A SU(2) KvB caloron gas model and confinement

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Motivation:

- Instantons = classical solutions of Euclidean Yang-Mills eq. of motion, discovered by Belavin, Polyakov, Shvarts and Tyupkin (1975)
  - successful in semi-classical calculations of hadronic correlation functions
  - solve $U(1)_A$ problem
  - explain spontaneous chiral symmetry breaking
  - but could not be related to confinement

- The same applies to the periodic instanton (= HS caloron), discovered by Harrington, Shepard in 1978.

- In 1998 time periodic instantons with arbitrary asymptotic holonomy were discovered by Kraan, van Baal, Lee, Lu

  - The possible contribution of KvB calorons to a confining potential has not been studied numerically so far. This and other aspects of a KvB caloron gas model are the aims of the present work.

Could KvB calorons be relevant in the QCD vacuum at $T \neq 0$?

- Gross, Pisarski, Yaffe(1980): Classical solutions with non-trivial, asymptotic holonomy are suppressed by a factor $e^{-const \cdot V}$, where V is the 3D volume.

  $\Rightarrow$ ignored for instanton model

- Diakonov(2004) calculated the holonomy dependence of the free energy of a non-interacting KvB caloron gas using the one-loop KvB caloron quantum weight.

- Trivial holonomy is only stable above a certain temperature but becomes unstable below.

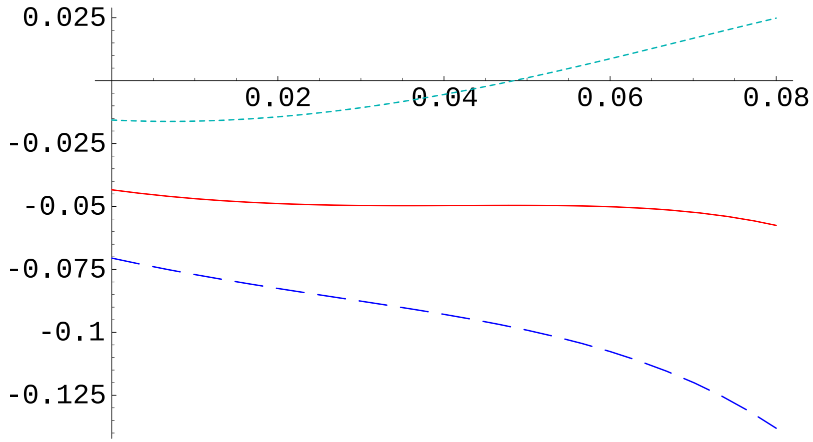
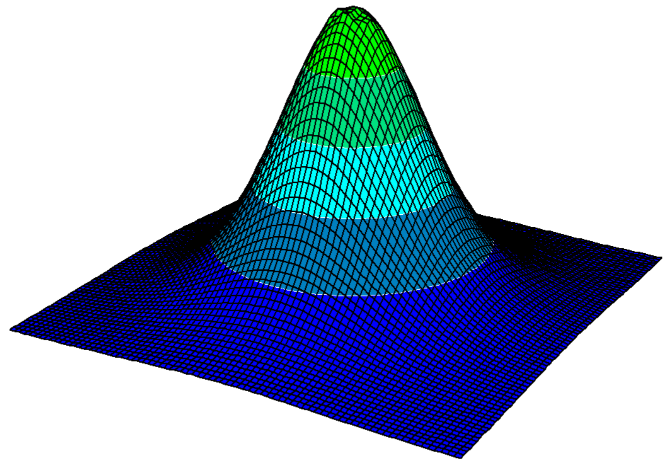


Figure taken from Diakonov et al. Free energy vs. asymptotic holonomy v for $T = 1.3\Lambda$ (dotted), $T = 1.125\Lambda$ (solid), $T = 1.05\Lambda$ (dashed) in dimensionless units, (v=0: trivial holonomy).

Summary: SU(2)-calorons:

- Caloron $\equiv$ instanton at finite temperature $T = \frac{1}{\beta}$, periodic in time.
- Exists for arbitrary holonomy $P_\infty \in SU(2)$
  with $P_\infty = e^{2\pi i \vec{\omega}\vec{\tau}} = \lim_{|\vec{x}|\to\infty} Pexp\left(i\int_0^\beta A_4(\vec{x},t)dt\right)$, $\omega = |\vec{\omega}|$, $\bar{\omega} = 0.5 - \omega$
  $\Rightarrow A_4$ does not vanish for $|\vec{x}| \to \infty$: $A_{4,\infty}^{per} = 2\pi T\vec{\omega}\vec{\tau}$

- Consists of 2 constituents with 3D-distance d.
- $d \ll \beta$: Constituents form one action lump
- $d \gg \beta$: Two separate, static BPS-monopoles
  Fraction of action in lumps is $\frac{\omega}{\bar{\omega}}$
- $\omega = 0$ or $\omega = 0.5$: Trivial holonomy
- $0 < \omega < 0.5$: Non-trivial holonomy



Action density in XZ-plane of two merged constituents, $\omega = \bar{\omega}$.

