Moments of generalized parton distribution functions viewed from chiral effective field theory.

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1. Form Factors of the nucleon to $\mathcal{O}(p^3)$
   - Definition
   - HBChPT
   - Chiral ChPT results up to order $\mathcal{O}(p^3)$
   - Generalized Parton Distributions

2. Generalized Form Factors to $\mathcal{O}(p^4)$
   - Definition
   - ChPT results up to order $\mathcal{O}(p^4)$
   - Forward limit
   - Conclusions and Outlook

Isovector Form Factors

Nucleon matrix element of the isovector component of the quark vector current $V^a_\mu = \overline{q}\gamma_\mu(\frac{\tau^a}{2})q$:

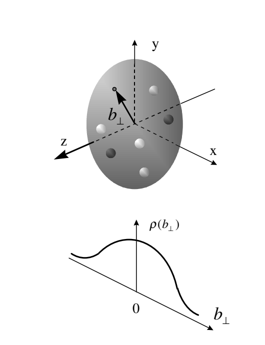
$$\langle N(p_2)|V^a_\mu|N(p_1)\rangle = \overline{u}(p_2)\left[F^\upsilon_1(q^2)\gamma_\mu + \frac{i}{2m_N}F^\upsilon_2(q^2)\sigma_{\mu\nu}q^\nu\right]u(p_1) \times \eta^\dagger\frac{\tau^a}{2}\eta$$

$$isovector \longrightarrow F^\upsilon_i = F_p - F_n$$

Sachs Form Factors

$$G^\upsilon_E(q^2) = F^\upsilon_1(q^2) + \frac{q^2}{4m_N{}^2}F^\upsilon_2(q^2)$$

$$G^\upsilon_M(q^2) = F^\upsilon_1(q^2) + F^\upsilon_2(q^2)$$



$$F^\upsilon_i(q^2) = F^\upsilon_i(0)\left[1 + \frac{1}{6}(r^\upsilon_i)^2q^2 + O(q^4)\right]$$

$$(r^\upsilon_i)^2 = \frac{6}{F_i(0)}\frac{dF^\upsilon_i(q^2)}{dq^2}\bigg|_{q^2=0}$$

Non-relativistic reduction

Nucleons treated non relativistically $\Longrightarrow$

- Heavy Baryon approximation
- non-relativistic reduction in the Breit frame

Breit frame for $ep \longrightarrow ep$

$$\vec{p} = +\vec{q}/2 \qquad \vec{P} = -\vec{q}/2$$
$$\vec{p'} = -\vec{q}/2 \qquad \vec{P'} = +\vec{q}/2$$
$$q = (0, \vec{q}) \Longrightarrow Q^2 = \vec{q}^{\,2}$$

$$p^\mu \to m\upsilon^\mu + r^\mu$$

$$\langle N(p_2)|V^a_\mu|N(p_1)\rangle = \frac{1}{\mathcal{N}_1\mathcal{N}_2}\overline{u}_\upsilon(r_2)\left[\tilde{G}_E(q^2)\upsilon_\mu + \frac{1}{m_N}\tilde{G}_M(q^2)[S_\mu, S_\nu]q^\nu\right]u_\upsilon(r_1) \times \eta^\dagger\frac{\tau^a}{2}\eta$$

$$u_\upsilon(r) = P^+_\upsilon u(p) = \frac{1}{2}(1+\not\upsilon)u(p)$$

$$\mathcal{N}_i = \sqrt{\frac{E_i + m_N}{2m_N}}$$

with the choice $\upsilon^\mu = (1,0,0,0)$

$$\tilde{G}_E(q^2) \equiv G_E(q^2)$$
$$\tilde{G}_M(q^2) \equiv G_M(q^2)$$

Heavy Baryon Chiral Perturbation Theory

$$\mathcal{L}_{\pi\pi}^{(2)} = \frac{1}{4}F_\pi^2 \left\{ Tr[\nabla_\mu U^\dagger \nabla^\mu U + \chi^\dagger U + \chi U^\dagger] \right\}$$

