Transverse spin densities in lattice QCD

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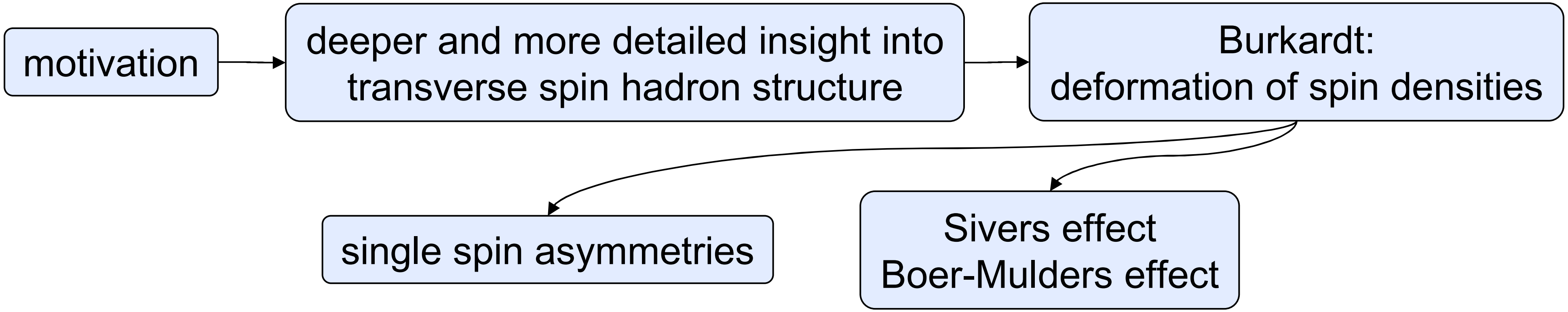
(QCDSF/UKQCD-collaborations)

supported by

Deutsche Forschungsgemeinschaft DFG

Overview



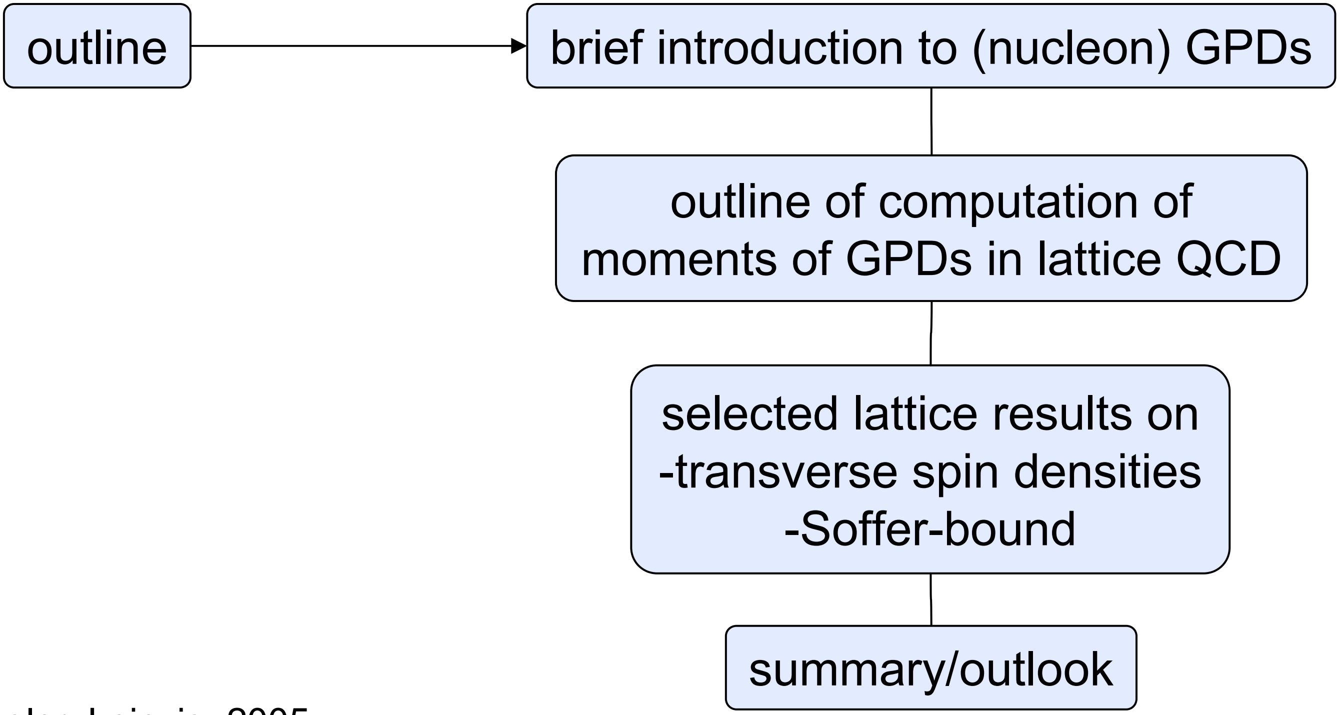
motivation

deeper and more detailed insight into transverse spin hadron structure

Burkardt:
deformation of spin densities

single spin asymmetries

Sivers effect
Boer-Mulders effect



outline

brief introduction to (nucleon) GPDs

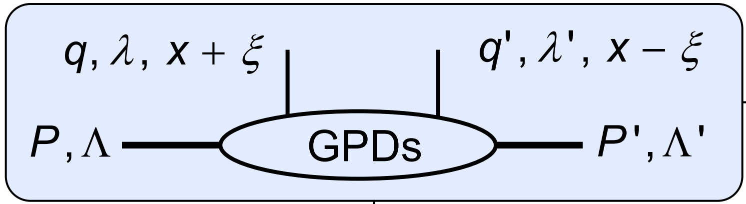
outline of computation of moments of GPDs in lattice QCD

selected lattice results on
-transverse spin densities
-Soffer-bound

summary/outlook

Brief introduction to GPDs

Müller,Robaschik,Geyer,
Dittes,Horejsi,1994
Ji, 1997, Radyushkin, 1997



$t = (\Delta \equiv P' - P)^2 \,\hat{=}$ momentum transfer squared

$\xi = -n \cdot \Delta / 2 \,\hat{=}$ longitudinal momentum transfer

8 real functions needed for a complete description of the nucleon quark structure at twist 2
(M. Diehl, EPJ C19, 2001)

