Numerical survey of the tunable condensate shape and scaling laws in pair-factorized steady states

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Stochastic Mass Transport

In stochastic mass transport models the focus is typically on the dynamics in order to understand generic properties of realistic instantiations. In this work we study the steady state of a transport model that is based on a process generalized from the zero-range process (ZRP) [1, 2], but with a pair-factorized steady state. It most notably features extended condensates with a qualitatively tunable envelope shape.

Pair-factorized steady states

The model consists of a number of indistinguishable particles distributed on a peridoc ring lattice. In the discrete time stochastic process a particle may leave a randomly selected site at every time step with a hopping rate $u(m_i|m_{i-1}, m_{i+1})$ and move to either direct neighbour. This dynamics is the same as in the ZRP with an added nearest-neighbour interaction.

This also leads to a steady state that is similar to that of the ZRP, but factorizes over pairs of sites instead of single sites:

$$P(\vec{m}) = P(m_1, \ldots, m_N) = \frac{1}{Z}\prod_{i=1}^{N} g(m_i, m_{i+1})$$

The hopping rate given by the weight functions:

$$u(m_i|m_{i-1}, m_{i+1}) = \frac{g(m_i - 1, m_{i-1})}{g(m_i, m_{i-1})}\frac{g(m_i - 1, m_{i+1})}{g(m_i, m_{i+1})}$$

The weights are assumed to separate into zero-range and local-range interactions:

$$g(m, n) = \sqrt{p(m)p(n)}K(|m - n|)$$

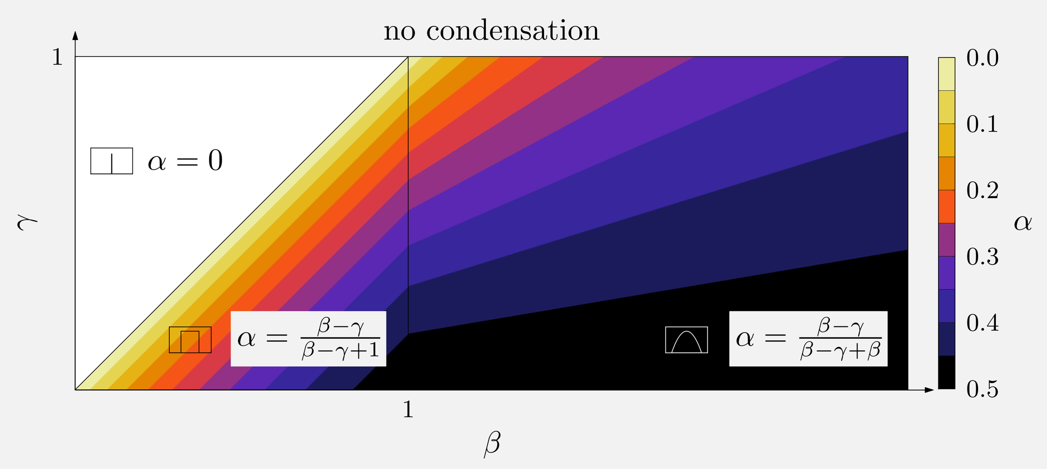
Well-behaving weight functions, where the zero-range interaction approaches a constant for large m and the short-range interaction falls off faster than any power law lead to an analytically known condensate shape and scaling.

To produce interesting behaviour these conditions are deliberately broken by introducing weights with tunable fall-off:

$$K(x) \sim \mathrm{e}^{-a|x|^{\beta}}, \quad p(m) \sim \mathrm{e}^{-bm^{\gamma}}$$

Predicted Properties

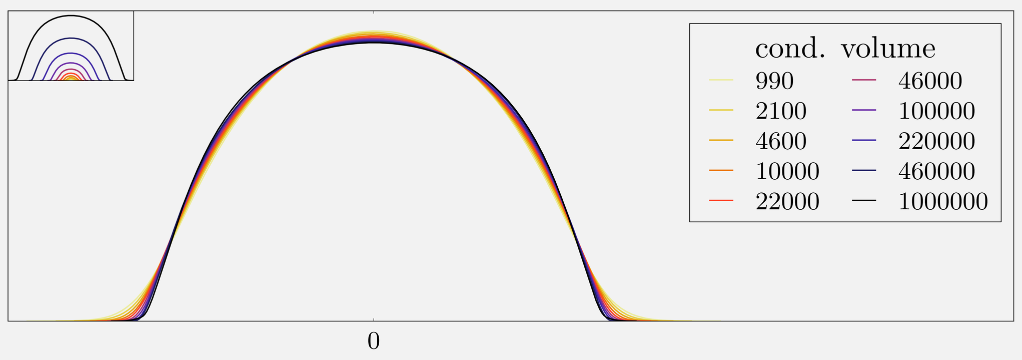
Using a large-volume limit approximation for this model B. Waclaw et al. predict four regimes of qualitatively different condensates in this model:



In the respective regime the condensate emerges as a peak occupying a single site or as an extended condensate with either rectangular or smooth bell-like envelope shape. Furthermore the scaling of the width was derived for the extended condensate shapes although these were not derived [3].

