FERMI SYSTEMS IN TWO DIMENSIONS AND FERMI SURFACE FLOWS

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A proof of Fermi liquid behaviour of weakly coupled Hubbard–like models in two spatial dimensions at positive temperature, in the sense of finiteness of the quasiparticle interactions and regularity of the selfenergy, is discussed. The proof is by a renormalization group flow in which the Fermi surface gets adjusted dynamically during the flow, so that no counterterms are needed. To show the required regularity properties of the selfenergy and the Fermi surface, the technique of improving power counting by single and double overlaps is implemented in a nonperturbative setting.

1. Introduction

In recent years, the foundations of Landau's Fermi liquid (FL) theory [1] have come under new investigation both from a mathematical and a theoretical physics point of view. The mathematical studies started when Feldman and Trubowitz [2], and Benfatto and Gallavotti [3], began to apply renormalization group (RG) methods of constructive quantum field theory to nonrelativistic fermion systems. The solid–state physics community began to reconsider the problem when, in the wake of the discovery of high–T_c superconductivity in the cuprate compounds, Anderson [4] conjectured that due to an *infrared catastrophe*, FL theory was invalid in two–dimensional fermion systems. Two dimensions are relevant because the cuprates are crystals with a layered structure and normal state transport between layers is almost absent. Anderson gave arguments that the two–dimensional fermion system should behave like the one–dimensional one no matter how small and short–range the interaction is. The Luttinger model of spinless fermions in one dimension, which has a linear dispersion relation, is exactly solvable by bosonization, and it exhibits anomalous decay exponents. By RG and other arguments [5, 6] it had been argued early on that at most values of the electron density, the long–distance behaviour of a one–dimensional electron system with a repulsive interaction is determined by that of an associated Luttinger system. Haldane coined the term *Luttinger liquid* (LL) for this universality class.

Although the high–T_c materials are far from being well–understood, the theoretical efforts to understand the observed deviations from FL behaviour led to a clarification of a number of points. Quasi–one–dimensional systems were studied using a combination of RG and bosonization methods [7, 8, 9]. They can exhibit a behaviour that differs very much from the Luttinger system. On the mathematical physics side, RG methods were used to prove a number of nontrivial statements. LL behaviour was proven for a class of one–dimensional systems in [10], and a number of results for 1d systems were subsequently proven[11]. In two dimensions, a crucial step in the development was [12], where the sector method was introduced. This allowed Feldman, Knörrer, and Trubowitz to prove existence of a zero temperature Fermi liquid with a $(p \to -p)$ – asymmetric dispersion relation [13, 14], and Disertori and Rivasseau to show FL behaviour of a two–dimensional system with a round Fermi surface [15].

Before starting to prove theorems about FL behaviour, one needs to define how to recognize it. For $p \to -p$ symmetric systems, a property proven in [14], namely that a jump in the occupation number density exists at zero temperature, is not expected to hold because, by the Kohn–Luttinger effect, superconductivity will set in and smooth out the step even in systems with a repulsive initial interaction. It is the asymmetry under $p \to -p$ which removes the Cooper instability in the model of [14], so that a jump remains at zero temperature. On the other hand, deviations from the predictions of FL theory, such as the T–linear resistivity, are observed in the cuprates well above the critical temperature (and these are *high–*T_c materials), so it seems natural to study FL behaviour above the critical temperature for symmetry breaking. A FL criterion at positive temperature was given in [16] and then verified for rotationally invariant systems in two dimensions by Disertori and Rivasseau [15].

The constructions in [14, 15] were done using counterterms that keep the Fermi surface fixed. The counterterm technique is convenient from a technical point of view, and it allows to formulate results about analyticity in the initial interaction. In the spherically symmetric case, the counterterm is a shift in the chemical potential, and its effect is simply to fix the Fermi momentum and hence the density. However, in the nonspherical case, the counterterm is a function of momentum and thus a nontrivial modification of the kinetic term.