$H(x,\xi,t), E$
$\widetilde{H}, \widetilde{E}$
$H_T, \overline{E}_T, \widetilde{H}_T, \widetilde{E}_T$

$$\int \frac{d\eta}{4\pi} e^{i\eta x} \langle P' | \bar{q}(-\frac{\eta n}{2}) \gamma^{\mu} \mathcal{U} q(\frac{\eta n}{2}) | P \rangle = \overline{U}(P') \left(\gamma^{\mu} H(x,\xi,\Delta^2) + i \frac{\sigma^{\mu\nu}\Delta_{\nu}}{2M} E(x,\xi,\Delta^2) \right) U(P')$$

vector

$$\int \frac{d\eta}{4\pi} e^{i\eta x} \langle P' | \bar{q}(-\frac{\eta n}{2}) \gamma^{\mu} \gamma^{5} \mathcal{U} q(\frac{\eta n}{2}) | P \rangle = \overline{U}(P') \left(\gamma^{\mu} \gamma^{5} \widetilde{H}(x,\xi,\Delta^2) + \frac{\gamma^{5}\Delta^{\mu}}{2M} \widetilde{E}(x,\xi,\Delta^2) \right) U(P)$$

axial-vector

$$\int \frac{d\eta}{4\pi} e^{i\eta x} \langle P' | \bar{q}(-\frac{\eta n}{2}) \sigma^{\mu\nu} \gamma_5 \mathcal{U} q(\frac{\eta n}{2}) | P \rangle = \overline{U}(P') \Big\{ \sigma^{\mu\nu} \gamma_5 \left(H_T(x,\xi,\Delta^2) - \frac{t}{2m^2} \widetilde{H}_T(x,\xi,\Delta^2) \right)$$
$$+ \frac{\in^{\mu\nu\alpha\beta} \Delta_{\alpha} \gamma_{\beta}}{2m} \overline{E}_T(x,\xi,\Delta^2) + \frac{\Delta^{[\mu} \sigma^{\nu]\alpha} \gamma_5 \Delta_{\alpha}}{2m^2} \widetilde{H}_T(x,\xi,\Delta^2) + \frac{\in^{\mu\nu\alpha\beta} \overline{P}_{\alpha} \gamma_{\beta}}{m} \widetilde{E}_T(x,\xi,\Delta^2) \Big\} U(P)$$

tensor

Basic features of GPDs

forward limit

$$H(x,0,0) = q(x) \hat{=} 1/\ 2\left((\rightarrow)\rightarrow + (\leftarrow)\rightarrow \right)$$

$$\widetilde{H}(x,0,0) = \Delta q(x) \hat{=} (\rightarrow)\rightarrow - (\leftarrow)\rightarrow$$

$$H_T(x,0,0) = \delta q(x) = h_1(x) \hat{=} \overset{\uparrow}{(\uparrow)} - \overset{\uparrow}{(\downarrow)}$$

„local“ limit

$$\int dx H(x,\xi,t) = F_1(t),$$

$$\int dx \widetilde{H}(x,\xi,t) = g_A(t),$$

$$\int dx H_T(x,\xi,t) = g_T(t) \text{ etc.}$$

decomposition of
the nucleon total spin

(O)AM of quarks
in the nucleon

$$\frac{1}{2} = J_N = \frac{1}{2}\left(< x >_{q+g} + E^{n=2}(0)\right) = \frac{1}{2}\Delta\Sigma + L_q + J_g$$

X. Ji

GPDs $H, E, \widetilde{H}$

probability interpretation
of quark distributions
in transverse coordinate space

$b_\perp \hat{=}$ distance of active quark to
the center of momentum $R_\perp$

$$q(x, b_\perp^2) = \int d^2\Delta_\perp e^{-i\Delta_\perp \cdot b_\perp} H(x, \xi = 0, \Delta_\perp^2) =$$

$$= \left\langle P^+, R_\perp = 0 \middle| \hat{O}(x, b_\perp) \middle| P^+, R_\perp = 0 \right\rangle$$

M. Burkardt

Parametrization in terms of tensor GPDs

Mellin moments $\int_{-1}^{1} dx\, x^n$ of matrix elements

$2\lfloor n/2 \rfloor + n + 3$ GFFs ✓

$$
\begin{aligned}
&A_{\mu\nu} S_{\nu\mu_1\ldots\mu_n} \left\langle P' \middle| \overline{\psi}(0)\sigma^{\mu\nu}\gamma_5 iD^{\mu_1}\ldots iD^{\mu_n}\psi(0) \middle| P \right\rangle = \\
&\quad A_{\mu\nu} S_{\nu\mu_1\ldots\mu_n} \overline{U}(P') \Bigg\{ \sum_{i=0,\text{even}}^{n} \Bigg(\sigma^{\mu\nu}\gamma_5 \Delta^{\mu_1}\ldots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\ldots\overline{P}^{\mu_n}\left[A_{Tn+1i}(\Delta^2) - \frac{t}{2m^2}\tilde{A}_{Tn+1i}(\Delta^2) \right] \\
&\qquad + \frac{\in^{\mu\nu\alpha\beta}\Delta_\alpha\gamma_\beta}{2m}\Delta^{\mu_1}\ldots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\ldots\overline{P}^{\mu_n}\overline{B}_{Tn+1i}(\Delta^2) \\
&\qquad + \frac{\Delta^{[\mu}\sigma^{\nu]\alpha}\gamma_5\Delta_\alpha}{2m^2}\Delta^{\mu_1}\ldots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\ldots\overline{P}^{\mu_n}\tilde{A}_{Tn+1i}(\Delta^2) \Bigg) \\
&\qquad + \sum_{i=0,\text{odd}}^{n} \frac{\in^{\mu\nu\alpha\beta}\overline{P}_\alpha\gamma_\beta}{m}\Delta^{\mu_1}\ldots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\ldots\overline{P}^{\mu_n}\tilde{B}_{Tn+1i}(\Delta^2) \Bigg\} U(P)
\end{aligned}
$$

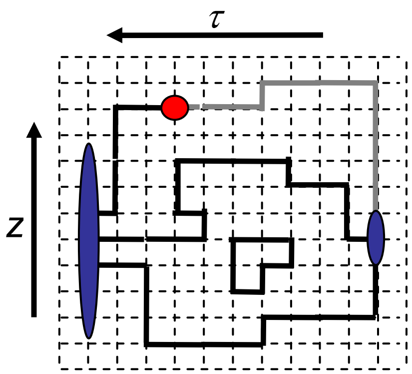
Ph.H., PLB 594(2004),
see also Chen/Ji, hep-ph/0404276

how to get back the moments of the GPDs from the GFFs?

$$H_T{}^n(\xi,\Delta^2) = \sum_{i=0,\text{even}}^{n}(-2\xi)^i A_{Tn+1i}(\Delta^2),\quad \overline{E}_T{}^n = \sum_{i=0,\text{even}}^{n}(-2\xi)^i \overline{B}_{Tn+1i}(\Delta^2)$$

$$\tilde{H}_T{}^n = \sum_{i=0,\text{even}}^{n}(-2\xi)^i \tilde{A}_{Tn+1i}(\Delta^2),\quad \tilde{E}_T{}^n = \sum_{i=0,\text{odd}}^{n}(-2\xi)^i \tilde{B}_{Tn+1i}(\Delta^2)$$