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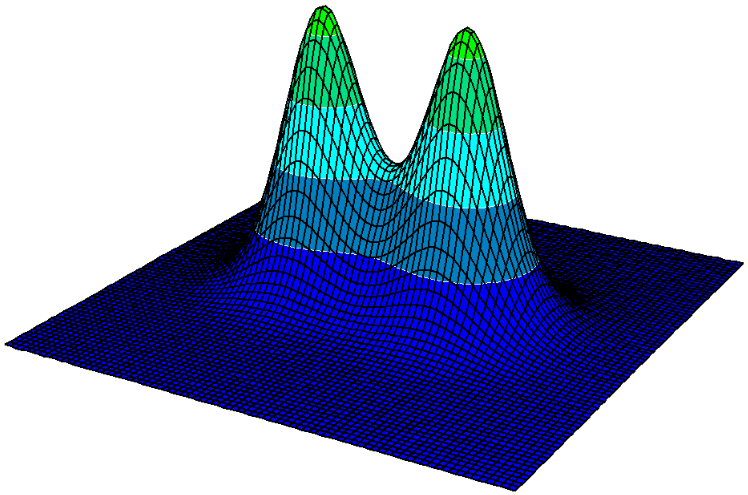

Action density in XZ-plane of two separating constituents, $\omega = \bar{\omega}$.

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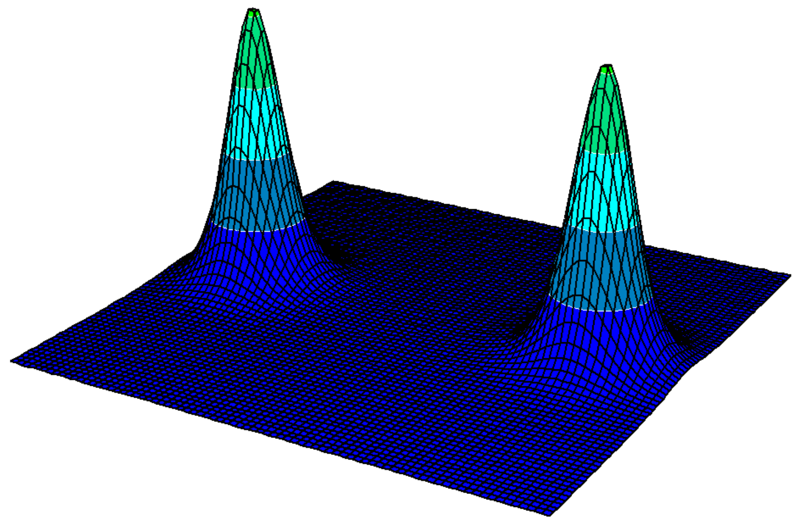


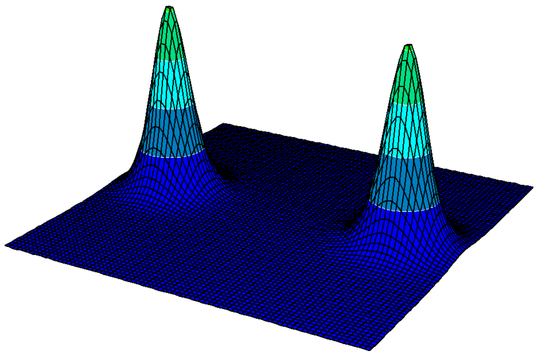
Action density in XZ-plane of two separated constituents, $\omega = \bar{\omega}$.

- 8 parameters per caloron $\forall P_\infty \in SU(2)$:

ρ	Determines constituent separation $d = \frac{\pi\rho^2}{\beta}$	$\longrightarrow$	1 parameter
x_0	Position	$\longrightarrow$	4 parameters
	SU(2)-rotation along $\vec{\omega}\vec{\tau}$	$\longrightarrow$	1 parameter
	Spatial rotation	$\longrightarrow$	2 parameters

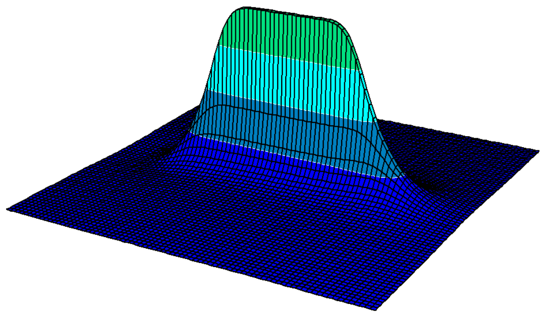
- Dirac string between constituents when separated:
  $\longrightarrow$ Very strong vector potential between constituents,
  $\longrightarrow$ fine-tuned to be free of action.