Because the counterterm changes the model, using it has to be justified to obtain complete constructions. This was done in [17, 18, 19, 20] by proving an *inversion theorem* that gives a complete justification for putting counterterms, as well as a one–to–one map between the Fermi surface of the free and the interacting system. See also [21, 22, 23]. However, the theorems proven in [17, 18, 19, 20] are perturbative, i.e. they apply to the perturbation expansion to any fixed order, but the bounds given there do not contain nonperturbative remainder estimates. These perturbative regularity theorems require very tight bounds on the fermionic selfenergy. The inversion theorem has not been proven nonperturbatively up to now.

In this paper we outline our nonperturbative result about regularity of the Fermi surface and Fermi liquid behaviour in two–dimensional interacting fermion systems. We construct the model without using counterterms. Instead of counterterms, we keep track of the change of the Fermi surface during the RG flow that we use to construct the effective action. Thus we can directly discuss the behaviour of a given system, without reference to an inversion theorem. The essential regularity problem does not get any easier in our method; it is only transferred into an inductive verification of regularity along the RG flow instead of a one–step inversion theorem. The details of our work will be published in several papers [24].

Thus deviations from FL behaviour for weak and short–range interactions are not generic in two dimensions but require special Fermi surfaces with nesting or van Hove singularities. Approximate RG flows have been used to consider models of the cuprates [26, 27, 28, 29, 30, 31]. Some of the essential features of these materials can be understood in a natural way using the RG.

Proving FL behaviour in the three–dimensional case, where FL theory has had extraordinary phenomenological success, remains an open mathematical problem, but partial results exist [25].

2. Many–fermion systems and Fermi liquid theory

We consider a system of fermions on the lattice. Continuum systems with a Hamiltonian consisting of a periodic Schrödinger operator and a two–body interaction can be treated as well, provided that high energies are removed, which is natural in a crystal. Using the lattice makes the setup easier, but it does not change the infrared problem that is at the core of the analysis. Let Γ be a discrete torus, say $\Gamma = \mathbb{Z}^d / L\mathbb{Z}^d$ where the sidelength L of the volume is very large. For a solid, $L = O(10^8)$ and we shall only be interested in statements that are uniform in L at large L and that hold also in the limit $L \to \infty$. The fermions obey the usual canonical anticommutation relations $\{a_{x,\alpha}, a^+_{x',\alpha'}\} = \delta_{\alpha,\alpha'}\delta_{x,x'}$. Here $\alpha \in \{-1, 1\}$ is the z component of the spin in units of $\hbar/2$. The Hamiltonian

$$H = H_0 + \lambda V = \sum_{x,y;\alpha=\pm} \mathrm{t}(x-y) a^+_{x,\alpha} a_{y,\alpha} + \lambda \sum_{x,y} : n_x \, v(x-y) \, n_y : \tag{1}$$

has a kinetic term H_0 and a density–density interaction v (here $n_x = a^+_{x,+}a_{x,+} + a^+_{x,-}a_{x,-}$ is the local density operator). More general interactions can be treated. We assume that $\mathrm{t}(-\xi) = \mathrm{t}(\xi) \in \mathbb{R}$ and that both the hopping amplitude $\mathrm{t}(\xi)$ and interaction $v(\xi)$ are short-range, i.e.

$$\exists \alpha > 0 : \qquad \sum_{\xi \in \mathbb{Z}^d} |\mathrm{t}(\xi)| \, |\xi|^\alpha < \infty \quad \text{and} \quad \sum_{\xi \in \mathbb{Z}^d} |v(\xi)| \, |\xi|^\alpha < \infty. \tag{2}$$

The problem is already nontrivial when t and v are finite–range, provided the density is such that there is a Fermi surface in the noninteracting ($\lambda = 0$) system. A typical model of this kind is the standard Hubbard model where the kinetic term is the discrete Laplacian and the fermions interact only when they occupy the same site, i.e. $v(\xi) = 0$ for $\xi \neq 0$.

We only treat the weak–coupling case where λ is very small.

We consider the grand canonical ensemble at inverse temperature $\beta = (k_B T)^{-1}$ and chemical potential μ. For an operator A on Fock space

$$\langle A \rangle = \frac{1}{Z} \mathrm{tr} \, \left(\mathrm{e}^{-\beta(H-\mu N)} \, A \right) \tag{3}$$

with Z chosen such that $\langle 1 \rangle = 1$.