Overview – computation of GPDs on the Lattice



$$C_{3pt}(\tau, P', P) = Tr\left\{\tilde{\Gamma}\left\langle 0\left|N(\tau_{snk}, P')O_\Gamma(\tau)\overline{N}(\tau_{src}, P)\right|0\right\rangle\right\}$$

choose nucleon source/sink

$$C_{3pt}^{\mu_1\mu_2\cdots}(\tau, P', P = P' - \Delta) \propto \sum_{x', x'', y, y'} e^{i\Delta x'} e^{iP' x''}$$
$$Tr\left\{\tilde{\Gamma}\, G(x'', y')K_\Gamma^{\mu_1\mu_2\cdots}(y', y, x')G(y, x = 0)\right\}$$
$$Tr\left\{\tilde{G}(x'', x = 0)G(x'', x = 0)\right\} + \ldots$$

$$C_{3pt}^{\mu_1\mu_2\cdots}(\tau, P', P = P' - \Delta) \propto \sum_{x', y, y'} e^{i\Delta x'}$$
$$Tr\left\{\Omega_{\tilde{\Gamma}}(P'; x = 0, y')K_\Gamma^{\mu_1\mu_2\cdots}(y', y, x')G(y, x = 0)\right\} + \ldots$$

1. first inversion $\rightarrow G(z, x = 0), \forall z$
2. second inversion $\rightarrow \Omega_{\tilde{\Gamma}}(P'; x = 0, y')$ for fixed $P', \tilde{\Gamma}$
3. glue together with the operator $K_\Gamma^{\mu_1\mu_2\cdots}(y', y, x')$

insert complete set of states+
use translation operators

parametrize matrix element of local
ops in terms of generalized form factors

$$\left\langle P'\left|\overline{q}(0)\Gamma D^{\mu_1} D^{\mu_2} \ldots D^{\mu_n} q(0)\right|P\right\rangle \propto$$
$$\overline{U}(P', S')\left(a_\Gamma{}^{\mu_1\mu_2\cdots} A_\Gamma(t) + b_\Gamma{}^{\mu_1\mu_2\cdots} B_\Gamma(t) + \ldots\right)U(P, S)$$

$$C_{3pt}^{\mu_1\mu_2\cdots}(\tau, P', P) \propto Tr\left\{\tilde{\Gamma}\left(\not{P}' + m\right)\left(a_\Gamma^{\mu_1\mu_2\cdots} A(t) + b_\Gamma^{\mu_1\mu_2\cdots} B(t) + \ldots\right)\left(\not{P} + m\right)\right\}$$

finally, we equate the lattice ratio 3pt/2pts
and the continuum parametrization for
different momenta and indices simultaneously

this gives an (overdetermined) set of
linear equations which is solved to obtain the GFFs

H(4) tensor operator index combinations on the lattice for n=1,2

$$n=1 \rightarrow \tau_1^{(6)} : O^{n=1}_{T\mu\nu},\ \mu<\nu \rightarrow$$

# of Ops $=6$

# of FFs $=3$

$$n=2 \rightarrow$$

$$\tau_1^{(8)} : O^{n=2}_{T1} = \frac{1}{2^{1/2}}\left(O_{T122} - O_{T133}\right),\ O^{n=2}_{T2} = \frac{1}{6^{1/2}}\left(O_{T122} + O_{T133} - 2O_{T144}\right)$$

$$O^{n=2}_{T3} = \frac{1}{2^{1/2}}\left(O_{T211} - O_{T233}\right),\ O^{n=2}_{T4} = \frac{1}{6^{1/2}}\left(O_{T211} + O_{T233} - 2O_{T244}\right)$$

$$O^{n=2}_{T5} = \frac{1}{2^{1/2}}\left(O_{T311} - O_{T322}\right),\ O^{n=2}_{T6} = \frac{1}{6^{1/2}}\left(O_{T311} + O_{T322} - 2O_{T344}\right)$$

$$O^{n=2}_{T7} = \frac{1}{2^{1/2}}\left(O_{T411} - O_{T422}\right),\ O^{n=2}_{T8} = \frac{1}{6^{1/2}}\left(O_{T411} + O_{T422} - 2O_{T433}\right)$$

$$\tau_2^{(8)} : O^{n=2}_{T1} = \left(\frac{2}{3}\right)^{1/2}\left(2O_{T\mu\{\nu\rho\}} + O_{T\nu\{\mu\rho\}}\right),\ O^{n=2}_{T2} = 2^{1/2} O_{T\nu\{\mu\rho\}},\quad \mu<\nu<\rho$$

$$\rightarrow$$

# of Ops $=16$

# of GFFs $=4$

Göckeler, Horsley, Ilgenfritz,
Perlt, Rakow, Schierholz, Schiller,
Phys.Rev.D54:5705-5714,1996
Göckeler, 2005

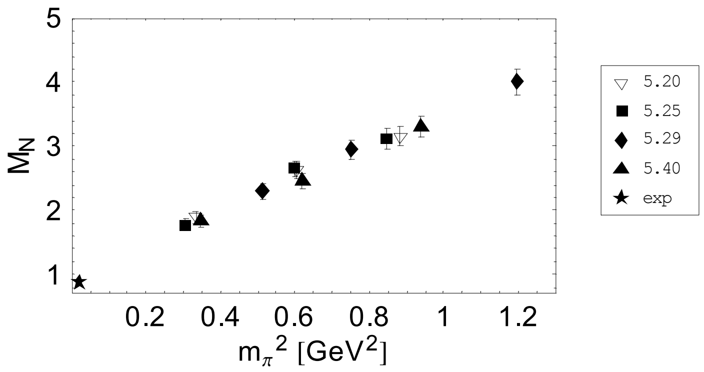
be aware of operator mixing for n>2:
Göckeler, Horsley, Perlt, Rakow, Schäfer,
Schierholz, Schiller, NPB 717, 2005

NP renormalization constants (Meinulf) for n=1,2 ✓

Lattice parameters – QCDSF/UKQCD-collaboration, 2005 (work in progress)

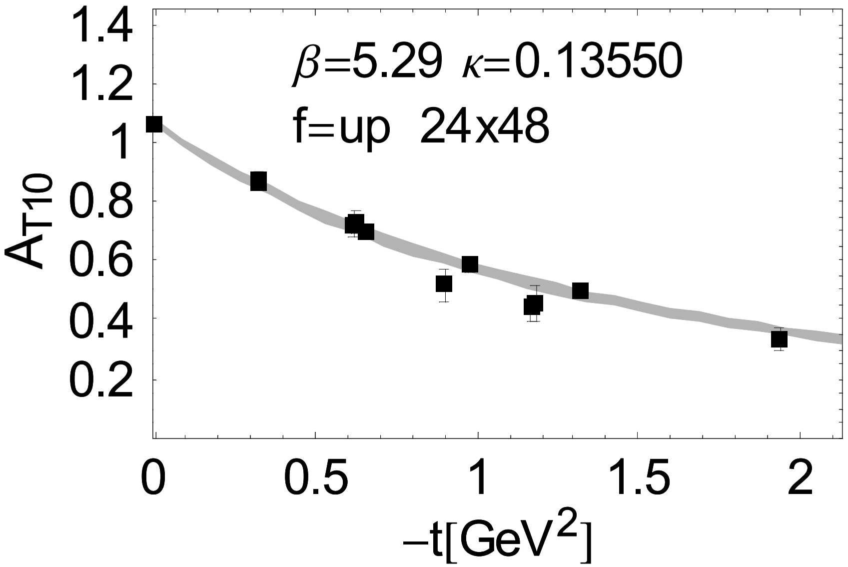
QCDSF - lattice - parameters :