Action density in XZ-plane for disociated caloron



$\longleftrightarrow$

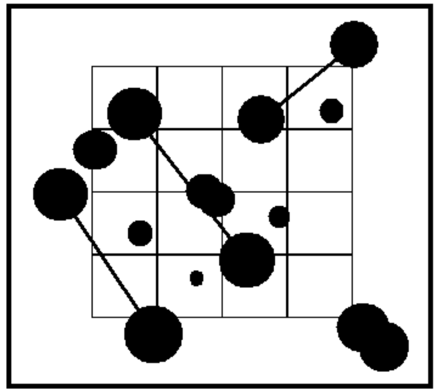
Corresponding vector potential $\sum_\mu |A_\mu|^2$



A SU(2) KvB caloron gas model:

- **Aim**: Simulate QCD vacuum as multi caloron system, measure string tension
- Caloron solutions only available for charge $|Q| = 1$ (except for special cases).
  $\Rightarrow$ Construction of approximate classical solutions by superposition of $A_\mu(x)$
- Caloron number determined by density n and physical volume
- All caloron parameters sampled randomly
- No additional weighing since configurations are considered as classical solutions
- Caloron size sampled according to a ρ-distribution $D(\rho, T)$

- Quantum fluctuations are accounted for by suitable ρ-distribution $D(\rho, T)$.
- Non-periodic lattice v embedded in bigger volume V, equal number of calorons and anti-calorons in V
- Physical scale enters the calculation through the **free parameters** $n(T)$, $D(\rho, T)$, $\omega(T)$, which have to be fixed by lattice observations.



Creating multi-caloron systems by superposition:

Calorons are solutions of Yang-Mills equation $F_{\mu\nu} = \tilde{F}_{\mu\nu}$, (not linear in A_μ).
$\Rightarrow$ Superposing N calorons $\sum_i^N A_\mu^{(i)}$ is not exact solution $\leftrightarrow$ interaction S_{int}.
$S_{int} = S(\sum_i^N A_\mu^{(i)}) - \sum_i^N S(A_\mu^{(i)}) \neq 0 \rightarrow$ good approximations if $S_{int} \approx 0$

Problem $S_{int} \gg S_0$**:**	**Solution:**
1. At non-trivial holonomy $A_{4,\infty} \neq 0$ $\Rightarrow$ interaction with distant calorons	$\rightarrow$ Superpose in algebraic gauge
2. If a caloron is located between two separated constituents $\Rightarrow$ interaction with Dirac string	$\rightarrow$ Move Dirac string from inside of caloron to outside by applying gauge transformation.
3. If action lumps of different calorons come too close $\Rightarrow$ interaction between action lumps	$\rightarrow$ Improve self-duality by pseudo-ADHM technique.

Selection of caloron density $n(T)$:

- Caloron density shall be fixed by topological susceptibility

$$\chi = \int d^4x < 0|T(q(x)q(0))|0 >= \lim_{\substack{a\to 0 \\ V\to\infty}} \frac{< Q^2 >}{V}$$

- Assuming that calorons and anti-calorons appear with same probability and are placed without correlation in the QCD vacuum, the topological susceptibiliy χ and the caloron density n are equal: $\chi = n$

- T-dependence of topological susceptibiliy $\chi(T)$ for SU(2) was measured by Alles et al. (1998)

- **For confined phase**: $\chi^{\frac{1}{4}} = 198 \pm 8 MeV$ (T-independent)

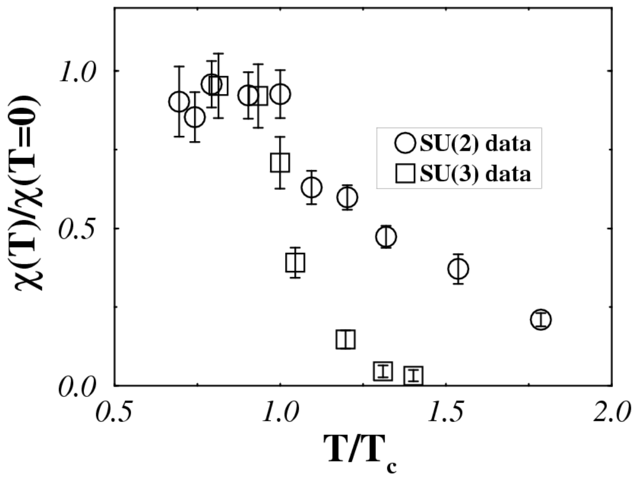


Figure taken from Alles et al.

