UNIVERSITY OF LEIPZIG
INSTITUTE FOR THEORETICAL PHYSICS
Department: Theory of Elementary Particles

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List of problems 4

10. The time-averaged potential of a neutral hydrogen atom is given by

$$\Phi = \frac{e}{4\pi\epsilon_0}\frac{1}{r}\exp(-\alpha r)\left(1+\frac{\alpha r}{2}\right)$$

where e is the magnitude of the electronic charge, and $\alpha^{-1} = a_0/2$, a_0 being the Bohr radius.
Calculate the vector of the electric field $\mathbf{E}$.
Find the distribution of charge (both continuous and discrete) that will give this potential.
Sketch the charge density.
Check that the total charge vanishes.

11. Derive the interaction energy from the electric fields of two charges q_1 and q_2 located at positions $\mathbf{x_1}$ and $\mathbf{x_2}$, respectively, as

$$W_{\rm int} = \frac{q_1 q_2}{16\pi^2\varepsilon_0}\int \frac{(\mathbf{x}-\mathbf{x}_1)\cdot(\mathbf{x}-\mathbf{x}_2)}{|\mathbf{x}-\mathbf{x}_1|^3|\mathbf{x}-\mathbf{x}_2|^3}\, d^3x$$

where the integration has to be taken over the whole space (see lecture). By changing the integration variable $\mathbf{x}$ to $\boldsymbol{\rho} = (\mathbf{x}-\mathbf{x}_1)/|\mathbf{x}_1-\mathbf{x}_2|$ show that

$$W_{\rm int} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|\mathbf{x}_1-\mathbf{x}_2|}\,.$$

In other words, check by explicit calculation that the following relation holds ($|\boldsymbol{\rho}| = \rho$)

$$\int \frac{\boldsymbol{\rho}\cdot(\boldsymbol{\rho}+\mathbf{n})}{\rho^3\,|\boldsymbol{\rho}+\mathbf{n}|^3}\, d^3\rho = 4\pi\,, \quad \mathbf{n} = \frac{\mathbf{x}_1-\mathbf{x}_2}{|\mathbf{x}_1-\mathbf{x}_2|}\,.$$

Here $\mathbf{n}$ is the unit vector in direction $\mathbf{x}_1-\mathbf{x}_2$.

12. A conducting sphere with total charge Q is cut into half. What force must be used to hold the two halves together? What is the direction of that force?
Hint: Use that the force exerting on a surface element da is

$$dF = \frac{\sigma^2}{2\varepsilon_0}da$$

where σ is the surface charge density.