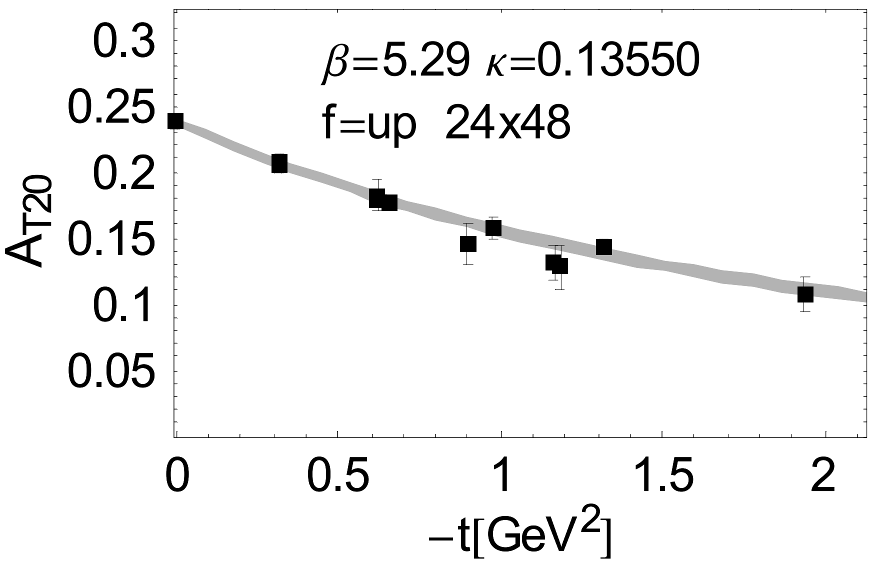
- Wilson - fermions with (NP) clover - improvement
- unquenched calculation, but only connected contributions
- lattice - size is $(L^3 x L_t)$ 16^3x32 for heavier and 24^3x48 for lighter quarks
- inverse lattice - spacing is $a^{-1} \approx 2$ GeV
- pions, nucleons are still quite heavy, $m_\pi \approx 500 \ldots 900$ MeV
- pretty large # of different β (couplings) and κ (quark masses) available
- up to 1200 configurations for some $\beta, \kappa -$ combinations
- data for finite volume analysis available
- lattice spacing fixed using $r_0 = 0.47 fm$
- three projectors $\tilde{\Gamma}_{unpol} = \frac{1}{2}(1+\gamma_0), \tilde{\Gamma}_{1,2} = \frac{1}{2}(1+\gamma_0)\gamma_5\gamma_{1,2}$
- three sink - momenta $p' = (0,0,0),(1,0,0),(0,1,0)$

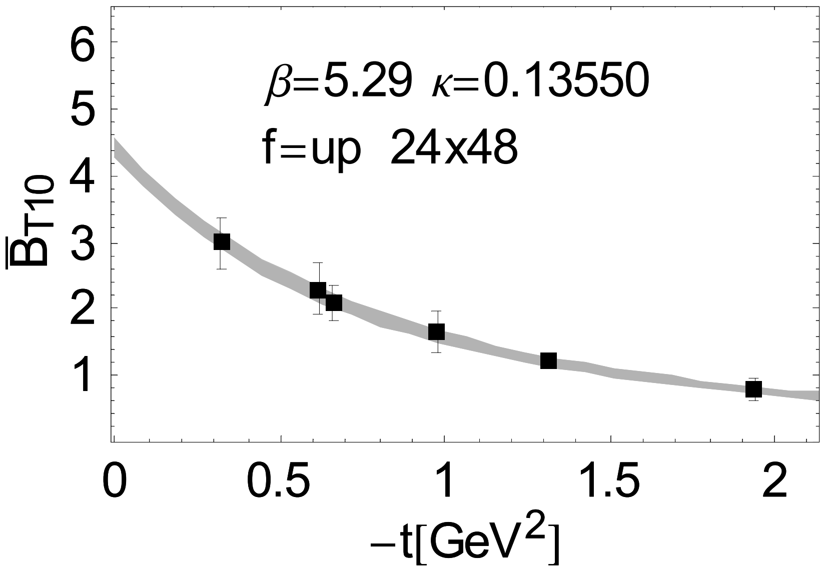


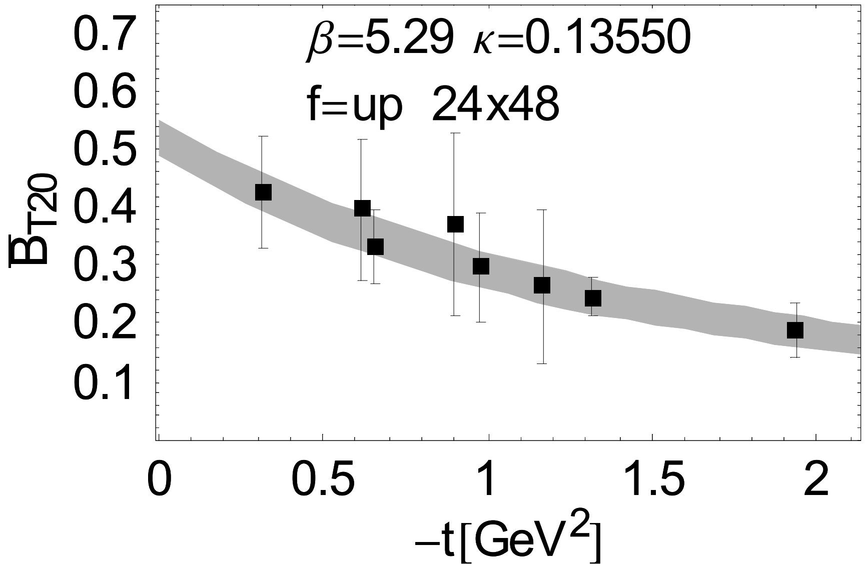
#	1	2	3	4	5	6	7	8	9	10	11	12
β	5.20	5.20	5.20	5.25	5.25	5.25	5.29	5.29	5.29	5.40	5.40	5.40
κ	.13420	.13500	.13550	.13460	.13520	.13575	.13400	.13500	.13550	.13500	.13560	.13610
a[fm]	0.12	0.11	0.099	0.11	0.097	0.091	0.1	0.096	0.09	0.082	0.079	0.075
L[fm]	1.96	1.68	1.59	1.69	1.56	2.19	1.66	1.53	2.16	1.97	1.88	1.79
m_π[Gev]	0.94	0.777	0.578	0.92	0.774	0.553	1.09	0.867	0.716	0.969	0.788	0.588
$m_\pi \times$ L	9.36	6.64	4.66	7.89	6.11	6.14	9.23	6.74	7.85	9.68	7.48	5.3