Temperature T	Caloron density $n(T)$
$\leq T_C$	$(198\, MeV)^4$
$1.10\, T_C$	$(178\, MeV)^4$
$1.20\, T_C$	$(174\, MeV)^4$
$1.32\, T_C$	$(165\, MeV)^4$
$1.54\, T_C$	$(157\, MeV)^4$
$1.79\, T_C$	$(136\, MeV)^4$

Model parameter $n(T)$ adapted from data of $\chi(T)$

Fixing of holonomy parameter $\omega(T)$:

- Holonomy parameter ω shall be determined by lattice measurements of average Polyakov loop

$$M(\vec{x}) = \frac{1}{2} Tr \prod_{t=1}^{N_t} U_4(\vec{x}, t)$$

- For dilute caloron gases the average Polyakov loop $< M >$ is connected to the holonomy parameter $< M >= cos(2\pi\omega)$.

- T-dependence of $< M >$ for SU(2) in limit $V \rightarrow \infty$ was extracted by Engels et al. (1999)

$$< M(T) >= B \cdot \left(\frac{T - T_C}{T_C}\right)^{\beta}, \quad T \rightarrow T_C^+$$

$$B = 0.825(1), \quad \beta = 0.327$$

- **For confined phase**: $\omega = 0.25$
$\rightarrow$ max. non-trivial holonomy (T-independent)

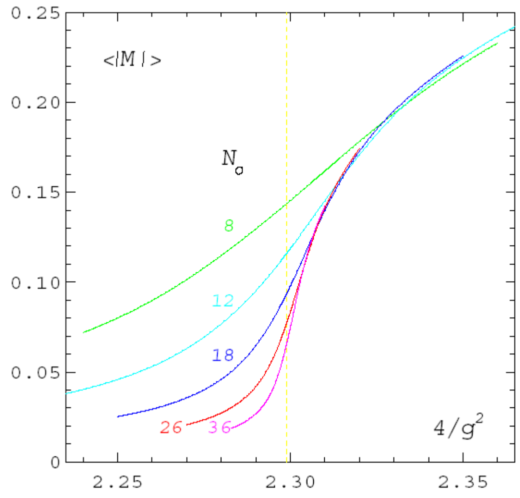
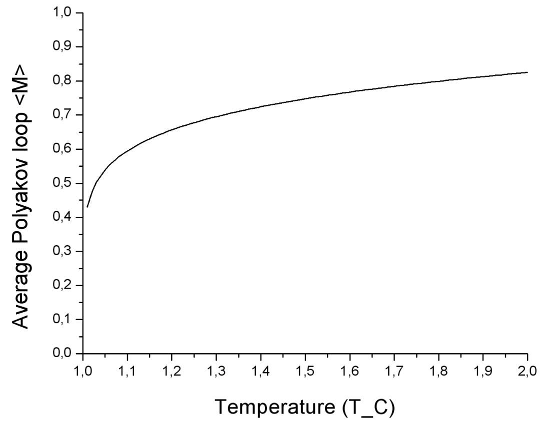


Figure taken from Engels et al.



T-dependence of $< M >$ to fix model parameter $\omega(T)$

Choice of size distribution $D(\rho, T)$:

- Quantum weight of KvB caloron (Diakonov, 2004):
  $Z = \int d^3z_1 \int d^3z_2\, e^{-\frac{8\pi^2}{g^2}} f(\rho), \quad \rho = \sqrt{\frac{\beta|z_1 - z_2|}{\pi}}, \quad z_1, z_2$ monopole positions

- For $\rho \ll \beta$: $f(\rho) \propto \rho^{b-5}, \quad b = \frac{11}{3} N_C$

- For $\rho \gg \beta$: $f(\rho) \propto e^{-VP(v) - \frac{2\pi^2}{\beta}\rho^2 P''(v)}, \quad P(v) = \frac{v^2 \bar{v}^2}{12\pi^2 T}, \quad P'' = \frac{d^2}{dv^2} P(v)$
  $\Rightarrow$ divergent for $v/(4\pi T) = \omega \in [\omega_-, \omega_+], \quad \omega_\pm = 0.25\,(1 \pm \sqrt{\frac{1}{3}})$
  $\Rightarrow$ ρ-distribution has to be cut off

- Similar problem as with instantons. Hard-core type interactions lead to exponential suppression of large sizes $\propto e^{-const \cdot \rho^2}$ (Diakonov $\&$ Petrov (1984), Müller-Preussker $\&$ Ilgenfritz (1981), Münster $\&$ Kamp (2000))

- Lattice observation helps: Average caloron size is approximately T-independent in confined phase $\bar{\rho} \approx 0.33 fm$ (Chu, Schramm, 1995).