$$U = exp\left[\frac{i\overrightarrow{\tau} \cdot \overrightarrow{\pi}}{F_\pi}\right]$$

$$\nabla_\mu U = \partial_\mu U - i(v_\mu + a_\mu)U + iU(v_\mu - a_\mu)$$

$$\chi = 2B(s + ip)$$

$$\mathcal{L}_{\pi N}^{(1)} = \overline{N}(iv \cdot D + g_A S \cdot u)N$$

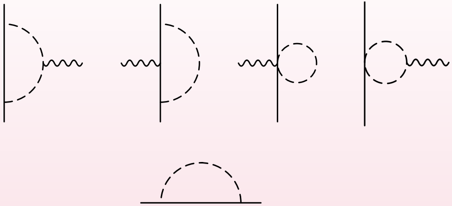
$$N_v \equiv e^{-imv\cdot x} P_{v_+}\Psi, \quad D_\mu \Psi = \partial_\mu \Psi + \Gamma_\mu \Psi$$

$$\Gamma_\mu = \frac{1}{2}[u^\dagger, \partial_\mu u] - \frac{i}{2}(v_\mu + a_\mu)u - \frac{i}{2}u(v_\mu - a_\mu)u^\dagger$$

$$u_\mu \equiv i(u^\dagger \nabla_\mu u - u \nabla_\mu u^\dagger)$$

$$S_\mu = \frac{1}{2}i\gamma_5 \sigma_{\mu\nu} v^\mu$$

Non-zero loop diagrams at order $\mathcal{O}(p^3)$



V.Bernard,N.Kaiser,J.Kambor,U.Meissner
Nucl.Phys.B388(1992)

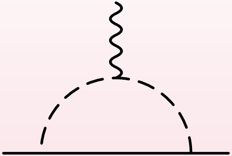
Power Counting Scheme of HBChPT:

$$D = 4N_L - 2I_M - I_B + \sum_{n=1}^{\infty} 2n\, N_{2n}^M + \sum_{n=1}^{\infty} n\, N_n^B$$

where

- $N_L \equiv$ number of independent loop momenta
- $I_M \equiv$ number of internal pion lines
- N_{2n}^M number of pion vertices originating from L_{2n}
- $N_M = \sum_{n=1}^{\infty} 2nN_{2n}^M \equiv$ total number of pion vertices
- $I_B \equiv$ number of internal nucleon lines
- $N_n^B \equiv$ number of baryonic vertices originating from $L_{\pi N}^{(n)}$
- $N_B = \sum_{n=1}^{\infty} \equiv$ number of baryonic vertices

Example of Loop Diagram calculation



defining

$$\tilde{m}^2 = m_\pi^2 - q^2 x(1-x).$$

$$L = \frac{\lambda^{d-4}}{16\pi^2}\left[\frac{1}{d-4} + \frac{1}{2}(\gamma_E - 1 - ln4\pi)\right]$$

$$A_{1a} = \int \frac{dl^4}{(2\pi)^4}\,\overline{u}(r_2)\ \frac{-g_a S\cdot(l+q)\tau^a}{F_\pi}\ \frac{i}{\upsilon\cdot(r-l)+i0^+}\ \frac{i\delta^{ab}}{(l+q)^2 - m_\pi^2 + i0^+}$$

$$e\epsilon^{c3b}\epsilon\cdot(l+l+q)\ \frac{i\delta^{dc}}{l^2 - m_\pi^2 + i0^+}\ \frac{g_a S\cdot l\tau^d}{F_\pi}\ u(r_1)$$