tensor GPDs – some „raw“ data

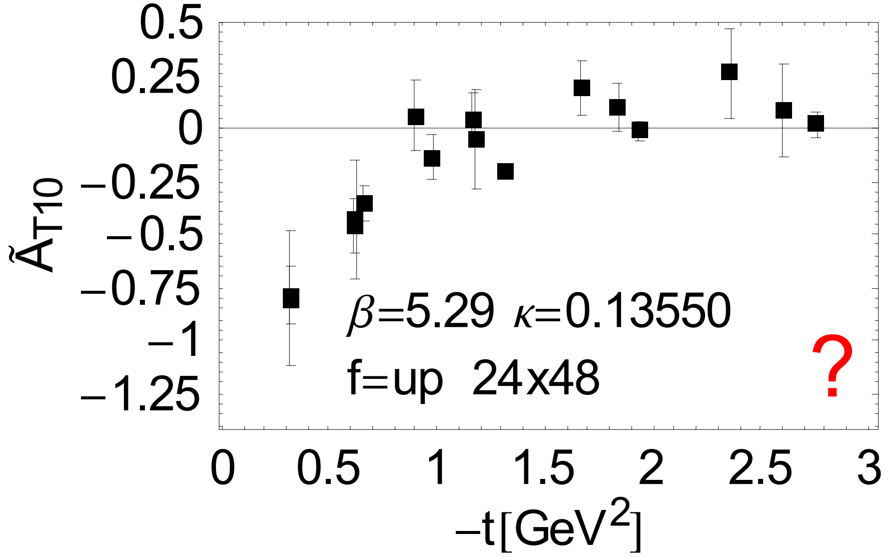


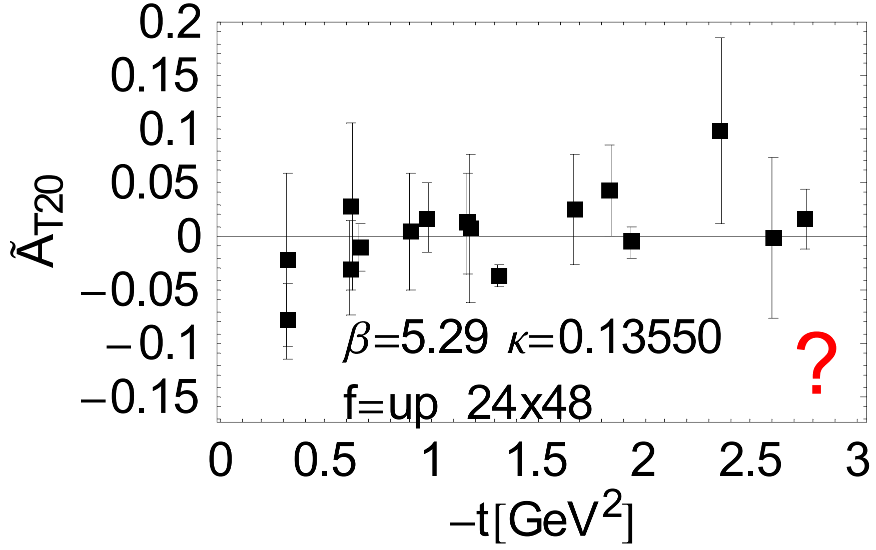


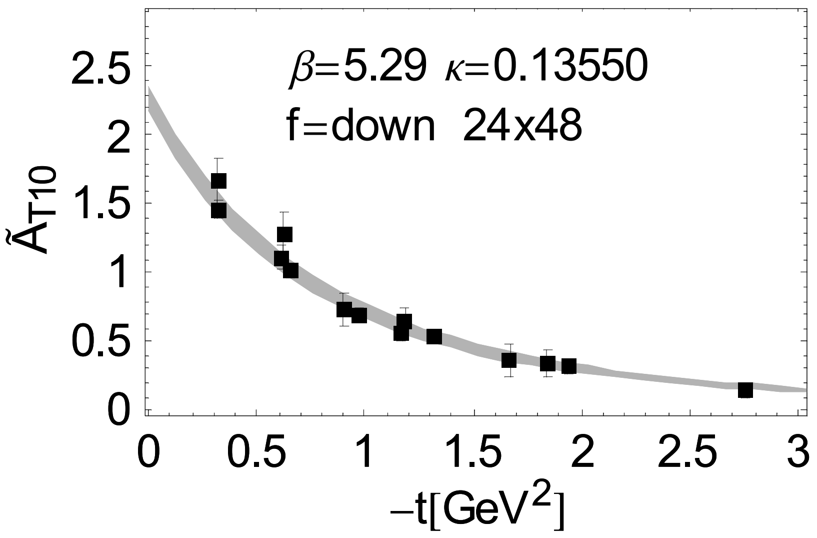


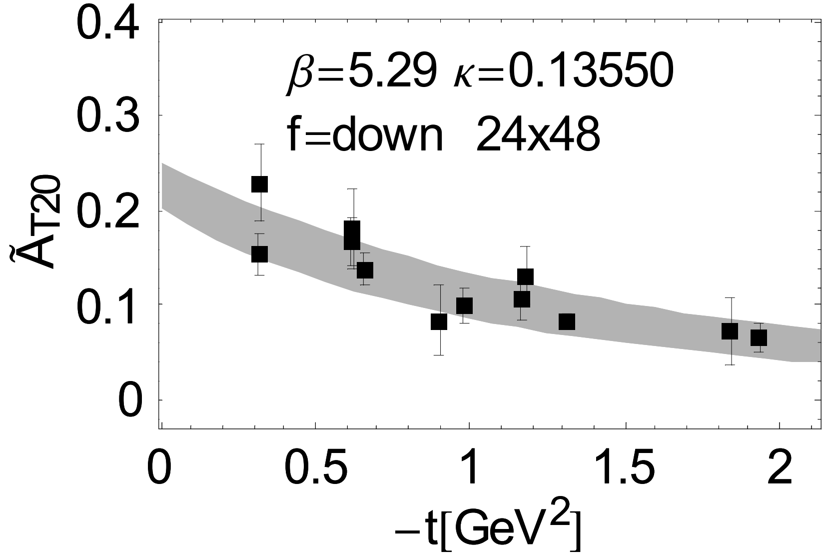


tensor GPDs – continued









The p-pole ansatz

p-pole ansatz $A(t) = \dfrac{A(0)}{\left(1 - t/m_p^2\right)^p}$ $\xrightarrow{\text{FT}}$ $A(b^2) = C(A(0),p)\, b^{p-1} K_{p-1}(m_p b)$

pro :

- describes almost all GFFs reasonably well over the whole range of momentum transfer
- simple relation to charge radius
- only 2(3) parameters
- simple Fourier - transform

contra :

- no sound theoretical foundation, purely phenomenological
- it is hard to determine m_p and p simultaneously from fit

(Mellin moments of) quark densities should be well behaved for $b \to 0$ and $b \to \infty$

