UNIVERSITY OF LEIPZIG
INSTITUTE FOR THEORETICAL PHYSICS
Department: Theory of Elementary Particles

TP2 2015

Lecturer: PD Dr. A. Schiller

List of problems 2

4. Calculate the divergence and the curl of the vectors

$$(\mathbf{a}\cdot\mathbf{r})\,\mathbf{b}\,, \quad (\mathbf{a}\cdot\mathbf{r})\,\mathbf{r}\,, \quad \mathbf{a}\times\mathbf{r}\,, \quad \phi(r)\,(\mathbf{a}\times\mathbf{r})\,, \quad \mathbf{r}\times(\mathbf{a}\times\mathbf{r})\,,$$

where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors and $\phi(r)$ is a scalar function of the magnitude r of the radius-vector $\mathbf{r}$.

5. Calculate the closed surface integrals

$$\oint_S \mathbf{r}\,(\mathbf{a}\cdot\mathbf{n})\,da\,, \quad \oint_S (\mathbf{a}\cdot\mathbf{r})\,\mathbf{n}\,da$$

using a variant of the divergence theorem, $\mathbf{n}$ is the outward pointing unit vector of the surface S around the volume V.

6. Using Dirac delta functions in the appropriate coordinates, express the following charge distributions as three-dimensional charge densities $\rho(\mathbf{x})$.

   (a) In cylindrical coordinates, a charge λ per unit length uniformly distributed over a cylindrical surface of radius b.

   (b) In cylindrical coordinates, a charge Q spread uniformly over a flat circular disc of negligible thickness and radius R.

   (c) The same as part (b), but using spherical coordinates.