Quark propagator estimation and static-light mesons

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Various approches to estimate propagator:

1. Naive way:

$$\eta^n_{i\alpha a} = D^{-1}_{i\alpha a\, j\beta b}\, \chi^n_{j\beta b} \quad (n = 1, \ldots, N)\,.$$

$$D^{-1}_{i\alpha a\, k\gamma c} \approx \frac{1}{N} \eta^n_{i\alpha a} \chi^{n\dagger}_{k\gamma c}\,,$$

where the χ^n are random source vectors.

2. Hybrid methods (eigenvalues, dilution, ...)

TrinLat Collaboration (Justin Foley et al.), hep-lat/0505023

A. Duncan, E. Eichten, Phys.Rev.D65:114502,2002

3. Maximal variance reduction (domain decomposition & pseudofermions)

UKQCD Collaboration (Chris Michael et al.), Nucl.Phys.Proc.Suppl.60A:55-60,1998

Our approach:

- Split the lattice in two disjoint regions: $\Lambda = R \,\dot{\cup}\, S$
- Block decomposition of D: $(\{r, r', \ldots\} \in R$; $\{s, s', \ldots\} \in S)$

$$D_{ij} = \begin{pmatrix} (\bar{D})_{rr'} & (\tilde{D})_{rs'} \\ (\tilde{D})_{sr'} & (\bar{D})_{ss'} \end{pmatrix}_{ij} .$$

- Then

$$\chi_r^n = D_{rj}\eta_j^n = \bar{D}_{rr'}\eta_{r'}^n + \tilde{D}_{rs}\eta_s^n .$$

- Solve for η_r^n and insert in naive estimation

$$\begin{aligned} D_{rs}^{-1} \quad & \approx \frac{1}{N}\eta_r^n \chi_s^{n\dagger} \\ & \approx \frac{1}{N}\left[\bar{D}_{rr'}^{-1}\left(\chi_{r'}^n - \tilde{D}_{r's'}\eta_{s'}^n\right)\right]\chi_s^{n\dagger} \\ & \approx -\frac{1}{N}\left(\bar{D}_{rr'}^{-1}\tilde{D}_{r's'}\eta_{s'}^n\right)\chi_s^{n\dagger} . \end{aligned}$$

- Resubstituting original η^n:

$$D^{-1}_{rs} \approx -\frac{1}{N}\bar{D}^{-1}_{rr'}\tilde{D}_{r's'}D^{-1}_{s'j}\chi^n_j\chi^{n\dagger}_s$$

$$\overset{N\to\infty}{=} -\bar{D}^{-1}_{rr'}\tilde{D}_{r's'}D^{-1}_{s's} \qquad \text{(exact !)}$$

Last expression can also be obtained from path integral and from linear algebra (Schur complement).

- Using γ_5-hermiticity:

$$\begin{aligned} D^{-1}_{rs} &= -\bar{D}^{-1}_{rr'}\tilde{D}_{r's'}\gamma_5 D^{-1\dagger}_{ss'}\gamma_5 \\ &\approx -\frac{1}{N}\bar{D}^{-1}_{rr'}\tilde{D}_{r's'}\gamma_5\left(D^{-1}_{sj}\chi^n_j\chi^{n\dagger}_{s'}\right)^\dagger\gamma_5 \\ &\approx -\frac{1}{N}\left(\bar{D}^{-1}_{rr'}\tilde{D}_{r's'}\gamma_5\chi^n_{s'}\right)\left(D^{-1}_{sj}\chi^n_j\right)^\dagger\gamma_5 \\ &\approx -\frac{1}{N}\psi^n_r\eta^{n\dagger}_s\gamma_5\,. \end{aligned}$$

