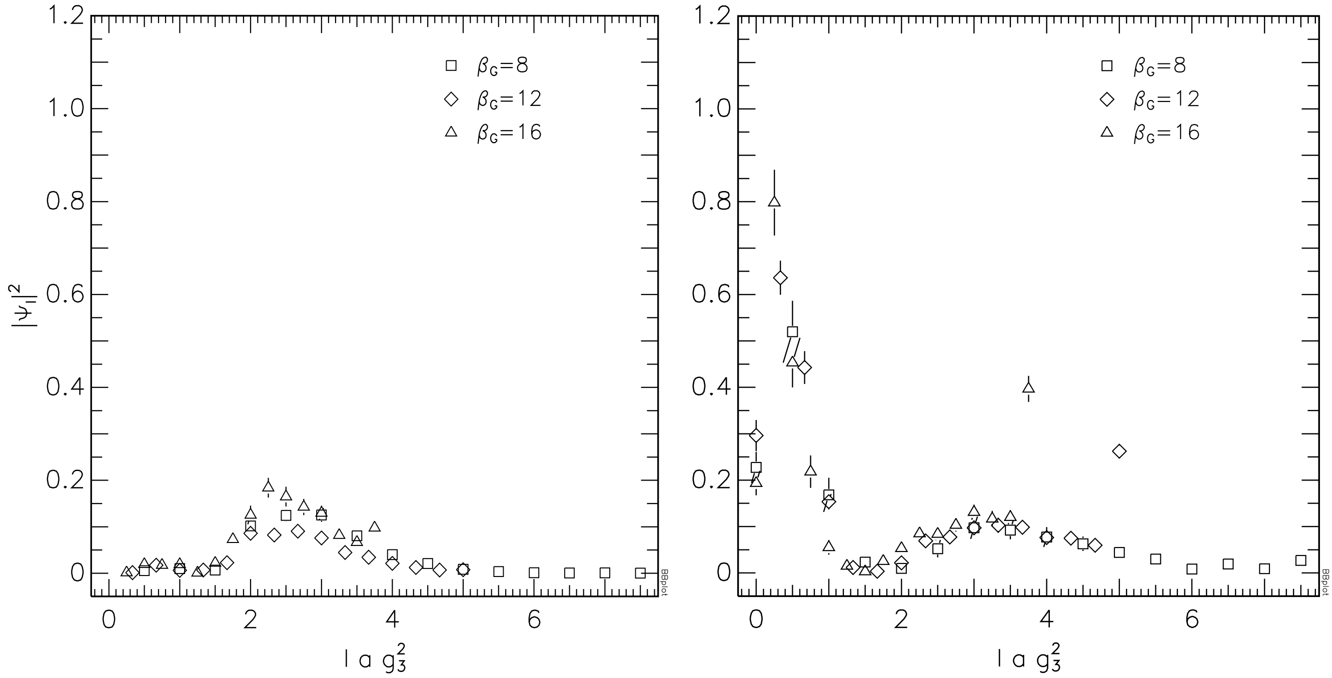


Higgs phase at $M_H^* = 70$ GeV

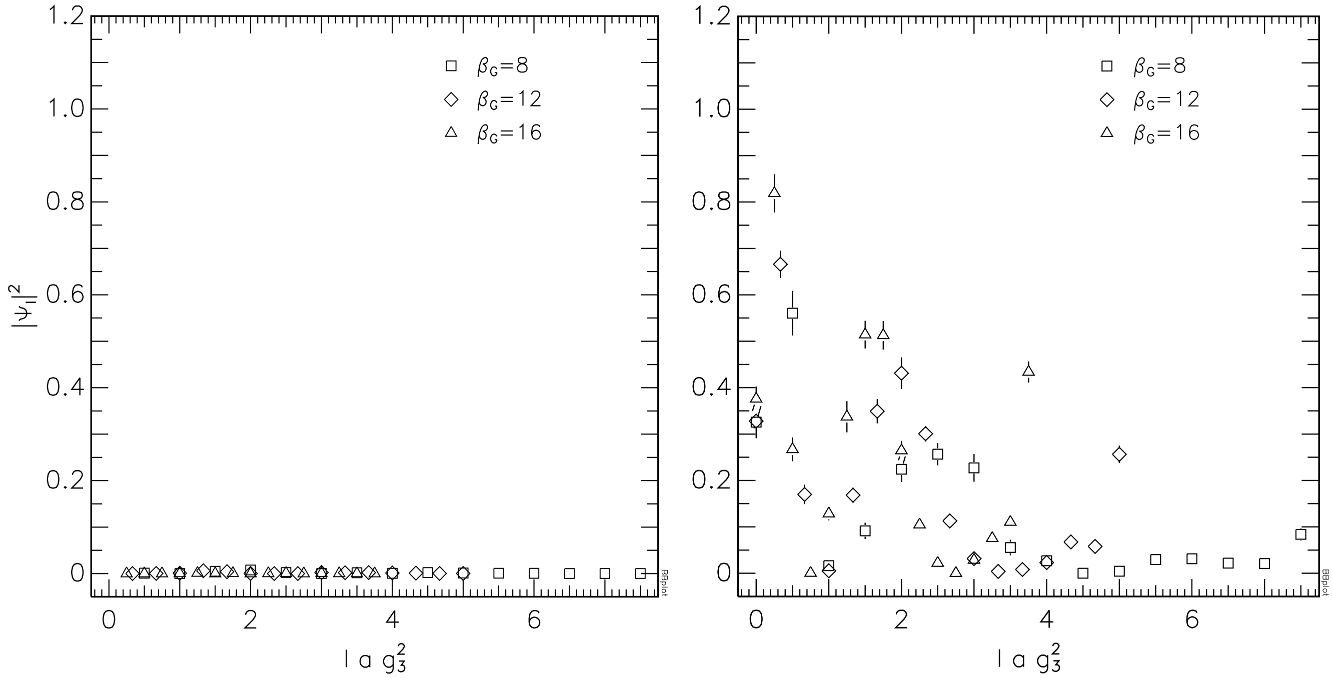
- Pure gauge matter W-ball excitations not expected to be present in spectrum, hypothesis numerically verified at light Higgs masses by the Oxford group
- Now situation changes due to influence of endpoint of 1st order phase transition at $M_H^* = 74$ GeV
- 1^{--} and 2^{++} channels are not qualitatively different from the symmetric phase (2^{++} without Wilson loops)
- 0^{++}: characteristic mixing between W-ball operators (with pure gauge d.o.f.) and operators projecting onto Higgs states
- Ground state: squared wave function similar to that in symmetric phase, no contributions from Wilson loops



0^{++} ground state, left: Wilson loops, right: extended Higgs operators $S_{x,\mu}(l)$

