UNIVERSITY OF LEIPZIG
INSTITUTE FOR THEORETICAL PHYSICS
Department: Theory of Elementary Particles

TP3 2017

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List of problems 6

16. Using the tensorial notation, determine the Poisson brackets formed from the Cartesian components
(i) of the momentum $\mathbf{p}$ and the angular momentum $\mathbf{J} = \mathbf{r} \times \mathbf{p}$ of a particle,
(ii) of $\mathbf{J}$.

17. Find the general solution for the motion of a particle $(x(t), p(t))$ the Hamilton function of which is given by $(m \equiv 1)$

$$H(x, p) = \frac{p^2}{2} + \frac{\omega_0^2 x^2}{2} + \lambda \left(\frac{p^2}{2} + \frac{\omega_0^2 x^2}{2} \right)^2 .$$

Hint: From the Hamilton's equation of motion derive first, that $p^2 + \omega_0^2 x^2$ is time independent. Express that constant through the found general solution.

18. The Hamilton function of a system has the form

$$H(q, p) = \frac{1}{2} \left(\frac{1}{q^2} + p^2 \, q^4 \right) .$$

Determine the Hamilton's equations of motion and the equation of motion for q.
Find a possible canonical transformation which reduces the Hamilton function to that of a harmonic oscillator

$$H'(Q, P) = \frac{1}{2} \left(Q^2 + P^2 \right) .$$

Show that the general solution for the transformed variables satifies the found equations of motion derived from the original Hamiltonian what is equivalent to find the solutions for $q(t)$ and $p(t)$.
Hint: Make a guess of a transformation $(q, p) \to (Q, P)$ and check whether that transformation is canonical.