UNIVERSITY OF LEIPZIG
INSTITUTE FOR THEORETICAL PHYSICS
Department: Theory of Elementary Particles

TP3 2017

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List of problems 4

10. (i) Derive the form of the Lagrange function (coordinate x and time t)

$$L = -mc^2 \sqrt{1 - \frac{1}{c^2}\left(\frac{dx}{dt}\right)^2}$$

under the transformation to the new coordinate q and the new "time" τ (λ is a constant):

$$x = q\cosh\lambda + \tau c \sinh\lambda\,, \quad t = \frac{q}{c}\sinh\lambda + \tau\cosh\lambda\,.$$

(Comment: L is the Lagrangian for a free relativistic particle, the transformation is a special Lorentz transformation.)
(ii) Derive the Lagrangian and the equation of motion for a particle in a field given by the potential energy function $V(x)$ introducing the "local" time $\tau = t - \lambda x$ (one-dimensional motion, λ is a constant).

11. Find the first integrals of the equations of motion for a particle of mass m and charge q in a magnetic field given by the vector potential (scalar potential $\Phi = 0$)
(i) of a constant magnetic dipole $\mathbf{m}_d$ at the origin

$$\mathbf{A} = \frac{\mu_0}{4\pi}\frac{\mathbf{m}_d \times \mathbf{r}}{r^3}\,.$$

Hints: Choose the direction of $\mathbf{m}_d$ along the z-direction. Cylindrical coordinates are eventually useful.
(ii) (α is a dimensionful constant)

$$A_\varphi = \frac{\alpha}{\rho}\,, \quad A_\rho = A_z = 0\,.$$

Why does case (ii) describe a free motion of the charged particle?

12. Assume that the residual mass of a rocket, without payload or fuel, is a given fraction λ of the initial mass including fuel (but still without payload). Show that the total take–off mass required to accelerate a payload m to velocity v is

$$M_0 = m\frac{1-\lambda}{\exp(-v/u) - \lambda}\,.$$

If $\lambda = 0.15$, what is the upper limit to the velocity attainable with an ejection velocity $u = 2.5$ km s^{-1}?