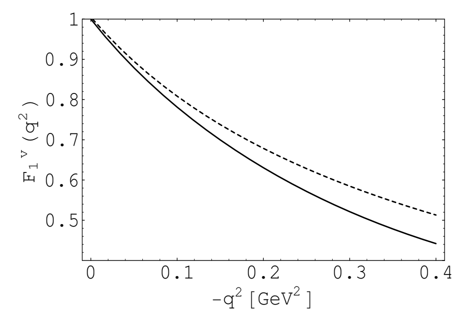
$$= i\frac{g_A^2}{(4\pi F_\pi)^2}\,\eta^\dagger\,\frac{\tau^i}{2}\,\eta\left\{\overline{u}(r_2)\,\epsilon\cdot\upsilon\,u(r_1)\left[\left(6m_\pi^2 - \frac{5}{3}q^2\right)\left(16\pi^2 L + log\frac{m_\pi}{\lambda}\right)\right.\right.$$

$$\left. + 2m_\pi^2 - \frac{2}{3}q^2 + \int_0^1 dx\,(3m_\pi^2 - 5q^2x(1-x))log\left[\frac{\tilde{m}^2}{m_\pi^2}\right]\right]$$

$$\left. - \overline{u}(r_2)[S_\mu, S_\nu]\epsilon^\mu_\nu q^\nu u(r_1)\int_0^1 dx\,4\pi\sqrt{\tilde{m}^2}\right\}$$

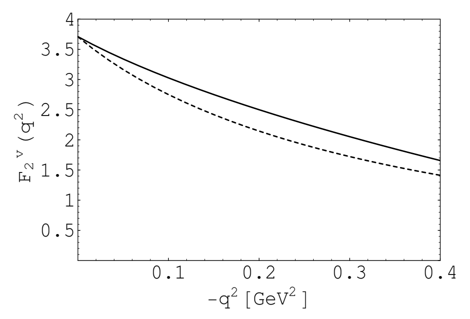
Dirac Form Factor

$$F_1^v(q^2) = 1 + \frac{1}{(4\pi F_\pi)^2}\Big\{ q^2\Big(-\frac{2}{3}g_A^2 - 2B_{10}^{(r)}\Big) + q^2\Big(-\frac{5}{3}g_A^2 - \frac{1}{3}\Big)\log\Big[\frac{m_\pi}{\lambda}\Big] + \int_0^1 dx\Big[m_\pi^2\Big(3g_A^2+1\Big) - q^2 x(1-x)\Big(5g_A^2+1\Big)\Big]\log\Big[\frac{\tilde{m}^2}{m_\pi^2}\Big]\Big\}$$



Pauli Form Factor

$$F_2^v(q^2) = \kappa_v \left\{ 1 - g_A^2 \frac{4\pi M_N}{(4\pi F_\pi)^2} \int_0^1 dx \left[\sqrt{\tilde{m}^2} - m_\pi \right] \right\}$$

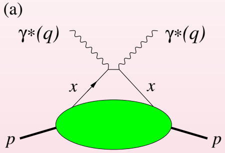


Generalized Parton Distributions

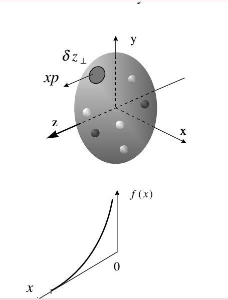
GPDs as generalization of Parton Distributions.

- DIS $ep \longrightarrow eX$

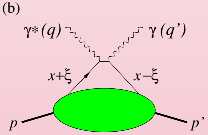
FACTORIZATION: hard partonic subprocess
+
parton distributions



Handbag for CS
PDs



- Case finite momentum transfer to the target



Handbag for DVCS
GPDs

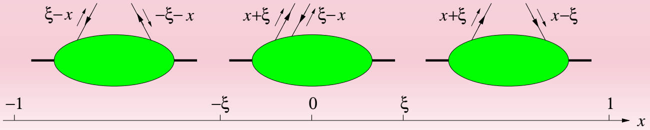
Notation:
Working in light-cone coordinates

$$v^{\pm} = \frac{1}{\sqrt{2}}(v^0 \pm v^3), \quad \mathbf{v} = (v^1, v^2)$$
$$\overline{p} = \frac{p+p'}{2}, \quad \Delta = p' - p, \quad t = \Delta^2$$
$$\xi = \frac{p^+ - p'^+}{p^+ + p'^+}$$

