

**UNIVERSITY OF LEIPZIG**  
**INSTITUTE FOR THEORETICAL PHYSICS**  
**Department: Theory of Elementary Particles**

TP3 2017

Lecturer: PD Dr. A. Schiller

List of problems 1

1. Find which of the following forces are conservative, and for those that are find the corresponding potential energy ( $a$  and  $b$  are constants,  $\mathbf{a}$  is a constant vector,  $r = |\mathbf{r}|$ )

$$F_x = ax + by^2, \quad F_y = az + 2bxy, \quad F_z = ay + bz^2, \\ \mathbf{F} = \mathbf{a} \times \mathbf{r}, \quad \mathbf{F} = \mathbf{a} r, \quad \mathbf{F} = \mathbf{a} (\mathbf{a} \cdot \mathbf{r}).$$

2. Compute the work done in taking a particle around the circle  $x^2 + y^2 = a^2$ ,  $z = 0$  if the force is (a)  $\mathbf{F} = y\hat{\mathbf{i}}$ , (b)  $\mathbf{F} = x\hat{\mathbf{i}}$ . ( $\hat{\mathbf{i}}$  – unit vector in  $x$  direction) What do you conclude about these forces? Check your conclusion.  
*Hint:* Use the parametrization  $x = a \cos \theta$ ,  $y = a \sin \theta$ ,  $z = 0$ .
3. Find the kinetic energy, and the Lagrange's equations of motion (assuming that a potential energy  $V$  exists) for a particle of mass  $m$  in terms of the following pairs of coordinates in a plane:
- (i) parabolic coordinates  $\xi = r + x$ ,  $\eta = r - x$  ( $r^2 = x^2 + y^2$ );
  - (ii) elliptic coordinates  $\lambda$ ,  $\theta$  defined by

$$x = \cosh \lambda \cos \theta, \quad y = \sinh \lambda \sin \theta, \quad (\lambda \geq 0, 0 < \theta \leq 2\pi).$$