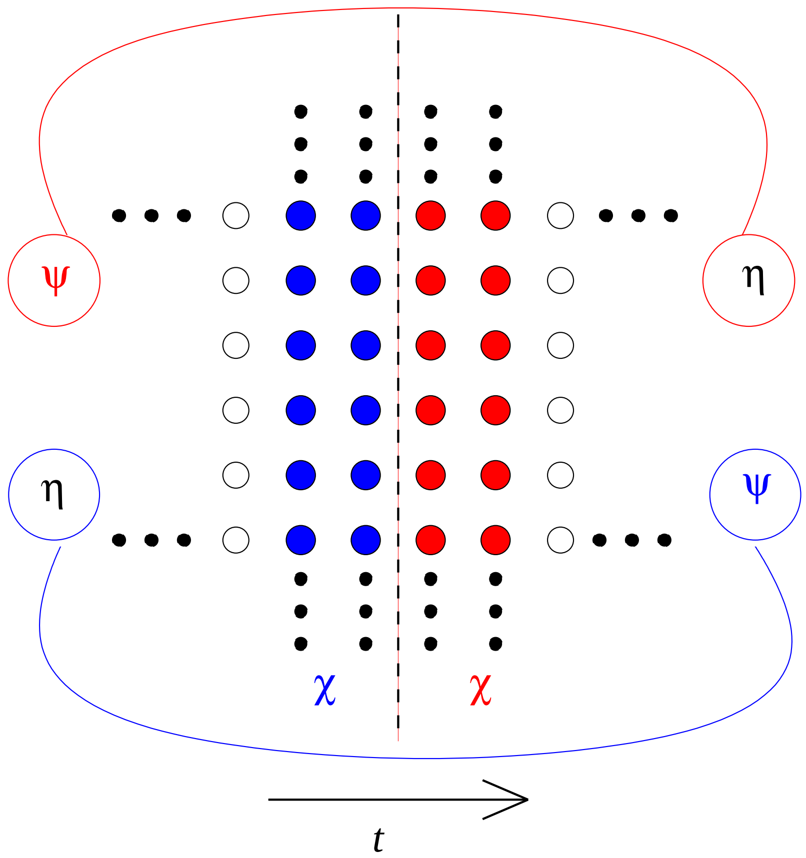
Our setup:

$L^3 \times T = 12^3 \times 24$

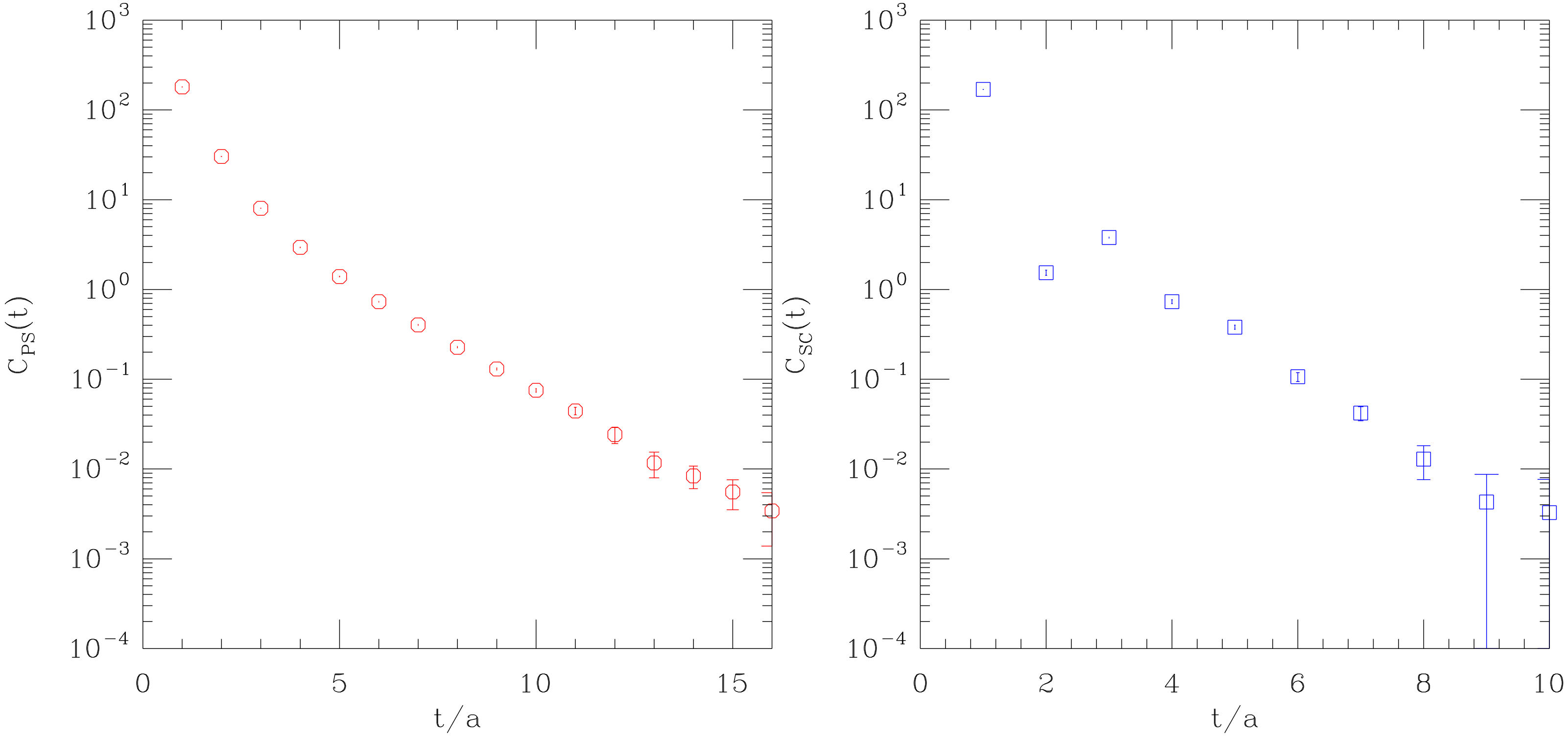
$\beta = 7.90$

$a^{-1} = 1340\text{MeV}$

$N = 48$

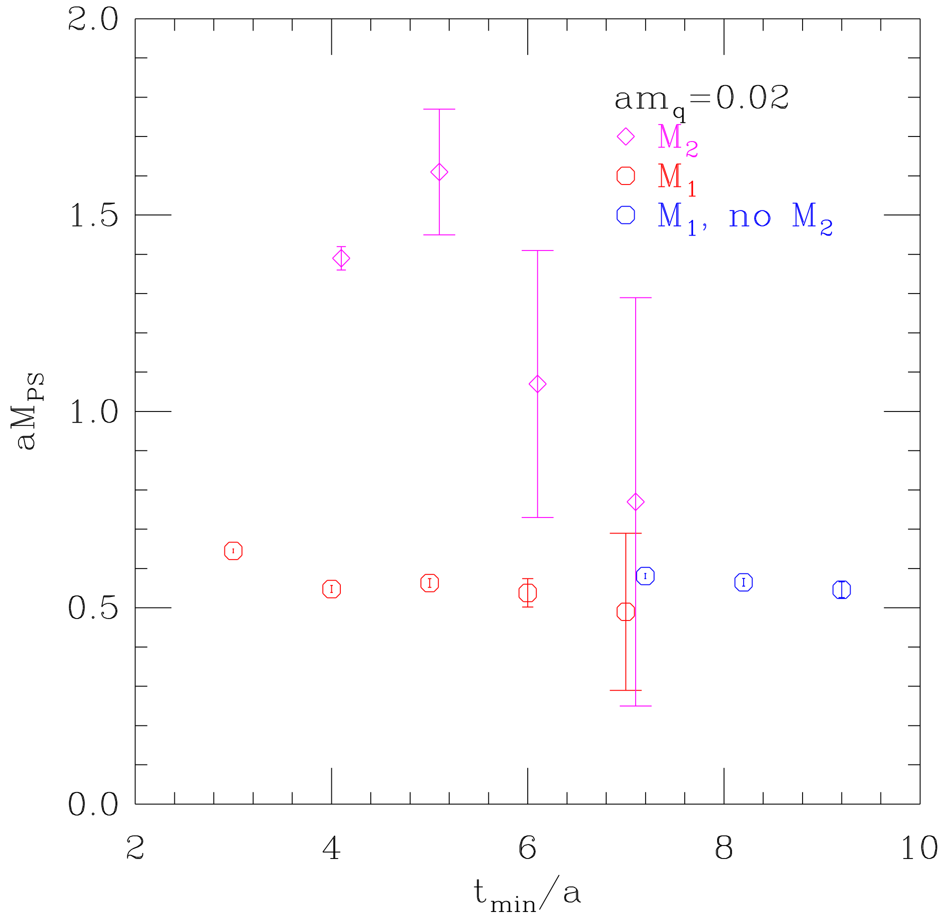


Pseudoscalar and scalar correlator



Using just one estimate and one local source !

Mass for the pseudoscalar from a two-exponential fit.



Future applications:

- Static-light excited states & NRQCD-light ground and excited states
- Hadronic decays
- Disconnected diagrams
- 3-point functions (extended quark propagator)