An essential object in the study of these systems is the Fermi surface. Let p be in the dual lattice $\Gamma^\sharp$ (for the infinite lattice, $\Gamma^\sharp = \mathbb{R}^d / 2\pi\mathbb{Z}^d = \mathcal{B}$ is the first Brillouin zone). By our assumptions on the hopping amplitude t, the dispersion relation $e_0(p) = \hat{\mathrm{t}}(p) - \mu$ is C^α and satisfies $e_0(-p) = e_0(p) \in \mathbb{R}$. The Fermi surface of the noninteracting system is $S_0 = \{p \in \mathcal{B} : e_0(p) = 0\}$. The difference $w = \sup_{p \in \mathcal{B}} e_0(p) - \inf_{p \in \mathcal{B}} e_0(p)$ is usually called the bandwidth. Let $\epsilon_0 > 0$ be a fixed energy scale which is smaller than w, e.g. $\epsilon_0 = w/10$. ϵ_0 is used as a reference scale for energies.

As everyone knows, for $\lambda = 0$

$$n_p = \langle a^+_{p,\alpha} a_{p,\alpha} \rangle = \frac{1}{1 + \mathrm{e}^{-\beta e_0(p)}} = \lim_{\varepsilon \downarrow 0} \frac{1}{\beta} \sum_{\omega \in \mathbb{M}_F} \frac{\mathrm{e}^{\mathrm{i}\omega\varepsilon}}{\mathrm{i}\omega - e_0(p)} \tag{4}$$

where $\mathbb{M}_F = \frac{\pi}{\beta}(2\mathbb{Z}+1)$ is the set of fermionic Matsubara frequencies. Thus in the limit $\beta \to \infty$, n_p has a jump across S_0.

Based on diagrammatic techniques one expects at $\lambda \neq 0$ that

$$n_p = \lim_{\varepsilon \downarrow 0} \frac{1}{\beta} \sum_{\omega \in \mathbb{M}_F} \frac{\mathrm{e}^{\mathrm{i}\omega\varepsilon}}{\mathrm{i}\omega - e_0(p) - \sigma(\omega, p)} \tag{5}$$

with Dyson's selfenergy σ, which depends on λ. Although the values of σ enter only at discrete frequencies $\omega \in \mathbb{M}_F$, σ is defined in a natural way for $\omega \in \mathbb{R}$ in the diagrammatic expansion. The existence and regularity of σ are among the main problems of a nonperturbative analysis.

A simple way to distinguish a FL from a LL would be to take the limit $\beta \to \infty$ and check if n_p has a jump across some submanifold S, the Fermi surface of the interacting system. This is what is proven in [14] for a class of systems with a dispersion relation $e_0(p)$ that is noninvariant under $p \to -p$, with counterterms that fix S to equal S_0. For the systems with $e_0(-p) = e_0(p)$ considered here, the ground state is expected to exhibit superconductivity or another form of symmetry breaking, so (5) is invalid. In other words, the σ in (5) will diverge at some temperature $T^* > 0$. In one dimension, σ becomes singular so that the small–ω behaviour of the denominator is no longer linear in ω but $|\omega|^\gamma$ with $\gamma < 1$.

The FL criterion proposed in [16] makes statements about σ as a function of λ and β in the region where $|\lambda| \log \beta$ is small, so that the Cooper instability is suppressed because the temperature is still too high for the formation of pairs. However, boundedness of σ is not enough; one has to show differentiability properties to prove that there is no anomalous decay in a FL. The function σ determines the full propagator of the model but not all of its properties. The higher correlation functions have to be controlled, too. In particular, the four–point function, from which Landau's quasiparticle interactions are derived, has to be studied. For more precise definitions, the reader is referred to [16] and [24]; here we just state our result for σ in the limit $L \to \infty$.

