7. A simple harmonic oscillator of mass $m$ and angular frequency $\omega_0$ is initially at rest at the origin. During the time interval from $0$ to $\tau$ it is subjected to an external force of the form

$$F(t) = c \, t.$$  

Find the motion of the oscillator for $0 < t < \tau$ and $t > \tau$. Identify $\tau$ with the period of the oscillator in the final answer.

8. A particle moves vertically under gravity and a retarding force proportional to the square of its velocity. Given that $v$ is its upward or downward speed, show that

$$\dot{v} = \mp g - kv^2,$$

respectively, where $k$ is a constant.

Find (derive!) that if the particle is moving upwards, its position at time $t$ is given by

$$z(t) = z_0 + \frac{1}{k} \ln \cos \left[ \sqrt{gk} (t_0 - t) \right],$$

where $t_0$ is the time at which it comes to rest and $z_0 = z(t = t_0)$ is its height then.

If its initial velocity at $t = 0$ is $u$ and $z(t = 0) = 0$, find $t_0$ and $z_0$ in terms of $u$.

(See lecture Wednesday, October 29)

(Not that ln always denotes the natural logarithm. You may find the identity $\ln \cos x = -(1/2) \ln(1 + \tan^2 x)$ (for $0 < x < \pi/2$) useful).

9. An oscillator with free oscillation period $\tau$ is critically damped and subjected to a periodic force with the 'saw-tooth' form

$$F(t) = c \left( t - n\tau \right), \quad (n - \frac{1}{2})\tau < t < (n + \frac{1}{2})\tau$$

for integer $n$ with $c$ a constant. Find the ratios of the amplitudes of oscillation at the angular frequencies $2\pi n/\tau$.

(See lecture Wednesday, October 29)