Definition:

$$F^q = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ix\overline{p}^+ z^-} \langle p'|\overline{q}(-\frac{1}{2}z)\gamma^+ q(\frac{1}{2}z)|p\rangle\Big|_{z^+=0, z=0}$$
$$= \frac{1}{2\overline{p}^+}\left[H^q(x,\xi,t)\overline{u}(p')\gamma^+ u(p) + E^q(x,\xi,t)\overline{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p)\right]$$

$x \in [-1, 1]$, $\xi \geqslant 0$



GPDs: properties and relation to Form Factors

- caso $\xi = t = 0$ [forward limit]

  $\Longrightarrow$ *GPDs* reduce to *PDs*

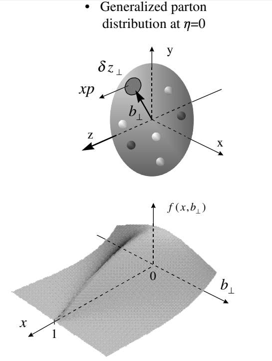
$$H^q(x,0,0) = q(x)$$

no corrisponding relation for E!

- case $\xi = 0, t \neq 0$ [purely transverse momentum transfer]

$$H^q(x,0,-\vec{\Delta}_\perp^2) = \int d^2r_\perp f(x,\mathbf{b}_\perp) e^{-\vec{\Delta}_\perp \cdot b_\perp}$$

$\Longrightarrow$ info about transverse structure of the target



O^{th} moments of GPDs

- $$\int_{-1}^{1} dx\, H^q(x,\xi,\Delta^2) = F_1^q(\Delta^2)\,,$$
  $$\int_{-1}^{1} dx\, E^q(x,\xi,\Delta^2) = F_2^q(\Delta^2)$$

Generalized Form Factors

higher x-moment of GPDs are form factors of the local twist-two operator

$$O^{(n),q}_{\mu,\mu_1\ldots\mu_n} = \frac{1}{n}\overline{q}\,\gamma_{\{\mu}\left(i\overleftrightarrow{D}_{\mu_1}\right)\ldots\left(i\overleftrightarrow{D}_{\mu_n\}}\right)q \;-\; traccia$$

- $n = 1$

1^{th} moments of GPDs

$$\int_{-1}^{1} dx\, x\, H^q(x,\xi,\Delta^2) = A^q_{2,0}(\Delta^2) + \xi^2 C^q_{2,0}(\Delta^2)$$

$$\int_{-1}^{1} dx\, x\, E^q(x,\xi,\Delta^2) = B^q_{2,0}(\Delta^2) - \xi^2 C^q_{2,0}(\Delta^2)$$

Matrix element of singlet twist-2 operator:

$$\begin{aligned}\langle p'|O^q_{\{\mu\nu\}}|p\rangle \equiv\;& \frac{i}{2}\langle p'|\overline{q}\gamma_{\{\mu}\overleftrightarrow{D}_{\nu\}}q|p\rangle \\ &= A^q_{2,0}(\Delta^2)\;\overline{u}(p')\gamma_{\{\mu}\overline{p}_{\nu\}}u(p) \\ &- B^q_{2,0}(\Delta^2)\;\frac{i}{2m_N}\;\overline{u}(p')\Delta^\alpha\sigma_{\alpha\{\mu}\overline{p}_{\nu\}}u(p) \\ &+ C^q_{2,0}(\Delta^2)\;\frac{1}{m_N}\;\overline{u}(p')u(p)\Delta_{\{\mu}\Delta_{\nu\}}\end{aligned}$$

where

$$\overline{p} = \frac{1}{2}(p'+p),\quad \overleftrightarrow{D}_\mu = \frac{1}{2}(\overrightarrow{D}_\mu - \overleftarrow{D}_\mu)$$