Theorem. *Let the spatial dimension $d = 2$ and (2) hold for a sufficiently large α. Assume that δ, the distance of the closest zero of ∇e_0 to S_0, is positive, that S_0 encloses a convex region, and that κ, the minimal curvature of S_0, is positive. Then there are $\lambda_0(\delta, \kappa) > 0$ and $c(\delta, \kappa) > 0$ such that for all (λ, β) that satisfy*

$$|\lambda| < \lambda_0 \qquad \textit{and} \qquad |\lambda| \log(\beta\epsilon_0) < c, \tag{6}$$

the selfenergy function $\sigma : \mathbb{R} \times \mathcal{B} \to \mathbb{C}$, $(\omega, p) \mapsto \sigma(\omega, p)$, exists and is a C^2 function of ω and p, whose second derivative is bounded on $\mathbb{R} \times \mathcal{B}$. σ and its first derivatives with respect to ω and p are C^1 in λ and vanish at $\lambda = 0$.

Remarks.
1. The Fermi surface of the interacting system, defined as $S = \{p \in \mathcal{B} : e_0(p) + \sigma(0, p) = 0\}$, has distance at most $O(\lambda)$ to S_0. The curvature of S is bounded and positive.
2. The quasiparticle weight $Z(p) = (1 + \mathrm{i}\partial\sigma/\partial\omega)^{-1}$ is close to 1. This rules out anomalous decay exponents.
3. In [24], we also prove statements about the higher correlation functions of the model. In particular, we show that the four–point function, which determines the quasiparticle

interactions of the FL, is bounded on the region given by (6). Together with items 1 and 2, this implies that the system is a Fermi liquid.
4. The second restriction in (6) removes the Cooper instability. According to BCS theory, for a local attractive interaction inducing s–wave pairing, the optimal constant c is $1/N(0)$ where $N(0)$ is the density of states at the Fermi surface S. If ϵ_0 is chosen such that the overlapping loop effect [17] gives an improvement already at scale ϵ_0 (see [23], Chapter 4), a proof that c is close to $1/N(0)$ seems feasible by a detailed analysis of ladder contributions, but has not been done yet. The scale ϵ_0 must not be too large because getting the correct constant requires a ladder resummation which is accurate only below a certain scale.
5. The condition (6) only involves $|\lambda|$, while of course the sign of λ is crucial for the existence of a solution to the BCS equations. It is expected (but still open) that one can show that for a repulsive interaction the second condition in (6) can be replaced by

$$\lambda^2 \log(\beta\epsilon_0) < c. \tag{7}$$

Proving this would again require a detailed analysis of ladder contributions.
6. Disertori and Rivasseau [15] have proven analyticity in λ for the case $e_0(p) = p^2 - 1$, using a counterterm to fix the density. Because of the spherical symmetry, the Fermi surface is fixed to be a circle. This largely simplifies the regularity problem treated here. The analyticity proof in [32] applies to nonspherical Fermi surfaces but does not provide enough regularity to solve the inversion problem for the counterterm function introduced there.
7. The all–order perturbative analysis of the regularity of σ done in [17, 18, 19, 20] was very much complicated by a peculiar lack of regularity of the selfenergy *at zero temperature*. Namely, the function σ is actually *not* C^2 at zero temperature. Its dependence on the variables parallel to the Fermi surface is C^2 [18, 19], but the second derivative in transversal direction is logarithmically divergent. This complication, present already in second order perturbation theory, does not arise under the restriction (6) because at positive temperature, the logarithmic divergence becomes a factor $\log\beta$, and a single such factor can be controlled by the coupling constant and (6). To extend our present analysis to the zero temperature asymmetric FL, we would have to perform a rather detailed higher–overlap classification extending that of [19]. Although possible, this is a further significant complication and left open for now.

