UNIVERSITY OF LEIPZIG
INSTITUTE FOR THEORETICAL PHYSICS
Department: Theory of Elementary Particles

TP3 2017

Lecturer: PD Dr. A. Schiller

List of problems 12 (35. and 36. required, 37. voluntary)

35. Show explicitly that two successive Lorentz transformations in the same direction are equivalent to a single Lorentz transformation with a velocity

$$v = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}} \, .$$

Identify the corresponding Lorentz factor.
This is an alternative way to derive the parallel–velocity addition law.

36. A coordinate system K' moves with a velocity $\mathbf{v}$ relative to another system K. In K' a particle has a velocity $\mathbf{u}'$ and an acceleration $\mathbf{a}'$. Find the parallel and transverse components of the acceleration $\mathbf{a}_\parallel$ and $\mathbf{a}_\perp$ in system K with respect to the direction given by $\mathbf{v}$.

37. voluntary

Under a general Lorentz transformation with relative velocity $c\boldsymbol{\beta}$ between the inertial frames K and K' ($\gamma = 1/\sqrt{1-\beta^2}$) the electric and magnetic part of the electromagnetic field transforms as follows (later shown in the lecture)

$$\begin{aligned} \mathbf{E}' &= \gamma \, (\mathbf{E} + c\boldsymbol{\beta} \times \mathbf{B}) - \frac{\gamma^2}{\gamma + 1} \, \boldsymbol{\beta} \, (\boldsymbol{\beta} \cdot \mathbf{E}) \, , \\ \mathbf{B}' &= \gamma \left(\mathbf{B} - \boldsymbol{\beta} \times \frac{\mathbf{E}}{c} \right) - \frac{\gamma^2}{\gamma + 1} \, \boldsymbol{\beta} \, (\boldsymbol{\beta} \cdot \mathbf{B}) \, . \end{aligned}$$

Show that $\mathbf{E}^2 - c^2 \, \mathbf{B}^2$ and $\mathbf{E} \cdot \mathbf{B}$ are invariant under that Lorentz transformation.
Hint: It might be useful to decompose the electric and magnetic parts into longitudinal and transverse components with respect to $c\boldsymbol{\beta}$ and consider the transformations of those components individually.