Connection to phenomenology

- caso $\xi = 0$ [limite forward]

$$H^q(x,0,0) = q(x)$$

$$A_2^q(0) = \langle x_q \rangle = \int_0^1 dx \;\; x\left(q_{\downarrow}(x) + q_{\uparrow}(x)\right)$$

- as for Form Factors

$$A_{2,0}^q = A_{2,0}^q(0)\left[1 + \frac{1}{6}\langle r^2 \rangle q^2 + O(q^4)\right]$$

$$(r_{A,2}^{v})^2 = \frac{6}{A_{2,0}(0)} \frac{dA_{2,0}^{v}(q^2)}{dq^2}\bigg|_{q^2=0}$$

NON-RELATIVISTIC REDUCTION

By performing the non-relativistic reduction we define three new structures:

$$\langle p'|O^q_{\{\mu\nu\}}\ |p\rangle = \frac{1}{\mathcal{N}_1\mathcal{N}_2}\overline{u}_\upsilon r_2\Big\{\mathcal{G}_1(t)(\upsilon_\mu\upsilon_\nu + \upsilon_\nu\upsilon_\mu - \frac{1}{2}g_{\mu\nu}) + \mathcal{G}_2(t)([S_\mu, S_\alpha]\Delta^\alpha\upsilon_\nu + \upsilon_\mu[S_\nu, S_\alpha]\Delta^\alpha) + \mathcal{G}_3(t)(\Delta_\mu\Delta_\nu + \Delta_\nu\Delta_\mu - \frac{1}{2}g_{\mu\nu}\Delta^2)\Big\}u_\upsilon(r_1) \times \eta^\dagger\frac{\tau^a}{2}\eta\,.$$

related to the Generalized Form Factors through the following expressions:

$$\mathcal{G}_1(t) = \langle\overline{u}u\rangle\sqrt{m_N^2 - \frac{t}{4}}\left[A^v_{2,0}(t) + \frac{t}{4m_N^2}B^v_{2,0}(t)\right]$$

$$\mathcal{G}_2(t) = \langle\overline{u}u\rangle\sqrt{m_N^2 - \frac{t}{4}}\left[\frac{1}{m_N}(A^v_{2,0}(t) + B^v_{2,0}(t))\right]$$

$$\mathcal{G}_3(t) = \langle\overline{u}u\rangle\left[\frac{1}{m_N}C^v_{2,0}(t)\right]$$

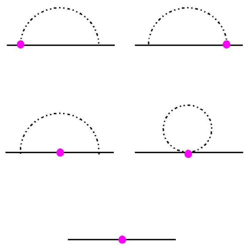
Loop diagrams up to order $\mathcal{O}(p^4)$

To this order the three structures take the form

$$\mathcal{G}_1(t) = \langle \overline{u}u\rangle \sqrt{m_N^2 - \frac{t}{4}}\left[A^v_{2,0}(t) + \frac{t}{4m_N^2}B^v_{2,0}(0) + \mathcal{O}(p^5)\right]$$

$$\mathcal{G}_2(t) = \langle \overline{u}u\rangle \sqrt{m_N^2 - \frac{t}{4}}\left[\frac{1}{m_N}(A^v_{2,0}(t) + B^v_{2,0}(0)) + \mathcal{O}(p^5)\right]$$

$$\mathcal{G}_3(t) = \langle \overline{u}u\rangle \left[\frac{1}{m_N}C^v_{2,0}(0) + \mathcal{O}(p^5)\right]$$



