UNIVERSITY OF LEIPZIG
INSTITUTE FOR THEORETICAL PHYSICS
Department: Theory of Elementary Particles

TP3 2017

Lecturer: PD Dr. A. Schiller

List of problems 9 (25. and 26. required, 27. voluntary)

25. A transverse electromagnetic wave (**E** and **H** components are perpendicular to the propagation direction and perpendicular to each other) is incident normally in vacuum on a perfectly absorbing flat screen.

    (a) From the law of conservation of linear momentum, show that the pressure (called radiation pressure) exerted on the screen is equal to the field energy per unit volume in the wave.

    (b) In the neighborhood of the earth the flux of electromagnetic energy from the sun is approximately 1.4 kW/m^2. If an interplanetary "sailplane" had a sail of 1 g/m^2 of area and negligible other weight, what would be its maximum acceleration in meters per second squared due to the solar radiation pressure?

26. Consider a possible solution to Maxwell's equation given by

$$\mathbf{A}(\mathbf{x},t) = \mathbf{A}_0\, \mathrm{e}^{\mathrm{i}\,(\mathbf{k}\cdot\mathbf{x}-\omega t)}\,, \quad \Phi(\mathbf{x},t) = 0\,,$$

    where **A** is the vector potential and Φ is the scalar potential. Further suppose $\mathbf{A_0}$, **k** and ω are constants in space-time. Give, and interpret, the constraints on $\mathbf{A_0}$, **k** and ω imposed by each of the Maxwell's equations in vacuum without sources.

27. voluntary, to collect an additional point:

    A simple model of propagation of radio waves in the earth's atmosphere or ionosphere consists of a flat earth at $z = 0$ and a nonuniform medium with $\varepsilon = \varepsilon(z)$ for $z > 0$. Consider the Maxwell equations under the assumption that the fields are independent of y and can be written as functions of z times $\exp[\mathrm{i}\,(kx - \omega t)]$.
    Show that the wave equation governing the propagation for $z > 0$ is

$$\frac{d^2F}{dz^2} + q^2(z)\,F = 0$$

    where

$$q^2(z) = \omega^2 \mu_0\, \varepsilon(z) - k^2$$

and $F = E_y$ for *horizontal* polarization, and

$$q^2(z) = \omega^2 \mu_0 \, \varepsilon(z) + \frac{1}{2\varepsilon} \frac{d^2\varepsilon}{dz^2} - \frac{3}{4\varepsilon^2} \left(\frac{d\varepsilon}{dz} \right)^2 - k^2$$

with $F = \sqrt{\varepsilon/\varepsilon_0} \, E_z$ for *vertical* polarization.
Note that the fields have also a vector component in the direction of propagation (no transverse waves!) not discussed here.