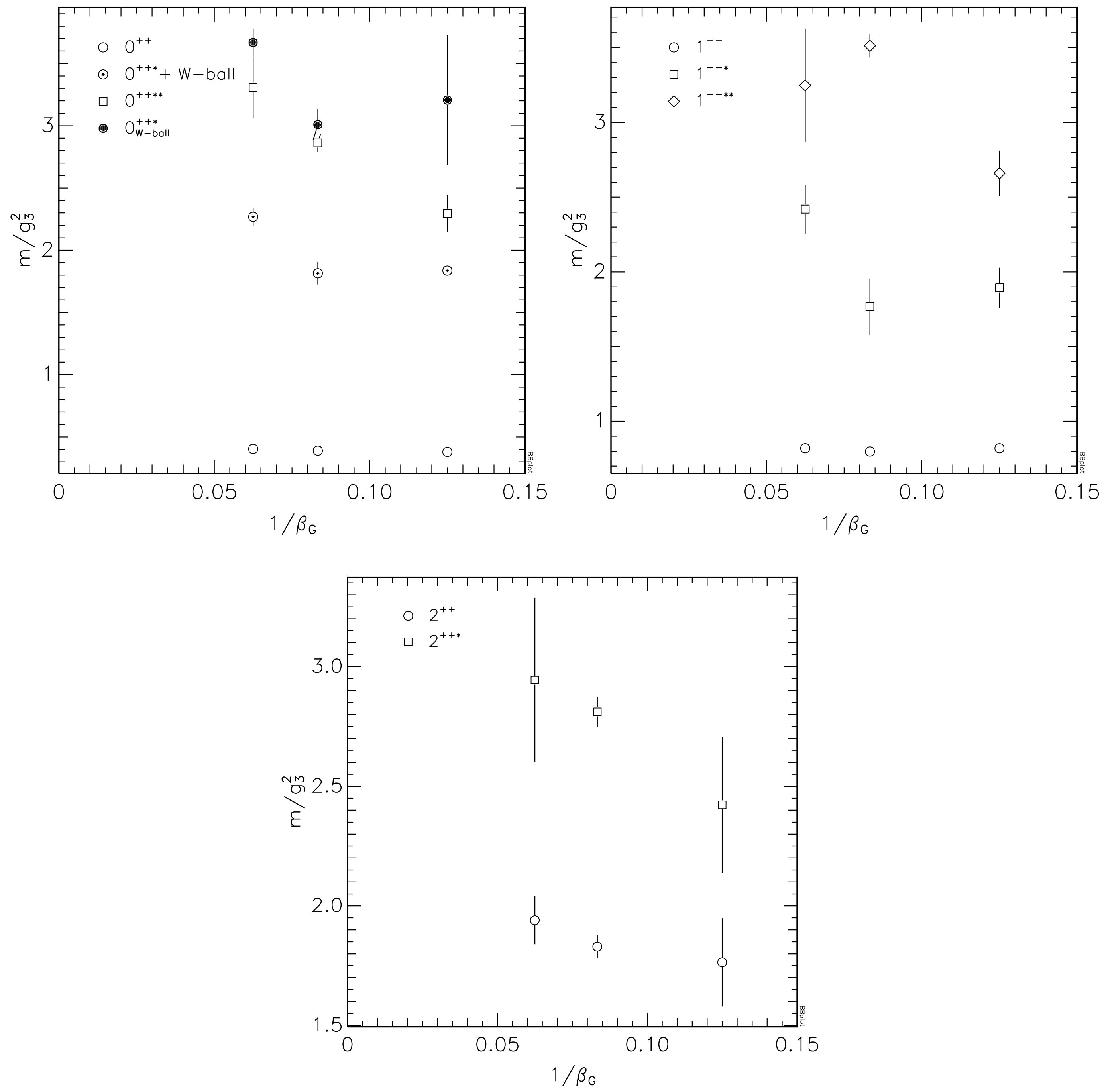


0^{++}: *Same for 1st excited state*



0^{++}: *Same for 2nd excited state*

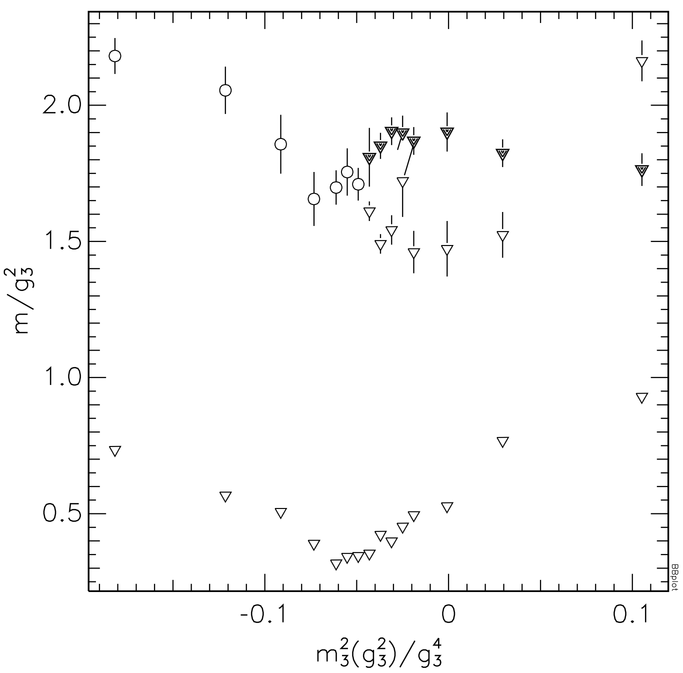
Masses vs. β_G at l.c.p.