Introduction of new mathematical tools:

$$V_{\mu\nu} = \tilde{v}_{\mu\nu}\frac{1}{2}(u^\dagger\tau^a u + u\tau^a u^\dagger)$$

$$\mathcal{L}^{(2)}_{\pi N}\ ,\ \mathcal{L}^{(3)}_{\pi N}\ ,\ \mathcal{L}^{(4)}_{\pi N}$$

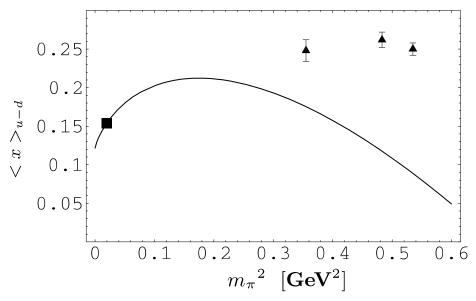
$\xi = t = 0$

The first structure $\mathcal{G}_1(t)$ reduces to

$$\mathcal{G}_1(0) = \langle \overline{u}u \rangle \; m_N \left[A^v_{2,0}(0) + \mathcal{O}(p^5) \right] \\ = \langle \overline{u}u \rangle \; m_N \; \langle x \rangle_{u-d}$$

and

$$\langle x \rangle_{u-d} = \tilde{c}_8 \left\{ 1 - \frac{(3g_A^2+1)}{(4\pi F_\pi)^2} m_\pi^2 \, log\left[\frac{m_\pi^2}{\lambda^2} \right] - \frac{2}{(4\pi F_\pi)^2} g_A^2 m_\pi^2 \right\} + 4m_\pi^2 \, S_{40}^{(r)}$$



Results

The forward limit HBChPT calculation, performed for the Generalized Form Factors $n = 1$ up to order $\mathcal{O}(p^4)$, give the following results:

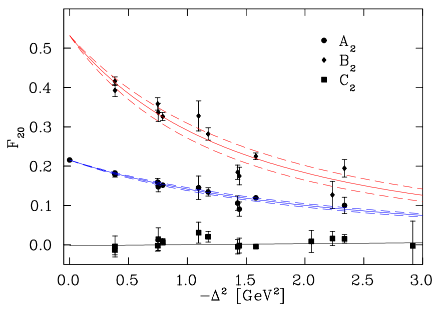
$$A^v_{2,0}(t) = \tilde{c}_8 \left\{ 1 - \frac{(3g_A^2 + 1)}{(4\pi F_\pi)^2} m_\pi^2 \, log \left[\frac{m_\pi^2}{\lambda^2} \right] - \frac{2}{(4\pi F_\pi)^2} g_A^2 m_\pi^2 \right\} + 4 m_\pi^2 S_{40}^{(r)} + S_{44} \, t + \mathcal{O}(p^5) \, .$$

$$B^v_{2,0}(0) = B_{40} \, m_N + \mathcal{O}(p^5)$$

$$C^v_{2,0}(0) = S_{42} \, m_N + \mathcal{O}(p^5)$$

Conclusions and Outlook

- Reproduction of HBChPT results for Dirac ($F_1^v(q^2)$) and Pauli ($F_2^v(q^2)$) Form Factors;
- Introduction of GPDs and definiton of the Generalized Isovector Form Factors ($A_{2,0}^v(t)$, $B_{2,0}^v(t)$, $C_{2,0}^v(t)$);
- Derivation of the non-relativistic Form Factors $\mathcal{G}_i(t)$;
- Construction of new Lagrangians and introduction of new counterterms;
- Forward limit up to order $\mathcal{O}(p^4)$, derivation of $< x >_{u-d}$ and comparison with phenomenological and lattice data.



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NEXT: OFF-FORWARD LIMIT UP TO ORDER $\mathcal{O}(p^5)$ AND MAPPING OF LOOP DIAGRAMS INTO $A_{2,0}^v(t)$, $B_{2,0}^v(t)$, $C_{2,0}^v(t)$