UNIVERSITY OF LEIPZIG
INSTITUTE FOR THEORETICAL PHYSICS
Department: Theory of Elementary Particles

TP3 2017

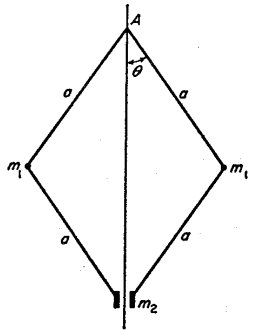
Lecturer: PD Dr. A. Schiller

List of problems 3

7. A particle of mass m and charge q can move along a vertical circle of radius R in the constant gravitational field of the earth. Another charge q is fixed to the lowest point of the circle.
   Find the equilibrium position and the frequency of small oscillations of the particle.

8. Find the Lagrangian and the equation of motion for the system shown in the Figure using the angle θ as generalized coordinate. The system freely rotates about the vertical axis with angular velocity Ω in the constant gravitational field of the earth. The rods of length a are assumed to be massless.
   Discuss qualitatively the motion for the case of equal masses $m = m_1 = m_2$ by analyzing the effective potential energy of the system $V_{\text{eff}}(\theta)$.
   Show that for equal masses m and $\Omega > \Omega_0 = \sqrt{2g/a}$ the frequency of small oscillations around the equilibrium angle (not equal zero!) is

$$\omega = \Omega \sqrt{\frac{\Omega^4 - \Omega_0^4}{3\Omega^4 - 2\Omega_0^4}}\,.$$



9. A light inextensible string passes over a light smooth pulley, and carries a mass $m_1 = 4m$ on one end. The other end supports a second light pulley with a string over it carrying masses $m_2 = 3m$ and $m_3 = m$ on the two ends.
   Using a suitable pair of generalized coordinates, write down the Lagrangian of the system and the Lagrange's equations of motions. Find the downward accelerations of the three masses $g/7$, $3g/7$ and -$5g/7$.
   Using the method of Lagrange multipliers, find the tensions in the strings for those particular mass values. Explain why the first pulley turns, although the total mass on each side is the same (see figure).