Heavy Quark Physics on Large Lattices

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Reminder: Weak quark currents

$$\mathcal{L} = -\frac{g_2}{\sqrt{2}} \left(\overline{u}_L \overline{c}_L \overline{t}_L\right) \gamma^\mu (V_{q_1 q_2}) \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} W_\mu^+ + \text{h.c.} \, ,$$

q_L $V - A$ projected quark spinors, diagonalizing mass matrix,
W_μ charged weak gauge boson, g_2 $SU(2)$ gauge coupling.

Weak interaction eigenstates $\neq$ mass eigenstates

CKM matrix

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

CP violation can be accommodated if imaginary part non-zero.

Relatively large uncertainties in off-diagonal elements involving heavy quarks.

Current values

PDG: $V_{cs} = 0.996(13)$, $V_{cd} = 0.224(12)$,
$V_{cb} = 41.3(1.5) \times 10^{-3}$, $V_{ub} = 3.67(47) \times 10^{-3}$,
$|V_{tb}V_{ts}^*| = -47(8) \times 10^{-3}$, $V_{td} = 0.0067(8) - i0.0031(4)$

Quenched charm physics on fine lattices

Calculation of (exclusive) semileptonic D (and D_s) decays from QCD. ($D \to \pi, K l\nu$, $D \to \rho, K^* l\nu$, rare D decays: $D \to \rho, K^* l\nu$)

Theoretical understanding of QCD background of decays of quarks within meson environment.

Determination of CKM matrix elements, e.g. V_{cd}, V_{cs}.

High precision charm physics from CLEO-c and CESR-C, and BaBar and Belle.

Lattice calculation of D, D_s and charmonium spectrum and comparison to experiment.

Spectra can be input for model calculations.

Can be simulated on fine lattices with high precision

Quenched b physics on fine lattices

$B \to \pi, Kl\nu,\ B \to \rho, K^* l\nu,\ B \to D, D^* l\nu,\ B \to \rho, K^* \gamma$

Extraction of CKM matrix elements

Search for new physics (heavy particles (t quarks) in loops: sensitivity to non-standard model heavy particles).

B, B_s meson spectrum

Still needs extrapolations or/and effective theory methods on the lattice

Semileptonic decays $\rightarrow$ tree-level information on CKM,
best information sources on weak current couplings.

Example: $D \rightarrow \pi, K$ exclusive decay form factors

$$\langle \pi, K(k) | V^\mu | D(p) \rangle = f_+(q^2)(p + k - q\Delta)^\mu + f_0(q^2) q^\mu \Delta$$

$q = p - k$ and $\Delta = (M_D^2 - m_{\pi,K}^2)/q^2$.

f_0 contribution $\propto m_l$, contribution to overall matrix element small.

Differential decay rate

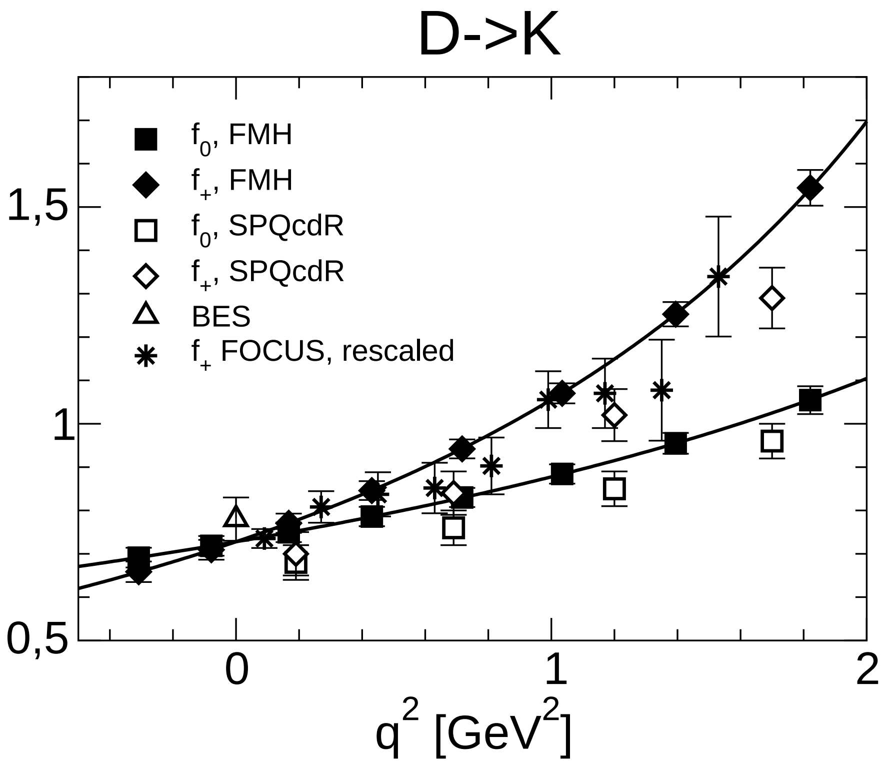
$$\frac{d\Gamma}{dq^2} \propto G_F^2 |V_{cd,s}|^2 |f_+(q^2)|^2.$$