→ for finite densities, we need

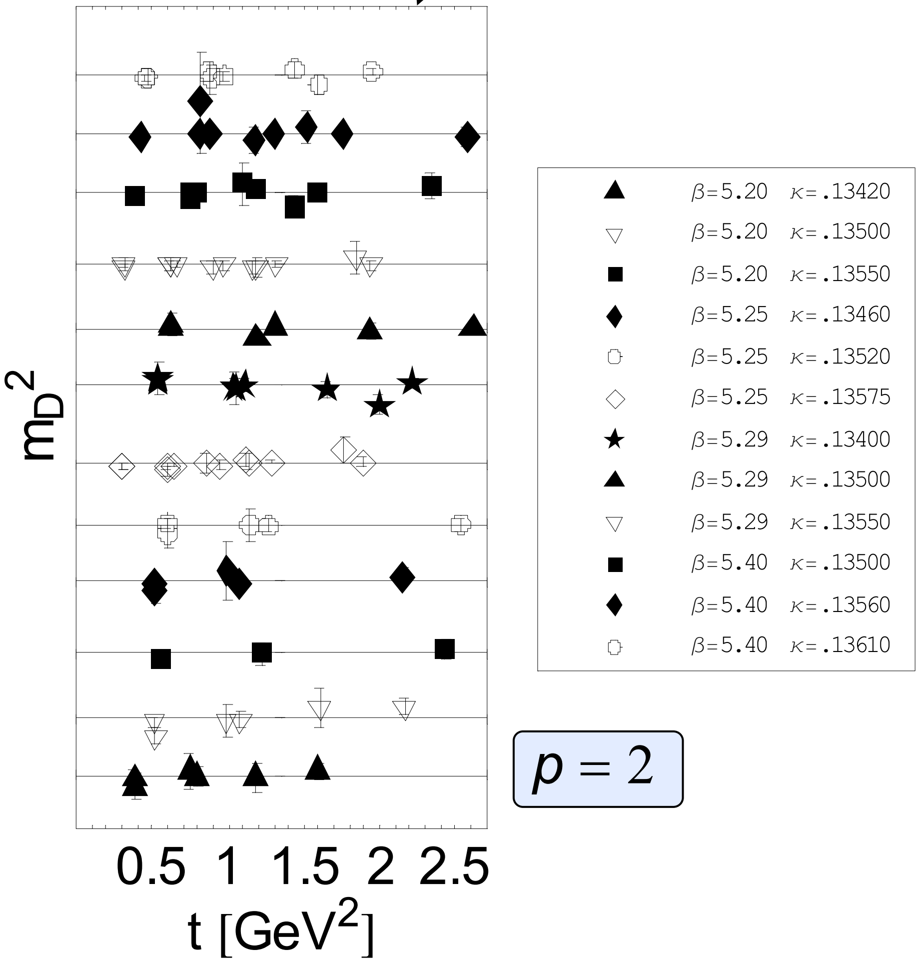
$$H, \widetilde{H}, H_T : \quad p > 1$$

$$E, \overline{E}_T : \quad p > \frac{3}{2}$$

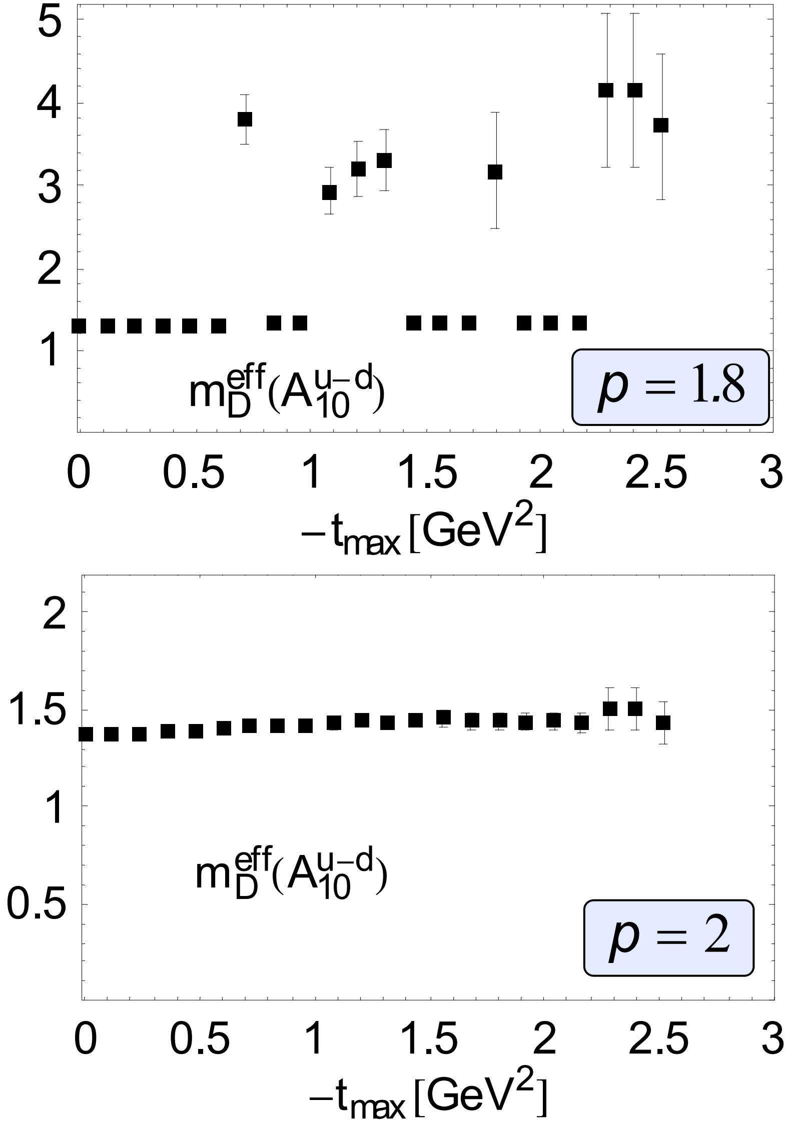
$$\widetilde{H}_T : \quad p > 2$$

Testing the dipole parametrization

$$A_{n0}(t)=\frac{A_{n0}(t)}{\left(1-t/m_{n,p}^2\right)^p}\to m_p^2=m_p^2\left(t,A_{n0}(t)\right)$$



