UNIVERSITY OF LEIPZIG
INSTITUTE FOR THEORETICAL PHYSICS
Department: Theory of Elementary Particles

TP3 2017

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List of problems 2

4. The position on the surface of a cone of semi-vertical angle α is specified by the distance r from the vertex and the azimuthal angle φ about the axis. Show (derive!) that the shortest path (or geodesic) along the surface between two given points is specified by a function $r(\varphi)$ obeying the equation

$$r\frac{d^2r}{d\varphi^2} - 2\left(\frac{dr}{d\varphi}\right)^2 - r^2\sin^2\alpha = 0\,.$$

Verify that this equation is satisfied if ($\sec\beta = 1/\cos\beta$)

$$r = r_0\sec[(\varphi-\varphi_0)\sin\alpha]\,.$$

5. Find the Lagrangian and the equation of motion for the following system placed in the uniform gravitational field (acceleration g): simple pendulum of mass m and length l whose point of support
(i) oscillates horizontally in the plane of motion of the pendulum according to the law $x = a\cos\omega t$.
(ii) rotates with constant angular frequency ω along a circle of radius a. The circle with the point of support lies is in the vertical plane (plane of motion of the pendulum).
Hint: To find the simplest form of the Lagrangian, remove a total derivative from L.

6. Find the Lagrangian and the equation of motion for a system in the constant gravitational field of the earth consisting of two vertical massless springs (each with spring constant k and relaxed length l, recoiling force: - k $\times$ extension of spring from equilibrium) fixed at one end and connected by a point particle of mass m (see Figure). The particle can move only vertically. Find the frequency of oscillations of the system.
At $t = 0$ the particle is at rest at distance l above the ground (not the equilibrium position!). Solve the equation of motion for those initial conditions.