3. Fermi surface flows

In this section we briefly outline the strategy of the proof. We study the system in its functional Grassmann integral representation obtained from the standard Trotter product formula (see, e.g. [23]). The generating function for the connected amputated Green functions is given by the convolution integral over Grassmann fields $\Psi = (\bar\psi, \psi)$

$$G(\Psi) = -\log \int \mathrm{d}\mu_{D_0}(\Psi')\, \mathrm{e}^{-V_0(\Psi+\Psi')}. \tag{8}$$

Here V_0 is determined by the original two–body interaction in the usual way and $\mathrm{d}\mu_{D_0}$ denotes the Grassmann Gaussian weight with covariance D_0. In the following we assume that the integration over the large frequency part has been done (see [14]). This leaves over an interaction V_0 which is the original two–body interaction plus a small correction analytic in λ, and D_0 is the propagator given by

$$D_0(\tau, x; \tau', x') = \frac{1}{\beta} \sum_{\omega \in \mathbb{M}_F} \int \mathrm{d}^2 p \, \mathrm{e}^{\mathrm{i}\omega(\tau-\tau') + \mathrm{i}p\cdot(x-x')} \frac{\chi_{0,<}(\omega, p)}{Q_0(\omega, p)} \tag{9}$$

with $Q_0(\omega, p) = \mathrm{i}\omega - e_0(p)$ and $\chi_{0,<}$ a cutoff function that vanishes for $|Q_0(\omega, p)| > \epsilon_0$. If the Gram constant of the propagator is finite, the functional integral defines connected Green functions in a Banach space of functions that are analytic in the fields [33, 34]. This is the case here because the positive temperature provides an infrared cutoff; however, the Gram bounds give a very bad temperature behaviour. Therefore, one uses an RG method to analyze the system. We now define a sequence of effective actions generated by RG transformations, with the property that, if convergent at all, the sequence will converge to G.

Let $M > 1$ be large enough and $\epsilon_j = \epsilon_0 M^{-j}$. Let $\chi^< + \chi^> = 1$ be a C^∞ partition of unity on $[0, \infty)$ such that $\chi^<(x) = 1$ for $x \le 1$ and $\chi^>(x) = 1$ for $x \ge 4$. Set $\chi_{1,<}(\omega, p) = \chi^<(\epsilon_1^{-2} |Q_0(\omega, p)|^2)$ and $\chi_{1,>} = 1 - \chi_{1,<}$. Accordingly, split $D_0 = D_0(\chi_{1,<} + \chi_{1,>}) = E_0 + F_0$. The fluctuation covariance $F_0 = Q_0^{-1} \chi_{0,<} \, \chi_{1,>}$ is supported on $\{(\omega, p) : \epsilon_1 \le |Q_0(\omega, p)| \le 2\epsilon_0\}$. Let

$$G_0(\Psi) = -\log \int \mathrm{d}\mu_{F_0}(\Psi') \, \mathrm{e}^{-V_0(\Psi+\Psi')} \tag{10}$$

then

$$G(\Psi) = -\log \int \mathrm{d}\mu_{E_0}(\Psi') \, \mathrm{e}^{-G_0(\Psi+\Psi')}. \tag{11}$$

By Theorem 1 of [34], G_0 is well–defined and analytic in the fields; we have

$$G_0(\Psi) = K_1 + (\bar{\psi}, q_0 \psi) + V_1(\Psi) \tag{12}$$

where V_1 contains the terms of order 4 and higher in Ψ and the q_0 in the quadratic form is translation invariant in space and Euclidian time. Its Fourier transform $\hat{q}_0$ satisfies $|\hat{q}_0(\omega, p)| \le const. \epsilon_1^{3/2}$, which is consistent with the behaviour of the propagator at scale ϵ_1 or ϵ_2, so that it may seem that we can just go on iterating functional convolutions to generate G_{j+1} from G_j. However, the quadratic piece in G_j obtained in this way grows, which in turn implies that the series diverges at some scale. One way to prevent this is to subtract the two–point insertions by introducing a counterterm, but this changes the model and requires a further justification. To avoid this, we move q_0 into the covariance, defining

$$Q_1(\omega, p) = Q_0(\omega, p) + q_0(\omega, p) \chi_{1,<}(\omega, p) \text{ and } D_1 = Q_1^{-1} \chi_{1,<}(\omega, p) \tag{13}$$

Consequently, with $A_1 = Q_1^{-1} Q_0 \chi_1$,

$$G(E_0 + F_0, V_0, \Psi) = K_0 + (\bar{\psi}, (q_0 + q_0 D_1 q_0)\psi) + G(D_1, V_1, A_1 \Psi) \tag{14}$$