global fit



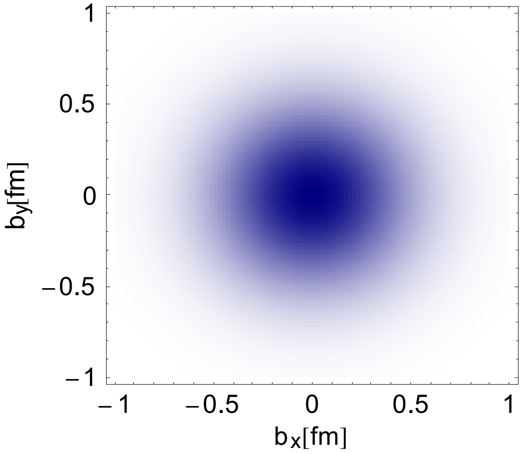
Spin densities in the transverse plane

spin density for transversely polarized quarks

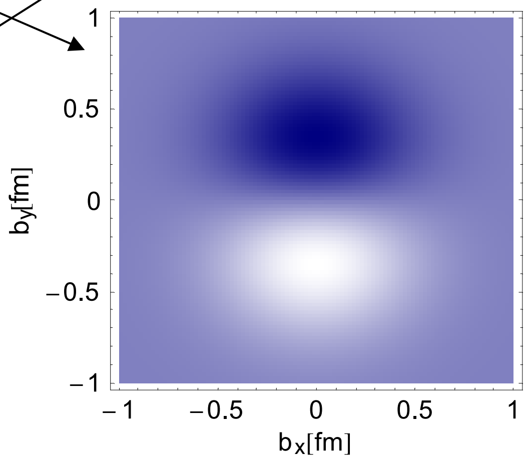
M.Diehl and Ph.H., EPJC 2005

$$\left\langle P^+, R_\perp = 0, \Lambda, S_\perp \left| \hat{\rho}^n(b_\perp, s_\perp) \right| P^+, R_\perp = 0, \Lambda, S_\perp \right\rangle =$$

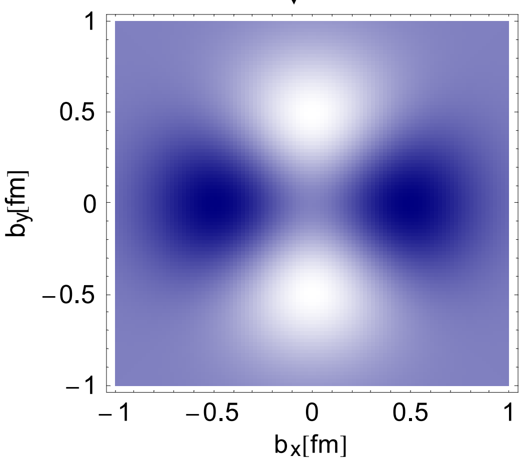
$$\frac{1}{2}\left[A_{n0} - \frac{S^i \epsilon^{ij} b^j}{M} B_{n0}' - \frac{s^i \epsilon^{ij} b^j}{M} \overline{B}_{Tn0}' + s^i S^i \left(A_{Tn0} - \frac{1}{4M^2} \Delta_b \tilde{A}_{Tn0} \right) + s^i \left(2b^i b^j - b^2 \delta^{ij} \right) S^j \frac{1}{M^2} \tilde{A}_{Tn0}'' \right]$$



monopole



dipole

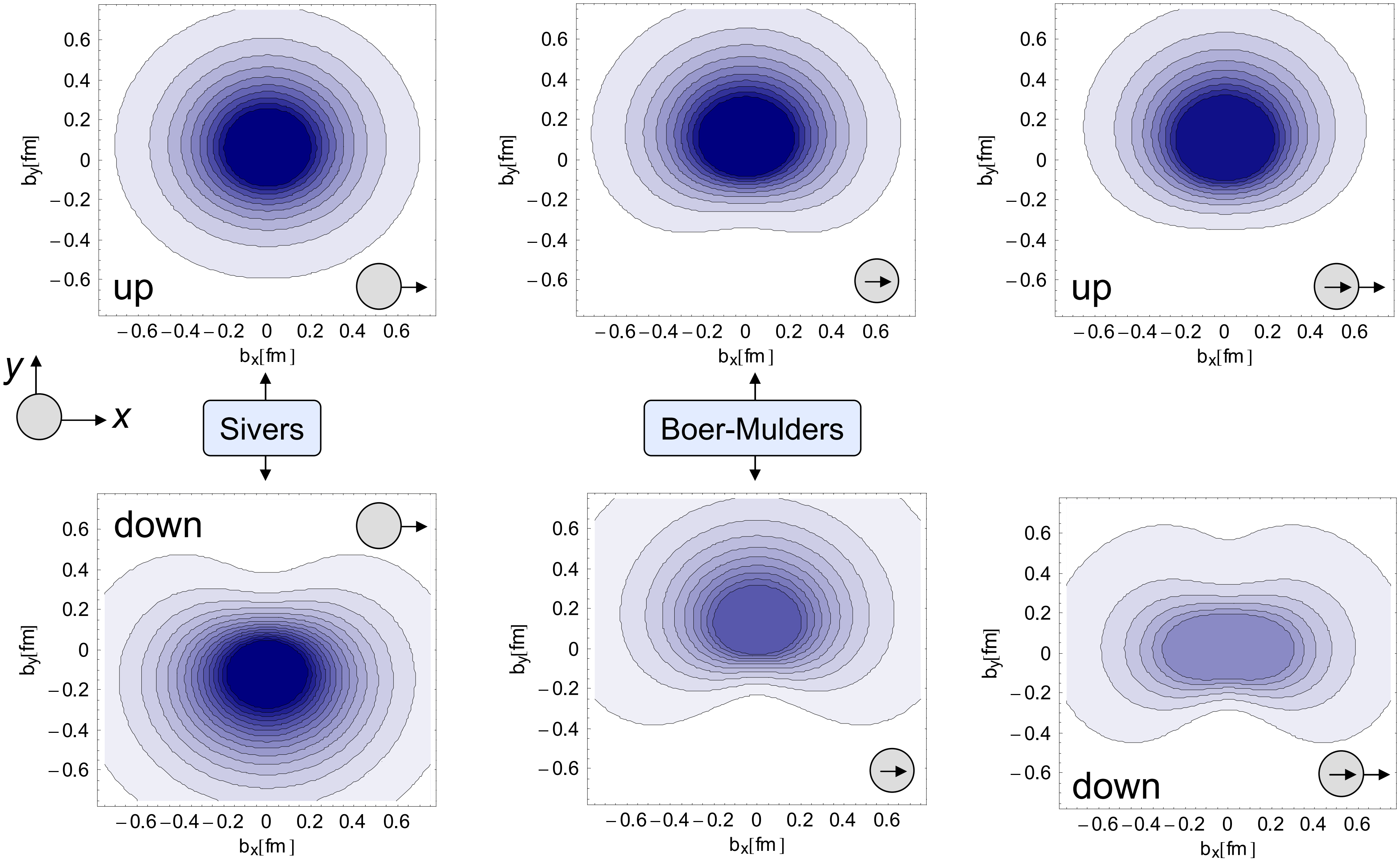


quadrupole

$$\hat{\rho}^n(b_\perp, s_\perp) \equiv \int_{-1}^{1} dx x^{n-1} \int \frac{d\eta}{2\pi} e^{i\eta x} \overline{q}(-\eta n/2) \gamma^+ \left[1 + (s_\perp \cdot \gamma_\perp) \gamma_5 \right] q(\eta n/2)$$

$$= \int_{-1}^{1} dx x^{n-1} \int \frac{d\eta}{2\pi} e^{i\eta x} \overline{q}(-\eta n/2) \left[\gamma^+ + i s_{\perp j} \sigma^{+j} \gamma_5 \right] q(\eta n/2)$$

