UNIVERSITY OF LEIPZIG
INSTITUTE FOR THEORETICAL PHYSICS
Department: Theory of Elementary Particles

TP2 2015

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List of problems 8

(22. and 23. required, use 24. to collect an additional point)

22. A localized charge distribution is nonvanishing only inside a sphere around some origin.
Show that outside the sphere the expansion of the potential $\Phi(\mathbf{x})$ in rectangular (Cartesian) coordinates ($|\mathbf{x}| = r$)

$$\Phi(\mathbf{x}) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} d^3x' = \frac{1}{4\pi\varepsilon_0} \left[\frac{q}{r} + \frac{\mathbf{p}\cdot\mathbf{x}}{r^3} + \frac{1}{2}\sum_{i,j} Q_{ij}\frac{x_i x_j}{r^5} + \cdots \right]$$

can be obtained by direct Taylor series expansion of $1/|\mathbf{x}-\mathbf{x}'|$. Here q, $\mathbf{p}$ and Q_{ij} are the total charge, electric dipole moment and the traceless quadrupole moment tensor, respectively.

23. A localized distribution of charge has a charge density

$$\rho(\mathbf{r}) = \frac{1}{64\pi} r^2 \mathrm{e}^{-r} \sin^2\theta \, .$$

(a) Make a multipole expansion of the potential due to this charge density and determine all the nonvanishing multipole moments. Write down the potential at large distances as a finite expansion in Legendre polynomials.
(b) Determine the potential explicitly at any point in space, and show that near the origin, correct to r^2 inclusive,

$$\Phi(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\left(\frac{1}{4} - \frac{r^2}{120}P_2(\cos\theta)\right) .$$

24. Consider a sphere of radius R which completely encloses a localized charge distribution $\rho(\mathbf{x})$.
Show that the volume integral of the electric field over the sphere becomes

$$\int_{r<R} \mathbf{E}(\mathbf{x})\, d^3x = -\frac{\mathbf{p}}{3\varepsilon_0}$$

where $\mathbf{p}$ is the electric dipole moment of the charge distribution with respect to the center of the sphere.
Hint: See e.g. Jackson, Chapter Multipole expansion, perform the intermediate steps.