Recent lattice results on $D \to K$

C. Aubin et al 2004, Fermilab, MILC, HPQCD (FMH): $N_f = 2_{light} + 1_{strange}$ staggered sea and light valence, $a^{-1} = 1.6$ GeV, $L = 2.5$ fm, $400 - 500$ cfgs.

A. Abada et al 2001, SPQcdR: $N_f = 0$, NP Wilson, $a^{-1} = 2.7$ GeV, $L = 1.7$ fm, 200 cfgs.
In agreement with other quenched lattice calculations: UKQCD, Fermilab, JLQCD.

experimental results from BES, FOCUS, CLEO



FMH data points sent by A. Kronfeld and M. Okamoto, priv. comm.

Leptonic decay constants:

$$b \to u: \quad B^- \to l^- \bar{\nu}_l, \quad BR \propto f_B |V_{ub}|^2 \tag{1}$$
$$c \to d: \quad D^+ \to l^+ \nu_l, \quad BR \propto f_D |V_{cd}|^2 \tag{2}$$

$\langle 0|A_\mu(x)|B(p)\rangle = -if_B p_\mu e^{-ipx}$, $A_\mu = \overline{q}\gamma_\mu\gamma_5 b$.

Experimental results:

$$f_{D_s} = 280(17)(25)(35) \text{ MeV (CLEO 1998)}$$
$$f_{D_s} = 285(19)(40) \text{ MeV (ALEPH 2002)}$$
$$f_D = 223(17)(3) \text{ MeV (CLEO 2005).}$$

Several recent lattice results for $f_{D_s} \sim 240 - 250$ MeV.

Simulation details:

Non-perturbatively improved clover quarks, Wilson gluons

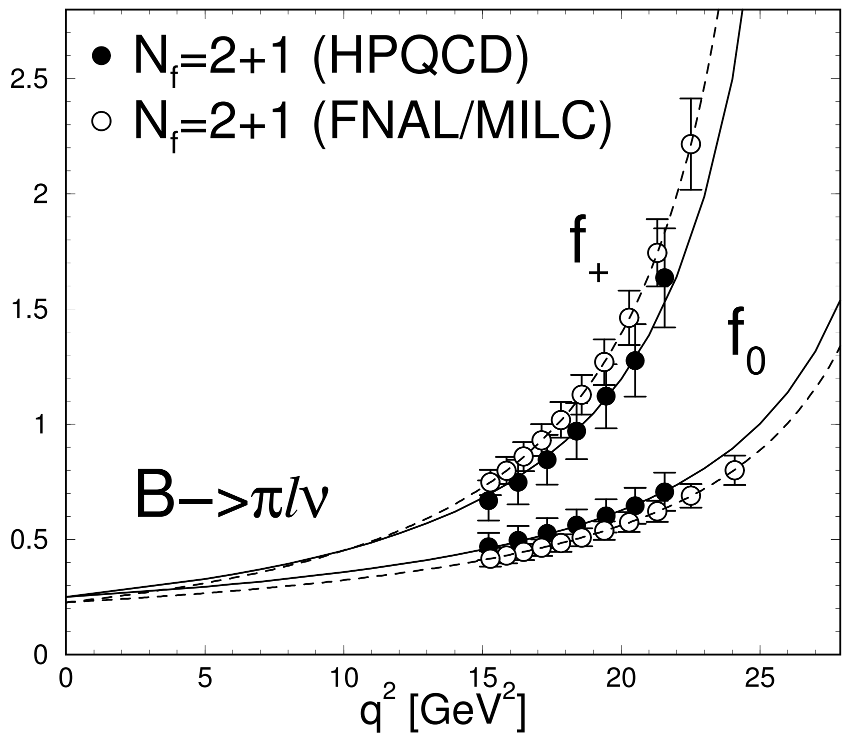
Lattice spacing a	~ 0.04 fm
Spatial extent	~ 1.6 fm

