The equation of state in lattice QCD: with physical quark masses towards the continuum limit

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Leipzig meeting

QCD thermodynamics at $\mu = 0$: unestablished

For $N_f = 2 + 1$, with physical quark masses:

- Order of transition
- T_c
- EoS

- Karsch, Laermann, Peikert, 2000:
  - staggered p4 action: very good at extreme high T
  - $N_t = 4$, $a \simeq 0.25 fm$ @ T_c: far from cont. lim.

    scaling, taste viloation
  - unphysical quark masses

    $m_{ud}^{sim} = m_{ud}^{phys}$,
    $m_s^{sim} \simeq 2m_s^{phys}$ @ T_c
    Change as T
  - finite step size for molecular dynamics
  - string tension for scale

- We do
  - Tree level improved gauge, stout-link improved KS (Morningstar & Peardon)
  - $N_t = 4, 6$
  - "physical quark mass"

    always keep
    $m_{ud}^{sim} = m_{ud}^{phys}$,
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  - exact algorithm: RHMC (Clark & Kennedy)
  - r_0 for scale

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Stout-Link Smearing

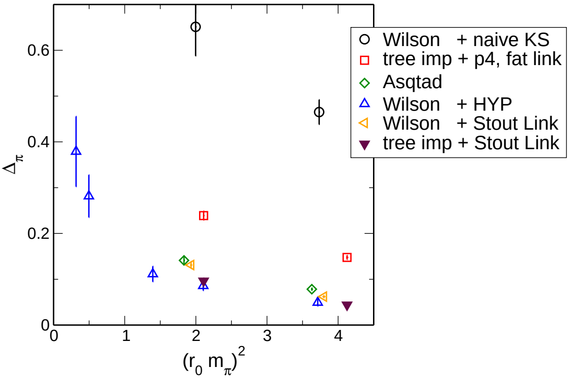
- Analitic in U_{orig}
  - Convensional molecular dynamics can be used
- Link smearing with simlar effect with HYP
  - reduces taste vilolation very well

Pion taste multiplet splitting

$$\Delta_\pi = \frac{m_{\pi'} - m_\pi}{m_\pi}$$

π: Goldstone pion

π': lightest non-Goldstone



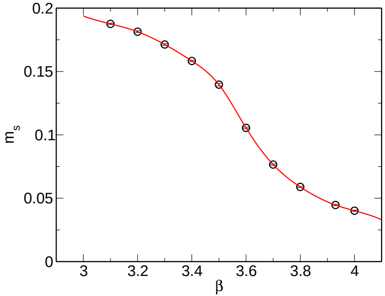
LCP

The line of constant physics

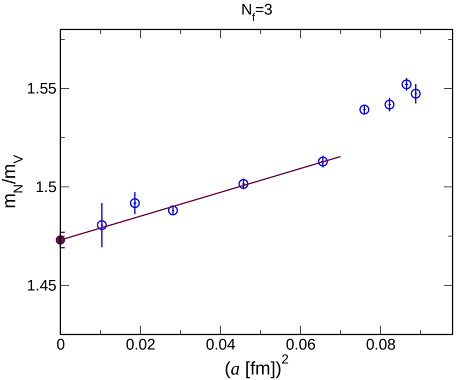
- $m_{ud}(\beta)$, $m_s(\beta)$.
- Fix m_s using $N_f = 3$ degenerate simulations and LO ChPT

$$\left.\frac{m_{PS}^2}{m_V^2}\right|_{m=m_s}^{N_f=3} \simeq \left.\frac{m_{PS(ss)}^2}{m_{V(ss)}^2}\right|_{m=(m_{ud},m_s)}^{N_f=(2+1)} = \frac{m_{\eta_s}^2}{m_\phi^2} = \frac{2m_K^2 - m_\pi^2}{m_\phi^2}.$$

- obtain $m_s(\beta)$
- $m_{ud} = m_s/25$
- should be checked in $N_f = 2+1$ simulation.