- For small T $\Rightarrow$ $\bar{\rho}/\beta \ll 1$ $\quad \Rightarrow$ calorons are non-dissociated $\Rightarrow$ exponential suppression from instanton calculation also applicable to calorons.

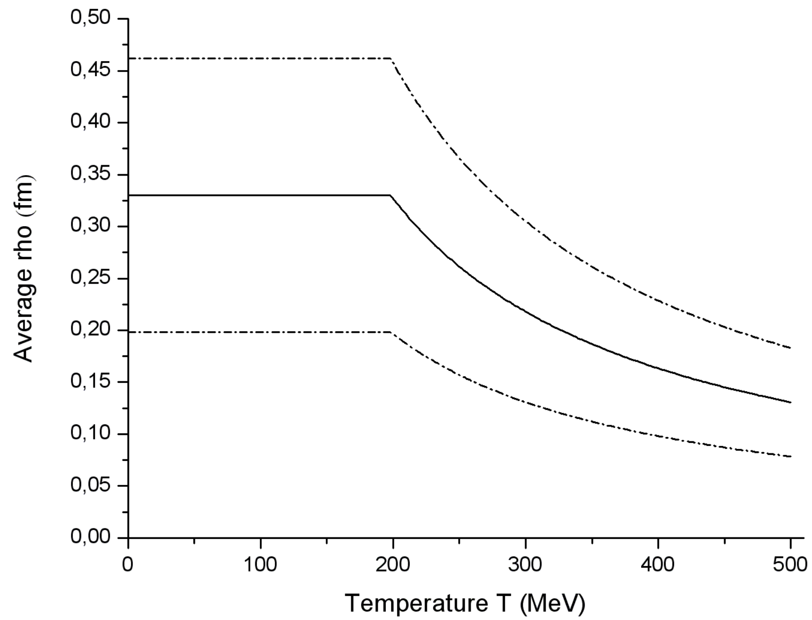
- For large T, but $T < T_C$ calorons start to dissociate into static monopoles with 3D extent $\approx \beta$ independent of ρ $\quad \Rightarrow$ suppression from caloron interaction unrealistic.

- 3D volume in $f(\rho) \propto e^{-VP(v)+\cdots}$ can be interpreted as ρ-dependent, *specific* caloron volume $V_{cal} = C_0(\omega)\pi\beta^2|z_1 - z_2|, \quad C_0(\omega) \approx 1$ leading again to an exponential suppression (Hofmann, 2005).

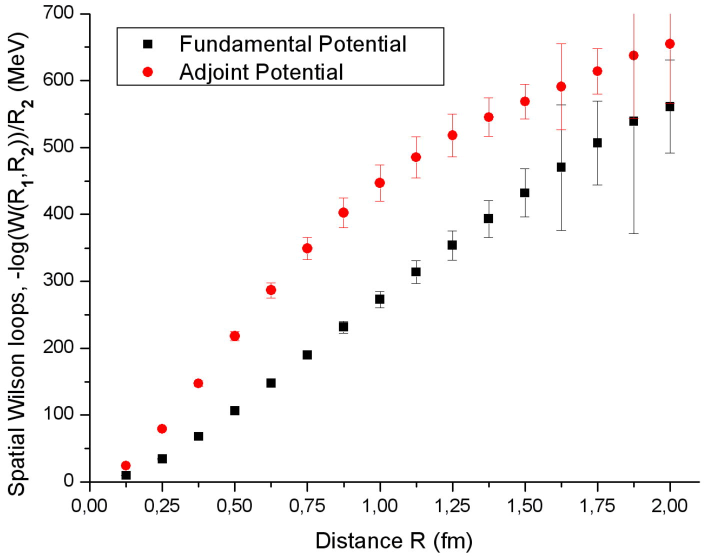
- For $T \gg T_C$ holonomy becomes trivial and $f(\rho) \propto e^{-\frac{4}{3}(\pi\rho T)^2}$

Temperature	ρ-distribution	fixation of parameters
$T < T_C$	$D(\rho, T) = a \cdot \rho^{b-5} \cdot exp(-c\rho^2)$	$\int D(\rho, T)d\rho = 1, \quad \bar{\rho} = 0.33fm$
$T > T_C$	$D(\rho, T) = a \cdot \rho^{b-5} \cdot exp(-\frac{4}{3}(\pi\rho T)^2)$	$\int D(\rho, T)d\rho = 1$