κ	M_π[GeV]
0.13519	0.55
0.13498	0.69
0.13472	0.87
0.13000	2.8
0.12900	3.2
0.12100	5.6

60 cfg's analyzed, errors on masses seem typically $< 1\%$.

Recent lattice results on $B \to \pi$ decay form factors:

(from M. Okamoto, Lattice 2005, Dublin, plenary talk)



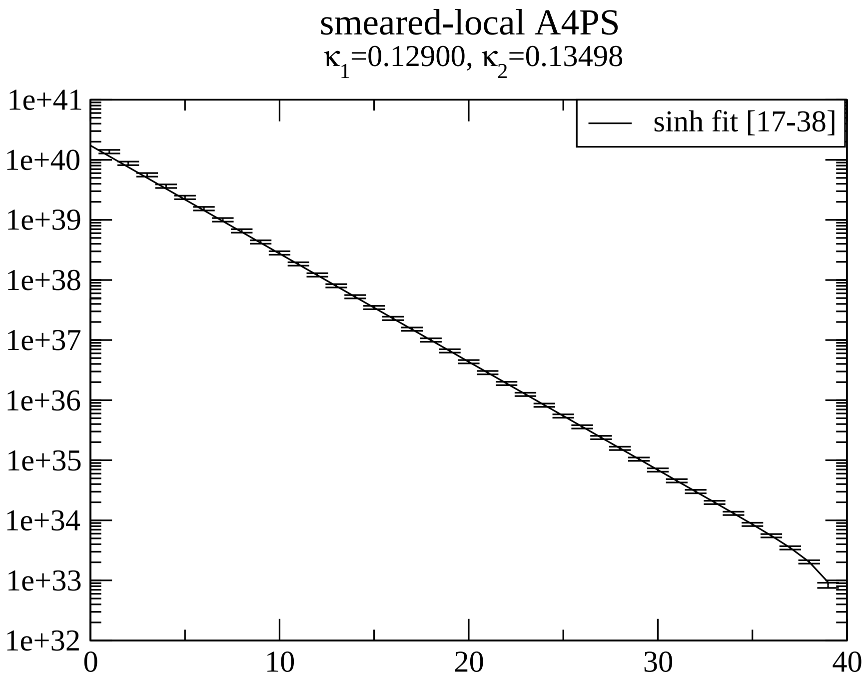
In our simulations trying to reach q^2 around ~ 5 GeV^2.

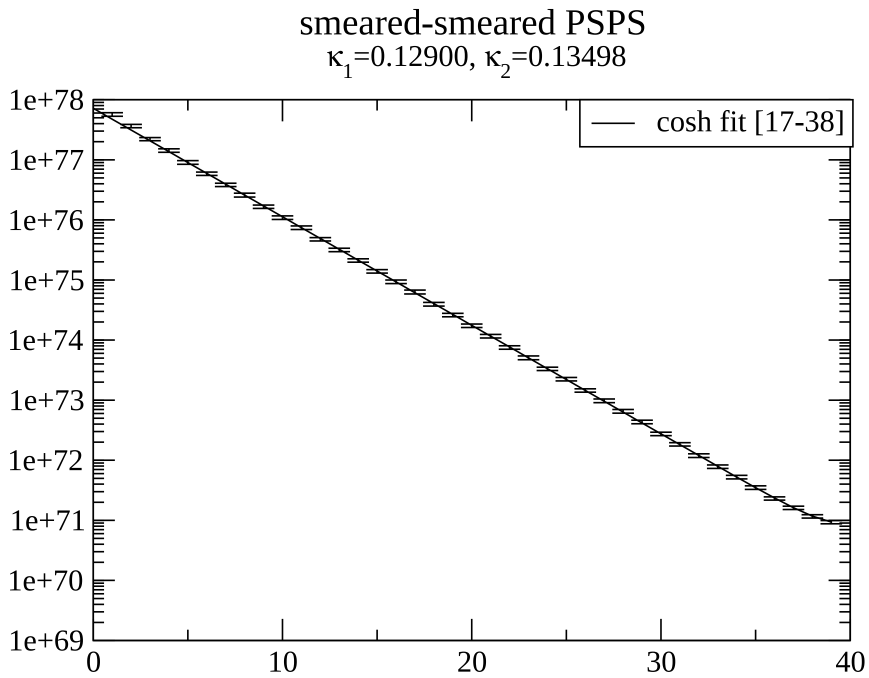
Amplitudes:

$$\frac{1}{\sqrt{M_{D_s}}} \langle 0 | A_4(0) | D_s(\vec{p}=0) \rangle = f_{D_s} \sqrt{M_{D_s}}$$

$f_{D_s}\sqrt{M_{D_s}} \propto Z_{SL}(\langle A_4 PS \rangle)/\sqrt{Z_{SS}(\langle PSPS \rangle)}$, PS pseudoscalar operator projecting on D_s.

Quark combination roughly charm-strange:

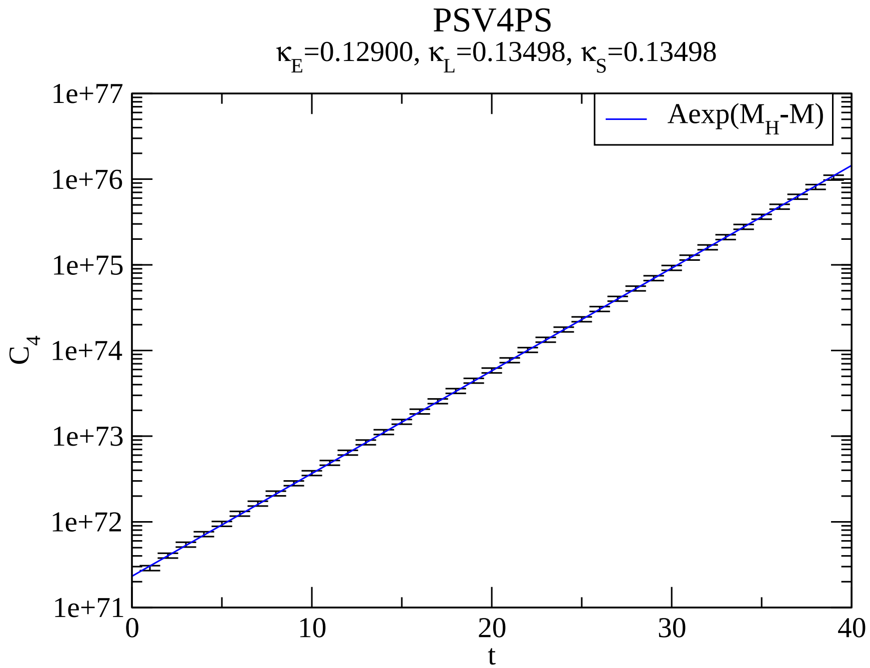




$$C_\mu(t_{ext}, t, \vec{p}, \vec{p}_H) \rightarrow \frac{\sqrt{Z_{SS}}(\langle HH\rangle)}{2E_H} e^{-tE} \frac{\sqrt{Z_{SS}}(\langle PSPS\rangle)}{2E} e^{-(t_{ext}-t)E_H} \langle PS(\vec{p}|V_\mu(0)|H(\vec{p}_H\rangle$$

PS light meson with energy E at time 0, mass M, H heavy-light meson with energy E_H, mass M_H at time t_{ext}.

Correlation function $C_4(t_{ext}, t, \vec{p} = 0, \vec{p}_H = 0)$



Summary

- Simulations of charmed and bottom meson matrix elements and spectra on very fine lattices: discretization errors expected to be small.
- Signals for charmed and light 2- and 3-point correlation functions appear reasonable.
- First results to appear in the near future.