preliminary results for the spin densities 1



Sivers

Boer-Mulders

Improved positivity bounds including tensor GPDs

M.Diehl/Ph.H., EPJC 2005

full spin matrix, $\xi = 0$

$$\begin{array}{c|cccc}
\Lambda',\lambda' \downarrow \;\; \Lambda,\lambda \rightarrow & ++ & -+ & +- & -- \\
\hline
++ & H+\tilde{H} & -ie^{-i\varphi}\frac{b}{M}E & ie^{i\varphi}\frac{b}{M}\left(E_T'+2\tilde{H}_T'\right) & 2\left(H_T-\frac{1}{4M^2}\Delta_b\tilde{H}_T\right) \\
-+ & ie^{i\varphi}\frac{b}{M}E & H-\tilde{H} & 2ie^{2i\varphi}\frac{b^2}{M^2}\tilde{H}_T'' & ie^{i\varphi}\frac{b}{M}\left(E_T'+2\tilde{H}_T'\right) \\
+- & -ie^{-i\varphi}\frac{b}{M}\left(E_T'+2\tilde{H}_T'\right) & 2ie^{-2i\varphi}\frac{b^2}{M^2}\tilde{H}_T'' & H-\tilde{H} & -ie^{-i\varphi}\frac{b}{M}E \\
-- & 2\left(H_T-\frac{1}{4M^2}\Delta_b\tilde{H}_T\right) & -ie^{-i\varphi}\frac{b}{M}\left(E_T'+2\tilde{H}_T'\right) & ie^{i\varphi}\frac{b}{M}E & H+\tilde{H}
\end{array}$$

positivity of eigenvalues

Soffer - bound $\left[H+\tilde{H} \geq 2\left|H_T\right|\right]_{t=0} \Leftrightarrow q+\Delta q \geq 2\left|\delta q\right|$

bounds by Pobylitsa and Burkardt

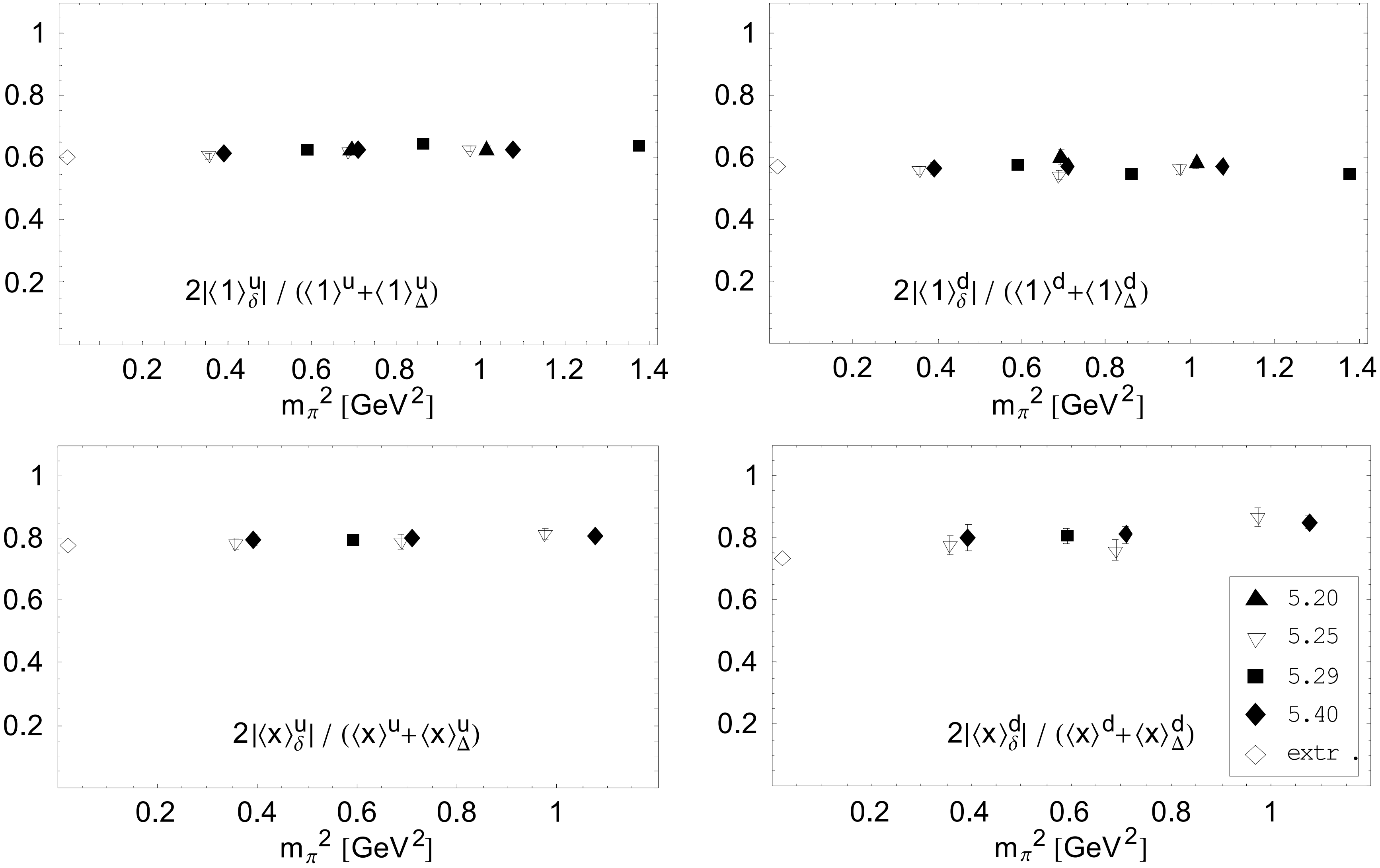
improved bounds including tensor GPDs

$$H \geq \left|H_T + \frac{1}{M^2}\tilde{H}_T' - \frac{1}{2M^2}\Delta_b\tilde{H}_T\right|$$

$$\left(H \pm H_T \pm \frac{1}{M^2}\tilde{H}_T' \mp \frac{1}{2M^2}\Delta_b\tilde{H}_T\right)^2 - \left(\tilde{H} \pm H_T \mp \frac{1}{M^2}\tilde{H}_T'\right)^2 \geq \frac{b^2}{M^2}\left(E \pm E_T' \pm 2\tilde{H}_T'\right)^2$$

Lowest two moments of the Soffer - bound $q + \Delta q \geq 2\left|\delta q\right|$ in lattice QCD

based on QCDSF hep-lat/0507001/PLB



Ph. Hägler, Leipzig, 2005

Summary/Outlook

preliminary lattice results on transverse spin densities show complex interplay of spin and coordinate degrees of freedom

implications for Sivers and Boer-Mulders functions (M. Burkardt)

Soffer-bound saturated to 60%-80%

plan to study the second moment of the transverse spin-densities

new positivity bounds on GPDs will be investigated