- T-dependence $\bar{\rho}(T)$ (solid), intervall of standard deviation $[\bar{\rho}-\sigma, \bar{\rho}+\sigma]$ (dashed)
- For $T \gg T_C$ $\bar{\rho}(T)$ is given by the HS caloron quantum weight.
- Both ansatzes are continously connected for $T_C \approx 200MeV$.
- **For confined phase**: $\bar{\rho} = 0.33\,fm$ (T-independent)



String tension σ in the confined phase:



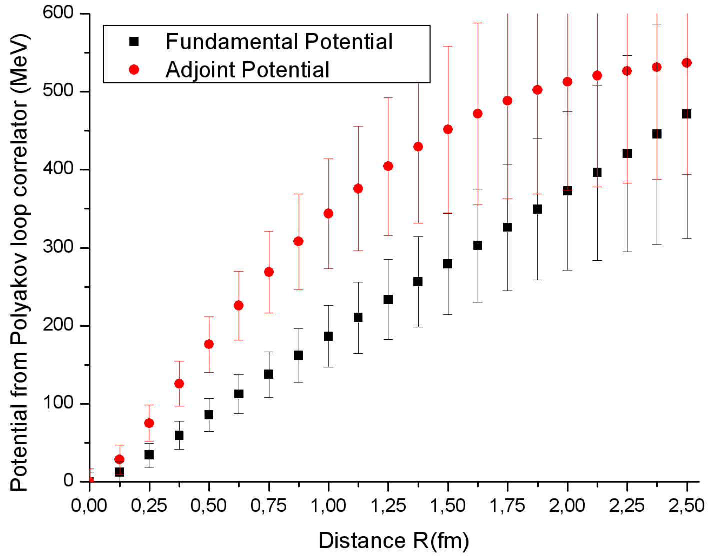
$V(R) = -log(W(R, R_2))/R_2$ from spatial Wilson loops in fundamental and adjoint representation.

Temperature T $= 198$ MeV $< T_C$.

Holonomy parameter $\omega = 0.25$

Caloron density $n^{\frac{1}{4}} = 198MeV$

String tension: $\sigma \approx 320\frac{MeV}{fm}$



$Q\bar{Q}$-potential calculated from Polyakov loop correlator in fundamental and adjoint representation.

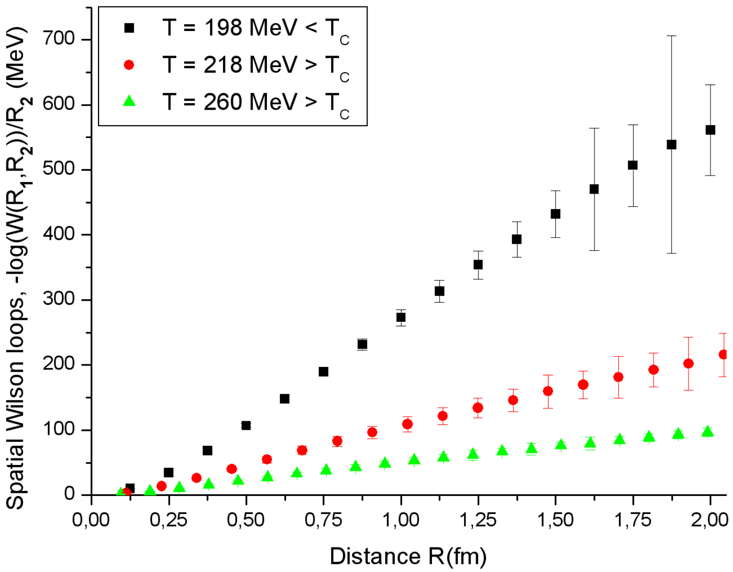
Temperature T $= 198$ MeV $< T_C$.

