UNIVERSITY OF LEIPZIG
INSTITUTE FOR THEORETICAL PHYSICS
Department: Theory of Elementary Particles

TP2 2015

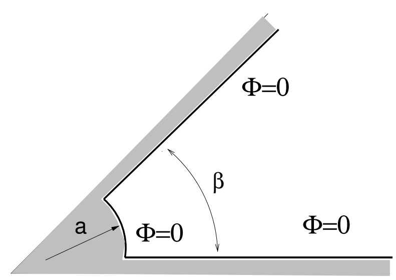
Lecturer: PD Dr. A. Schiller

List of problems 6

(16. and 17. required, use 18. to collect an additional point)

16. A hollow cube has conducting walls defined by six planes $x = 0$, $y = 0$, $z = 0$, and $x = a$, $y = a$, $z = a$. The walls $z = 0$ and $z = a$ are held at a constant potential V. The other four sides are at zero potential.
(a) Find the potential $\Phi(x, y, z)$ at any point inside the cube.
(b) Find the surface–charge density on the surface $z = a$.
Hint: Consider the two non-zero constant potentials on the sides by superposition of problems with only one non-zero potential on a side (see lecture).

17. The two–dimensional region $\rho \geq a$, $0 \leq \varphi \leq \beta$, is bounded by conducting surfaces at $\varphi = 0$, $\rho = a$, and $\varphi = \beta$ held at zero potential, as indicated in the sketch. At large ρ the potential is determined by some configuration of



charges and/or conductors at fixed potentials.
(a) Write down a solution for the potential $\Phi(\rho, \varphi)$ that satisfies the boundary conditions for finite ρ.
(b) Keeping only the lowest nonvanishing terms, calculate the electric field components E_ρ and E_φ and also the surface–charge densities $\sigma(\rho, 0)$, $\sigma(\rho, \beta)$, and $\sigma(a, \varphi)$ on the three boundary surfaces.
(c) Consider $\beta = \pi$ (a plane conductor with a half–cylinder of radius a on it). Show that far from the half–cylinder the lowest order terms of part (b) give a uniform electric field normal to the plane. Sketch the charge density on and in the neighborhood of the half-cylinder.

18. The potential at position $\mathbf{x}$ of a point-like electric dipole $\mathbf{p}$ located at position $\mathbf{x}'$ is given by (compare lecture 3.1.4 for dipole at the origin)

$$\Phi(\mathbf{x}) = \frac{1}{4\pi\varepsilon_0} \frac{\mathbf{p} \cdot (\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3}, \quad \mathbf{x} \neq \mathbf{x}'.$$

Suppose the dipole is fixed at a distance z_0 along the z-axis and at an orientation θ with respect to that axis (i.e., $\mathbf{p} \cdot \mathbf{e}_z = p\cos\theta$). Suppose the xy plane is a conductor at zero potential.
Give the charge density $\sigma(x, y)$ on the conducting place induced by the dipole.
Calculate the total induced charge on the conducting plane.
Hint: To satisfy the boundary conditions use as image an image point-like dipole. To find the correct orientation of that dipole vector, consider a dipole as a pair of a negative and a positive charge of same strength q in the limit of a vanishing length vector pointing from $-q$ to q.

Voluntary:
Find the positions of vanishing σ on the x-axis and y-axis and sketch the behavior of the distribution along those axes.
What happens to those positions when the dipole is orientated parallel or perpendicular to the conducting plane?
What is the curve of vanishing charge density $\sigma(x, y) = 0$ on the conducting plane for $\theta < \pi/2$ and $\theta = \pi/2$?