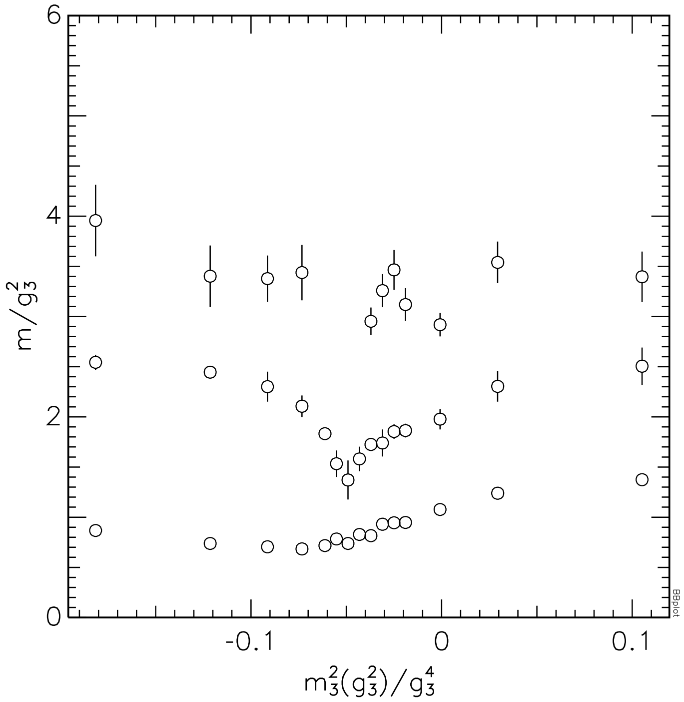


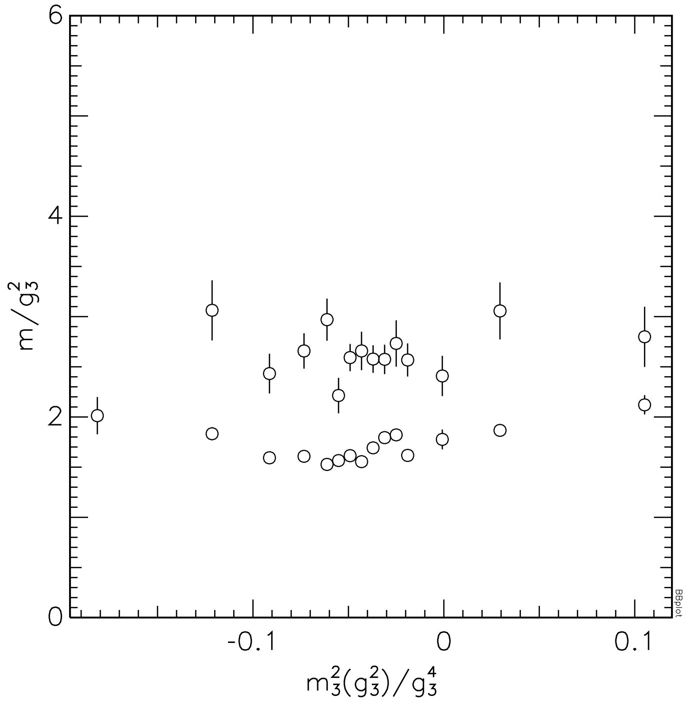
Spectral Change along Rapid Crossover at $M_H^* = 100$ GeV

- Crossover at $M_H^* = 100$ GeV characterized by a smooth change of thermodynamic properties and percolation transition
- Have noticed before that spectra of two phases differ with respect to contributions of Wilson loops (gauge d.o.f.) to excited states in different channels
- Study this change passing crossover line changing β_H (T) at fixed $\beta_G = 12$
- 1^{--} channel is not influenced qualitatively by crossing the crossover line
- Similarities at high temperature side of the crossover compared to $M_H^* = 70$ GeV
- Scalar and gauge sector are approximately decoupled as long as come near to crossover line from the high T side
- lowest mass W-ball state (full triangle) is roughly independent of β_H
- Closer to crossover line first Higgs excitation is moving to W-ball ground state and at a certain point a growing admixture to this Higgs excitation by contributions from Wilson loop operators is observed
- At higher β_H (lower T) the admixture from pure gauge d.o.f. has disappeared again

Spectra change near crossover in channels 0^{++}, 1^{--} and 2^{++}







0^{++}: open triangles - Higgs, full triangles - W-ball,
open circles - Higgs with mixing of gauge d.o.f.

Summary

- We have complemented our numerical study of electroweak p.t. $\rightarrow$ ground state and excited states in three J^{PC} channels
- Interest focused on Higgs screening masses
- Investigation with help of cross correlation matrix functions between different operators in the same channel
- Significantly improved our old ground state mass estimates
- Many techniques known today which allow to construct source operators with improved projection properties
  These improved operators take the extended character of states into account, although in a summary way
- Using cross correlation technique extended character can be specified in a more detailed and systematic way:
  with preselected operator set with same quantum numbers method finds projection strength of each operator onto first few states
- Our used operators classified in a very simple way (extended Higgs links and Wilson loops without internal structure) with clear reference corresponding length in lattice units
- $\rightarrow$ interpret results in terms of configuration space wave function
- Continuum limit of screening masses and wave functions has to be accompanied by correspondingly larger lattices with same physical volume

- $\rightarrow$ would require much higher statistics to reach necessary accuracy and stability in diagonalisation procedure
  $\rightarrow$ reason to include, in next step of method improvement, smearing of fields in the operators and/or construction of blocked operators
- At $M_H^* = 70$ GeV W-ball states of symmetric phase do not disappear in the Higgs phase as long as one keeps near to the transition line, first W-ball state appears as an admixture in the first excited Higgs state
- In the crossover region we could indicate mixing of first W-ball with first Higgs excited state in more detail by following spectral evolution across the crossover line
- Decoupling of gauge and scalar sector was proven over most of parameter space on high temperature side