Before proceeding, we have to verify that Q_1 has the same regularity properties as Q_0, that its zero set is $\{0\} \times S_1$ where $S_1 = \{p \in \mathcal{B} : Q_1(0, p) = 0\}$ is contained in a neighbourhood of S_0 of thickness of order $\lambda \epsilon_1$ and that S_1 is a C^2 curve with positive curvature. *If all this is the case*, we can set $\chi_{2,<}(\omega, p) = \chi^<(\epsilon_2^{-2} |Q_1(\omega, p)|^2)$ and $\chi_{2,>} = 1 - \chi_{2,<}$, split $D_1 = \chi_{2,<} + \chi_{2,>} = E_1 + F_1$, integrate F_1, and iterate the above steps, to obtain a sequence of effective actions that converge to $G(D_0, V_0)$. The so constructed sequence of D_j has shrinking supports and goes to zero. At positive temperature, the sequence terminates after $J = \log_M \frac{2\beta\epsilon_0}{\pi}$ steps because $D_j = 0$ for $j > J$. If the interaction V_j remains small enough for all $j \le J$, we finally get $G(0, V_J, A_J\Psi) = V_J(A_J\Psi)$. The explicit quadratic part in (14) iterates to become the full propagator, and the amputation operator A_J becomes the ratio of the full to the free covariance.

Verifying the above–mentioned regularity and smallness properties in every step j is the hard part of the problem. Because the quartic part of V_j grows marginally, we need the condition (6) to keep V_j small for $j \le J$. The procedure sketched roughly above has to be supplemented and modified in a number of ways to get the desired bounds: following [15], we use the Iagolnitzer–Magnen arch expansion to generate overlapping loops [17] from the fermion determinants in the tree expansion for the selfenergy. To keep the flow in a set of of dispersion relations of a fixed degree of differentiability, we use smoothing operator techniques. Finally, as in all works on two–dimensional fermion systems, we use the sector method of [12]. Because the sector estimates are weaker than the momentum space estimates of a perturbative analysis, we need to exhibit double overlaps to prove the C^2 property. Higher derivatives of Q_j also exist but they grow like inverse powers of ϵ_j.

The thus generated sequence of Fermi surfaces, $(S_j)_j$, the *Fermi surface flow*, was calculated to second order in the flowing coupling function in [29] for the example of the (t, t')–Hubbard model. In the density region considered there, t' tends to decrease.

4. Outlook

We have outlined our proof of Fermi liquid behaviour in two dimensions by an RG method in which the Fermi surface is allowed to flow. This avoids counterterms and the associated inversion problem of their justification. Several open problems have already been mentioned in the remarks following the theorem. In our opinion, it should be possible to get c close to the optimal constant $1/N(0)$, to obtain the $1/\lambda^2$ in the exponent instead of $1/\lambda$ for repulsive systems, and to show the existence of d–wave attractions near to antiferromagnetic instabilities.

We believe that by an extension of our method one can also treat the $p \to -p$ asymmetric model of [14] at temperature zero without counterterms. This will, however, require a further extension of our overlap classification results because of the above–mentioned subtle differences in tangential and transversal regularity at $T = 0$.

Although our work shows that counterterms can be avoided, the inversion theorem itself remains of great interest because such it establishes a bijective map between dispersion relations of noninteracting and interacting systems. At this time it seems that a nonperturbative proof of such a theorem would require a refined classification of contributions going beyond triple overlaps.

Last but not least we believe that the applications will continue to provide great stimulation for this field. In particular, the study of Fermi surfaces at or near to van Hove singularities leads to a number of interesting mathematical questions.

Acknowledgments

Financial support by the Max–Planck society is gratefully acknowledged.