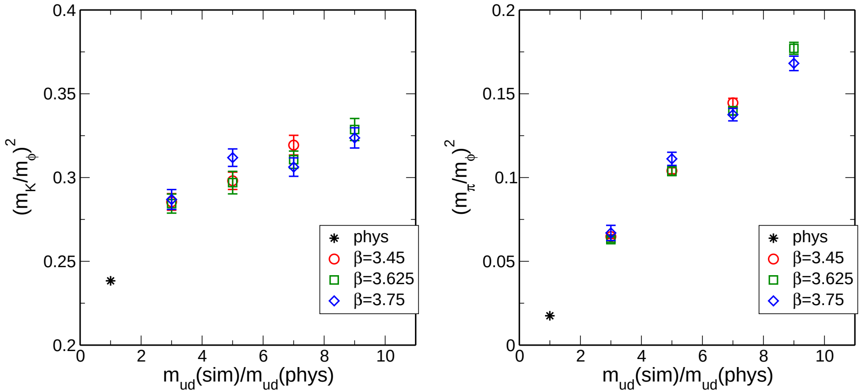
$N_f = 3$ Spectrum
scaling of N/ρ ratio



- $a \simeq 0.25$ fm $\rightarrow 0 : 2 \sim 3\%$ effect

$N_f = 2 + 1$ Spectrum

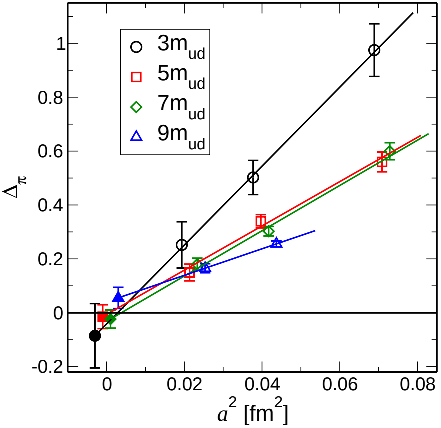
$\beta = 3.45, 3.625, 3.75$: $a \simeq \{5, 7, 9\}/T_c$, $m_s^{sim} = m_s^{phys}$ by LCP



- Discrepancy to the physical spectrum $\lesssim 5 - 8\%$
- Will be tested with finer lattice.

$N_f = 2 + 1$ Spectrum: Taste Symmetry

$\Delta_\pi = \frac{m_{\pi'} - m_\pi}{m_\pi}$



- Linear extrapolation in reasonable χ^2
- Taste symmetry under control

Simulation Procedure

- many β's (16 pts for $N_t = 4$, 14 pts for $N_t = 6$)
- given β, $m_s^{sim} = m_s^{phys}(\beta)$ fixed. $\rightarrow m_{ud}^{phys} = m_s^{phys}/25$.
- $T \neq 0$: $m_{ud}^{sim} = m_{ud}^{phys}$
  - $N_s = 3N_t$.
- $T = 0$: $m_{ud}^{sim} = \{3, 5, 7, 9\} \times m_{ud}^{phys}$
  - keeping $L_s m_\pi > 3$
- several β's checked: finite size effect less than stat. err.

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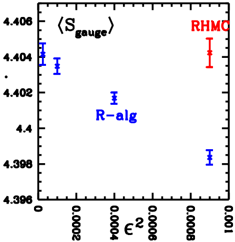
EoS procedure

$$\left.\frac{p}{T^4}\right|_{(\beta_0,m_0)}^{(\beta,m)} = -\left.\frac{f}{T^4}\right|_{(\beta_0,m_0)}^{(\beta,m)}$$

$$= -N_t^4 \int_{(\beta_0,m_0)}^{(\beta,m)} d(\beta, m_{ud}, m_s) \begin{pmatrix} \langle -S_g/\beta \rangle \\ \langle \bar{\psi}\psi_{ud} \rangle \\ \langle \bar{\psi}\psi_s \rangle \end{pmatrix}.$$



$$\left.\frac{p}{T^4}\right|^{(\beta,m)} = \left.\frac{p}{T^4}\right|_{(\beta_0,m_0)}^{(\beta,m)} (T \neq 0) - \left.\frac{p}{T^4}\right|_{(\beta_0,m_0)}^{(\beta,m)} (T = 0).$$

We use exact algorithm: no need of step size extrapolation.

T=0: no $m_{ud}^{sim} = m_{ud}^{phys}$ data.

