UNIVERSITY OF LEIPZIG
INSTITUTE FOR THEORETICAL PHYSICS
Department: Theory of Elementary Particles

TP3 2017

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Merry Christmas and a Happy New Year

List of problems 10 (28., 29. and 30. required, 31. voluntary)

28. Linearly polarized light of the form

$$E_x(z,t) = E_0\, \mathrm{e}^{\mathrm{i}\,(kz-\omega t)}$$

is incident normally onto a material which has index of refraction n_R for right-hand circularly polarized light and n_L for left-hand circularly polarized light.
Calculate the intensity and polarization of the reflected light.

29. A harmonic plane wave of frequency ν is incident normally on an interface between two dielectric media of indices of refraction n_1 and n_2. A fraction ρ of the energy is reflected and forms a standing wave when combined with the incoming wave.
Recall that on reflection the electric field changes its phase by π for $n_2 > n_1$.
(a) Find an expression for the total electric field $\mathbf{E}$ as a function of the distance d from the interface (z-direction). Determine the positions of the maxima and minima of the time averaged $\langle \mathbf{E}^2 \rangle = \langle E^2 \rangle$.
(b) From the behaviour of the electric field, determine the phase change on reflection of the magnetic field. Find $B(z,t)$ and $\langle B^2 \rangle$.
(c) When O. Wiener did such an experiment using a photographic plate in 1890, a band of minimum darkening of the plate was found for $d = 0$. Was the darkening caused by the electric or the magnetic field?

30. An approximate monochromatic plane wave packet in one dimension has the instantaneous form, $u(x,0) = f(x)\,\mathrm{e}^{\mathrm{i}\,k_0 x}$, with $f(x)$ the modulation envelope. For the forms
(a)

$$f(x) = N\,\mathrm{e}^{-\alpha^2 x^2/4}\,,$$

(b)

$$f(x) = N \text{ for } |x| < a\,, \quad f(x) = 0 \text{ for } |x| > a$$

calculate the wave-number spectrum $|A(k)|^2$ of the packet and sketch $|u(x,0)|^2$ and $|A(k)|^2$.
Evaluate explicitly the rms deviations from the mean Δx and Δk (defined in terms of the intensities $|u(x,0)|^2$ and $|A(k)|^2$) and test the inequality $\Delta x\,\Delta k \geq 1/2$.

31. voluntary

A circularly polarized plane wave moving in the z direction has a finite extent in the x and y directions. Assuming that the amplitude modulation is slowly varying (the wave is many wavelengths broad), show that the electric and magnetic fields are given approximately by

$$\begin{aligned} \mathbf{E}(x,y,z,t) &\approx \left[E_0(x,y)\,(\mathbf{e}_1 \pm \mathrm{i}\,\mathbf{e}_2) + \frac{\mathrm{i}}{k}\left(\frac{\partial E_0}{\partial x} \pm \mathrm{i}\,\frac{\partial E_0}{\partial y}\right)\mathbf{e}_3\right] \mathrm{e}^{\mathrm{i}\,kz-\mathrm{i}\,\omega t}\,, \\ \mathbf{B} &\approx \mp\mathrm{i}\sqrt{\mu\varepsilon}\,\mathbf{E} \end{aligned}$$

where $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ are unit vectors in the x, y, z directions.
Hint: Since the wave has a finite extent in the x and y directions, the wave is not transverse. Assuming the wave is dominated by the transverse polarization, but has a small longitudinal part, approximate it as follows

$$\mathbf{E}(x,y,z,t) \approx [E_0(x,y)\,(\mathbf{e}_1 \pm \mathrm{i}\,\mathbf{e}_2) + F_0(x,y)\,\mathbf{e}_3]\,\mathrm{e}^{\mathrm{i}\,(kz-\omega t)}\,.$$

Using the Maxwell equations, find $F_0(x,y)$ and the magnetic field.