References

1. L. D. Landau, *Soviet Physics JETP* ***3***, 920 (1956)
2. J. Feldman and E. Trubowitz, *Helv. Phys. Acta* ***63***, 157 (1990), ibid. **64**, 213 (1991)
3. G. Benfatto, G. Gallavotti, *J. Stat. Phys.* ***59***, 541 (1990)
4. P.W. Anderson, *The Theory of Superconductivity in the High-T_c Cuprates*, Princeton University Press (1997), and references therein
5. J. Solyom, *Adv. Phys.* ***28***, 201 (1979)
6. F.D.M. Haldane, *J. Phys.* ***C 14***, 2585 (1981)
7. L. Balents, and M.P.A. Fisher, *Phys. Rev.* ***B 53***, 12133 (1996).
8. H. Lin, L. Balents, and M.P.A. Fisher, *Phys.Rev.**B 58***, 1794 (1998).
9. For a review, see M.P.A. Fisher, cond-mat/9806164 and references therein.
10. G. Benfatto, G. Gallavotti, A. Procacci, B. Scoppola, *Comm. Math. Phys.* ***160***, 93 (1994)
11. F. Bonetto, V. Mastropietro, *Comm. Math. Phys.* ***172***, 57 (1995)
12. J. Feldman, J. Magnen, V. Rivasseau, and E. Trubowitz, *Helv. Phys. Acta* ***65***, 679–721 (1992)
13. J. Feldman, H. Knörrer, D. Lehmann, E. Trubowitz, in *Constructive Physics*, V. Rivasseau (ed.), Springer Lecture Notes in Physics, 1995
14. J. Feldman, H. Knörrer, E. Trubowitz, math-ph/0209040–0209049, to appear
15. M. Disertori and V. Rivasseau, *Commun. Math. Phys.* ***215***, 251,291 (2000)
16. M. Salmhofer, *Commun. Math. Phys.* ***194***, 249–295 (1998)
17. J. Feldman, M. Salmhofer, and E. Trubowitz, *J. Stat. Phys.* ***84***, 1209–1336 (1996)
18. J. Feldman, M. Salmhofer, and E. Trubowitz, *Comm. Pure Appl. Math.* ***LI***, 1133–1246 (1998)
19. J. Feldman, M. Salmhofer, and E. Trubowitz, *Comm. Pure Appl. Math.* ***LII***, 273–324 (1999)
20. J. Feldman, M. Salmhofer, and E. Trubowitz, *Comm. Pure Appl. Math.* ***LIII***, 1350–1384 (2000)
21. M. Salmhofer, *Rev. Math. Phys.* ***10***, 553–578 (1998)
22. J. Feldman, M. Salmhofer, and E. Trubowitz, Renormalization of the Fermi Surface, in XII^{th} *International Congress of Mathematical Physics.*
23. M. Salmhofer, *Renormalization: An Introduction.* Texts and Monographs in Physics. Springer-Verlag, Berlin-Heidelberg-New York, 1999.
24. W. Pedra and M. Salmhofer, to appear
25. M. Disertori, J. Magnen, and V. Rivasseau, *Henri Poincaré Acta* ***2***, 733 (2001)
26. N. Furukawa, M. Salmhofer, and T.M. Rice, *Phys. Rev. Lett.* ***81***, 3195 (1998)
27. D. Zanchi, and H.J. Schulz, *Europhys. Lett.* ***44***, 235 (1997); *Phys. Rev.* ***B 61***, 13609 (2000); *Europhys. Lett.* ***55***, 376 (2001)
28. C.J. Halboth, and W. Metzner, *Phys. Rev.* ***B 61***, 7364 (2000); *Phys. Rev. Lett.* ***85***, 5162 (2000)
29. C. Honerkamp, M. Salmhofer, N.Furukawa and T.M. Rice, *Phys. Rev.* ***B 63***, 035109 (2001)
30. C. Honerkamp, and M. Salmhofer, *Phys. Rev.* ***B 64*** 184516 (2001); *Phys. Rev. Lett.* ***87***, 187004 (2001)
31. M. Salmhofer, and C. Honerkamp, *Prog. Theo. Physics* ***105***, 1 (2001)
32. G. Benfatto, A. Giuliani, V. Mastropietro, cond-mat/0207210
33. A. Lesniewski. *Commun. Math. Phys.* ***108***, 437–467 (1987)
34. M. Salmhofer, C. Wieczerkowski, *J. Stat. Phys.* ***99***, 557 (2000)