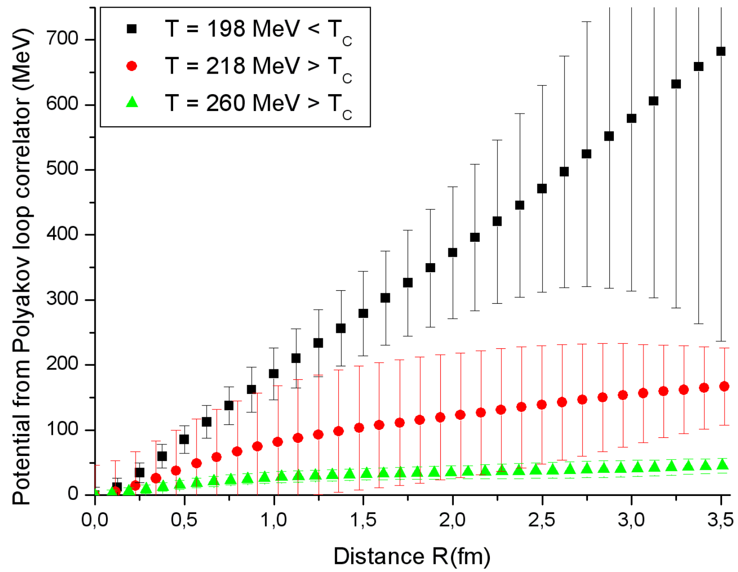
Holonomy parameter $\omega = 0.25$

Caloron density $n^{\frac{1}{4}} = 198MeV$

String tension: $\sigma \approx 200\frac{MeV}{fm}$

String tension σ in the deconfined phase:





$V(R) = -log(W(R, R_2))/R_2$ from spatial Wilson loops at different temperatures.

T=218 MeV $> T_C$, $\omega = 0.19$: $\sigma \approx 100\frac{MeV}{fm}$

T=260 MeV $> T_C$, $\omega = 0.15$: $\sigma \approx 50\frac{MeV}{fm}$

For comparison: T $=$ 198 MeV $< T_C$, $\omega = 0.25$: $\sigma \approx 320\frac{MeV}{fm}$

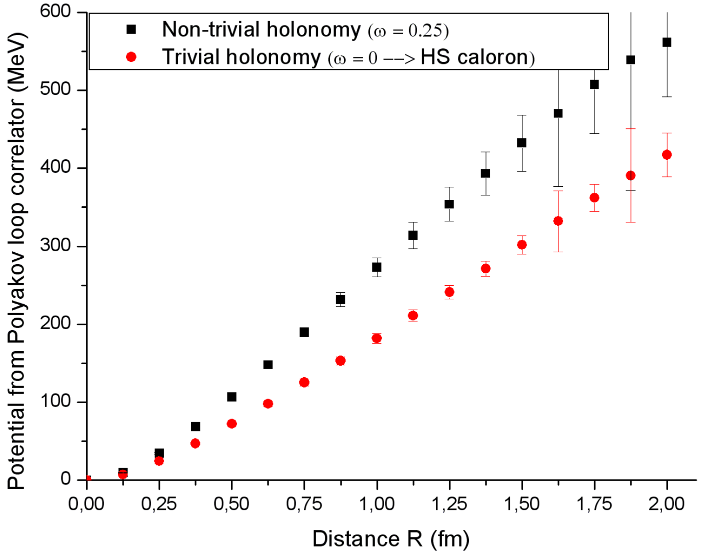
$Q\bar{Q}$-potential V calculated from Polyakov loop correlator at different temperatures.

T=218 MeV $> T_C$, $\omega = 0.19$: V stops rising $\Rightarrow$ no string tension

T=260 MeV $> T_C$, $\omega = 0.15$: V stops rising $\Rightarrow$ no string tension

For comparison: T $=$ 198 MeV $< T_C$, $\omega = 0.25$: $\sigma \approx 200\frac{MeV}{fm}$

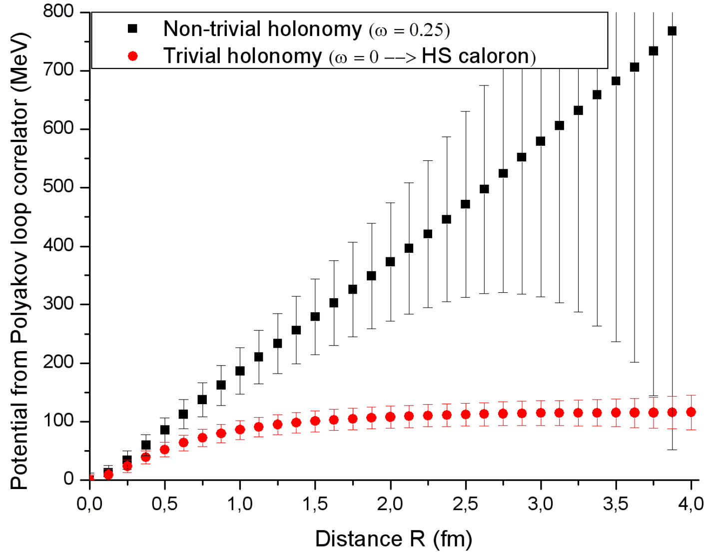
Comparison KvB calorons ($\omega = 0.25$) and HS calorons ($\omega = 0$):



$V(R) = -log(W(R, R_2))/R_2$ from spatial Wilson loops for contrast of $\omega = 0.25$ and $\omega = 0$ at T = 198 MeV.

