UNIVERSITY OF LEIPZIG
INSTITUTE FOR THEORETICAL PHYSICS
Department: Theory of Elementary Particles

TP2 2015

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List of problems 1

1. Verify the identities

$$\begin{aligned}(\mathbf{A}\times\mathbf{B})\cdot(\mathbf{C}\times\mathbf{D}) &= (\mathbf{A}\cdot\mathbf{C})\,(\mathbf{B}\cdot\mathbf{D}) - (\mathbf{A}\cdot\mathbf{D})\,(\mathbf{B}\cdot\mathbf{C})\\ (\mathbf{A}\times\mathbf{B})\times(\mathbf{C}\times\mathbf{D}) &= (\mathbf{A}\cdot(\mathbf{B}\times\mathbf{D}))\,\mathbf{C} - (\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}))\,\mathbf{D}\\ &= (\mathbf{A}\cdot(\mathbf{C}\times\mathbf{D}))\,\mathbf{B} - (\mathbf{B}\cdot(\mathbf{C}\times\mathbf{D}))\,\mathbf{A}\end{aligned}$$

2. (i) Write in invariant vectorial form

$$\varepsilon_{inl}\,\varepsilon_{irs}\,\varepsilon_{lmp}\,\varepsilon_{stp}\,a_n\,a_r\,b_m\,c_t\,.$$

(ii) Using the totally antisymmetric tensor ε_{ijk} write the product

$$(\mathbf{a}\cdot[\mathbf{b}\times\mathbf{c}])\,(\mathbf{a}'\cdot[\mathbf{b}'\times\mathbf{c}'])$$

as sum of terms which contains only scalar products of the appearing vectors.

3. (i) Using Cartesian coordinates x, y, z, calculate

$$\mathrm{grad}\,r\,,\quad \mathrm{div}\,\mathbf{r}\,,\quad \mathrm{curl}\,\mathbf{r}\,,\quad \mathrm{grad}\,(\mathbf{c}\cdot\mathbf{r})\,,\quad (\mathbf{c}\cdot\boldsymbol{\nabla})\mathbf{r}\,,$$

where $\mathbf{r}$ is the radius vector, $\mathbf{c}$ is a constant vector.
Hint: For the magnitude of the radius vector use $r = |\mathbf{r}| = \sqrt{\mathbf{r}\cdot\mathbf{r}} = \sqrt{x^2+y^2+z^2}$.

(ii) Using the differential vector operator $\boldsymbol{\nabla}$ and the rules of differentiation and multiplication of vectors (without using Cartesian components) show that the following identities are valid

$$\begin{aligned}\mathrm{grad}\,(\varphi\psi) &= \varphi\,\mathrm{grad}\,\psi + \psi\,\mathrm{grad}\,\varphi\,,\\ \mathrm{div}\,(\varphi\mathbf{A}) &= \varphi\,\mathrm{div}\,\mathbf{A} + \mathbf{A}\cdot\mathrm{grad}\,\varphi\\ \mathrm{curl}\,(\varphi\mathbf{A}) &= \varphi\,\mathrm{curl}\,\mathbf{A} - \mathbf{A}\times\mathrm{grad}\,\varphi\end{aligned}$$