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Extrapolation only needed for $\langle \bar{\psi}\psi_{ud} \rangle (m_{ud}^{phys}) \leftarrow m_{ud}^{sim} = \{3, 5, 7, 9\} \times m_{ud}^{phys}$.

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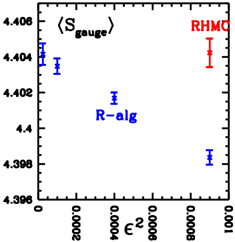

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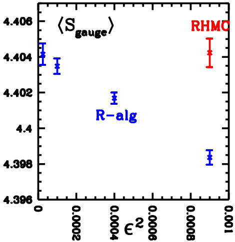

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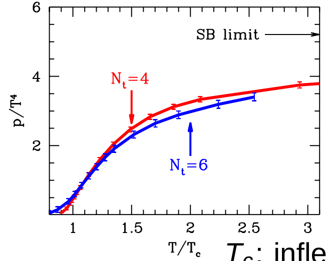
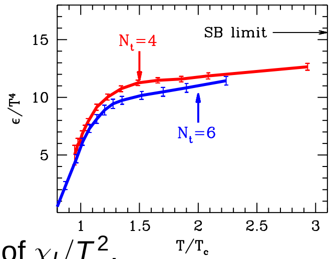
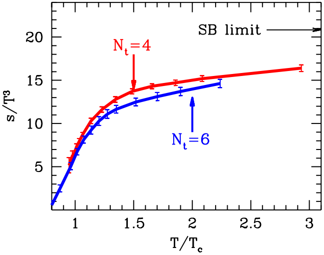
EoS

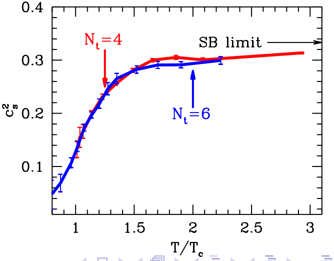
Multiplied with c_{SB}/c_{N_t} ($c_{SB} = \lim_{N_t \to \infty} c_{N_t}$), $N_t = 4$(red), 6(blue).



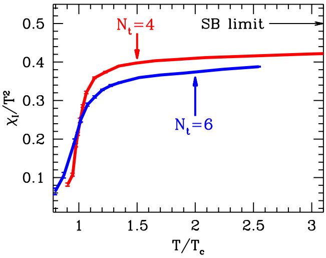


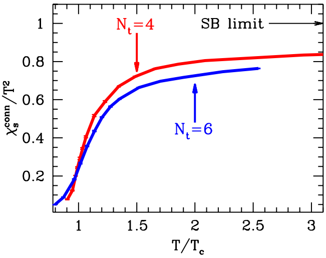
T_c: inflection point of χ_l/T^2.



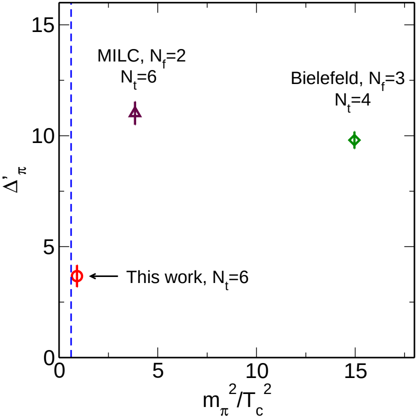


Quark number susceptibility





How close to the physical and continuum limit ?



Assuming $T_c = 173$ MeV.

$$\Delta'_\pi = \frac{m^2_{\pi'} - m^2_\pi}{T_c^2} = (m^2_{\pi'} - m^2_\pi) N_t^2$$

($T = 0$ masses, measured at $\beta_c(N_t)$)

Summary

- Stout-link smearing improvement was used to reduce the taste violation. Works quite well.
- We have obtained LCP with $N_f = 3$ simulation
- $N_f = 2 + 1$:
  - reasonable agreement to real world light meson spectrum
  - controlled flavor symmetry breaking
- The equation of state was calculated with $N_t = 4$ and 6 lattices.
- For the reliable continuum extrapolation, $N_t = 8$ simulation is needed.