$\omega = 0.25$: $\sigma \approx 320\frac{MeV}{fm}$

$\omega = 0.00$: $\sigma \approx 210\frac{MeV}{fm}$

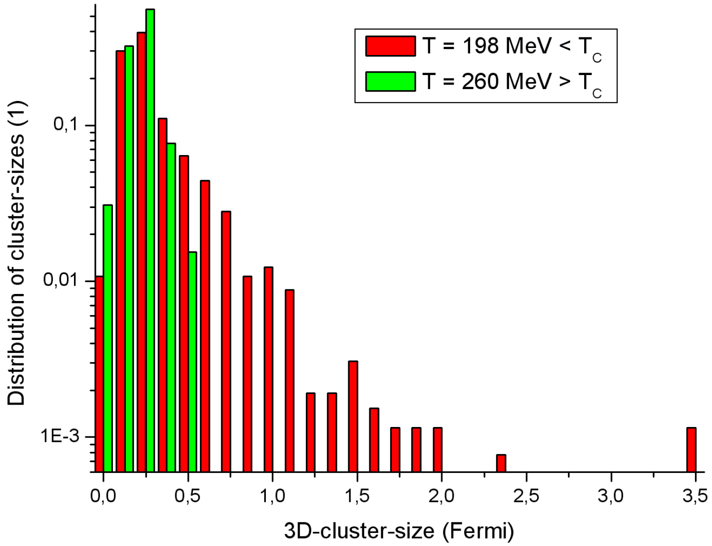


$Q\bar{Q}$-potential V calculated from Polyakov loop correlator for contrast of $\omega = 0.25$ and $\omega = 0$ at T = 198 MeV.

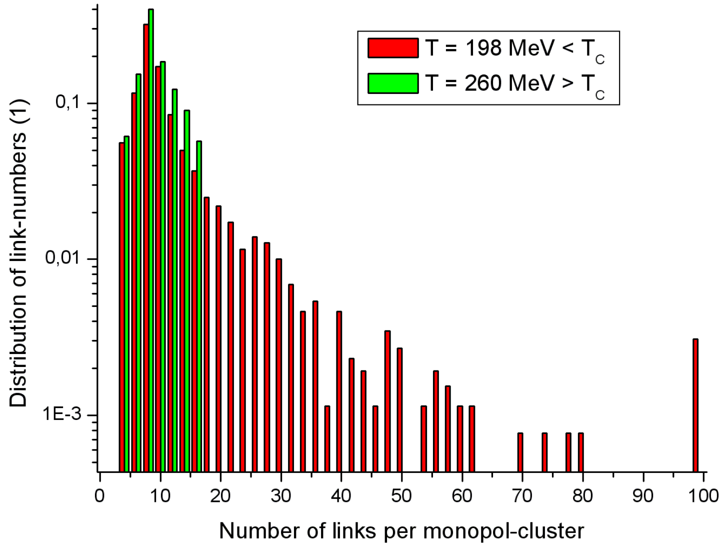
$\omega = 0.25$: $\sigma \approx 320\frac{MeV}{fm}$

$\omega = 0.00$: V stops rising $\Rightarrow$ no string tension

Analysis of monopol clusters:



Maximal 3D-extent of monopol clusters after fixing the maximally abelian gauge. Some large clusters are found in the confined phase (T=198 MeV $< T_C$, ω=0.25,$n^{\frac{1}{4}}$=198 MeV), but not for a deconfining temperature T=260 MeV, ω=0.15,$n^{\frac{1}{4}}$=165 MeV).



Number of links per monopol cluster after fixing the maximally abelian gauge. Some large clusters are found in the confined phase (T=198 MeV $< T_C$, ω=0.25, $n^{\frac{1}{4}}$=198 MeV), but not for a deconfining temperature T=260 MeV, ω=0.15,$n^{\frac{1}{4}}$=165 MeV).

Conclusion and outlook:

- String tensions $\sigma \approx 200 - 320\, MeV/fm$ can be obtained from a KvB caloron gas model. Compare with $T_C/\sqrt{\sigma} = 0.709$ for pure SU(2) (Teper et al.).
  $\Rightarrow$ Expected string tension $\sigma \approx 400 MeV/fm$ for $T_C \approx 200 MeV$

- The $Q\bar{Q}$ potential from the Polyakov loop correlator runs into a plateau for $T > T_C$ mainly because of $\omega(T) \rightarrow 0$.

- Caloron gas with non-trivial holonomy yields better results in confined phase than caloron gas with trivial holonomy (HS calorons).
  $\rightarrow$ HS calorons can not reproduce linear rising $Q\bar{Q}$ potential from Polyakov loop correlator in confined phase.

- Some large monopole clusters can be observed in confined phase.

- More statistic has to be collected.