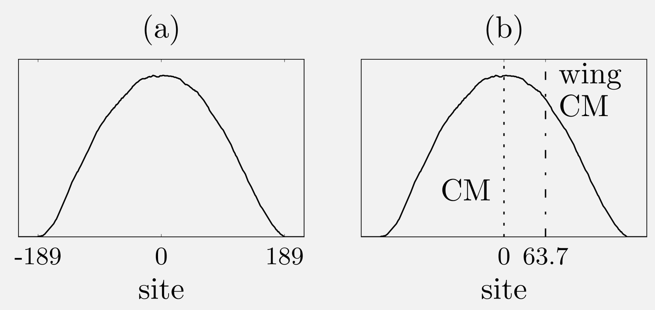
Characteristic Condensate Shape

The average condensate shape is measured by adding the individual shapes after aligning them with their respective centers of mass.



For large system sizes the condensate shape rescaled to unit width and volume converges to a characteristic shape for a given parameterization. This allows to employ more precise methods to estimate its scaling properties.

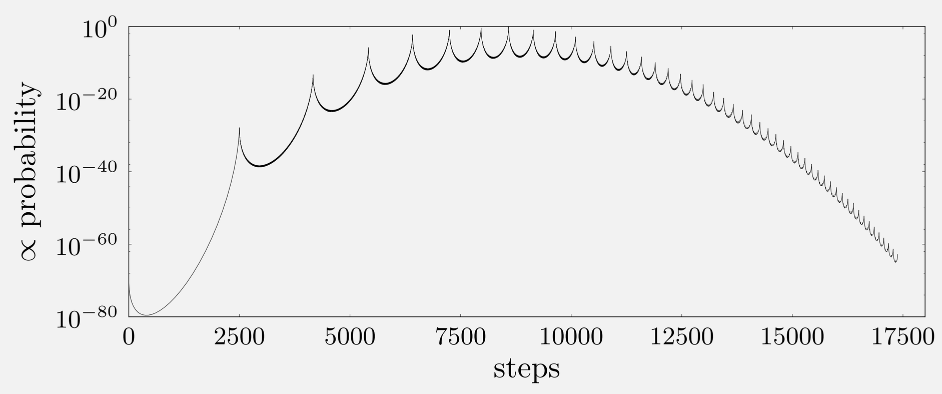
Determining the condensate width



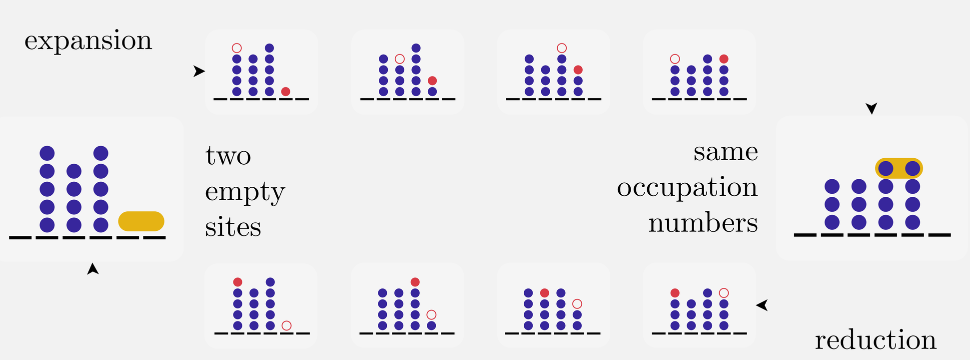
The width is easily determined by the points where the condensate drops below the background level. However, this method is prone to fluctuations at the condensate boundary. This influence is avoided by measuring an effective condensate width by computing the center of mass of the left and right condensate parts from its main center of mass.

Collective Updates

Purely local simulation methods become ineffective in the rectangular condensate regime, where regions of high probabilities (corresponding to rectangular condensates of different widths) are separated by highly suppressed transitional states. We illustrate this by taking the shortest path through state space between these regions.



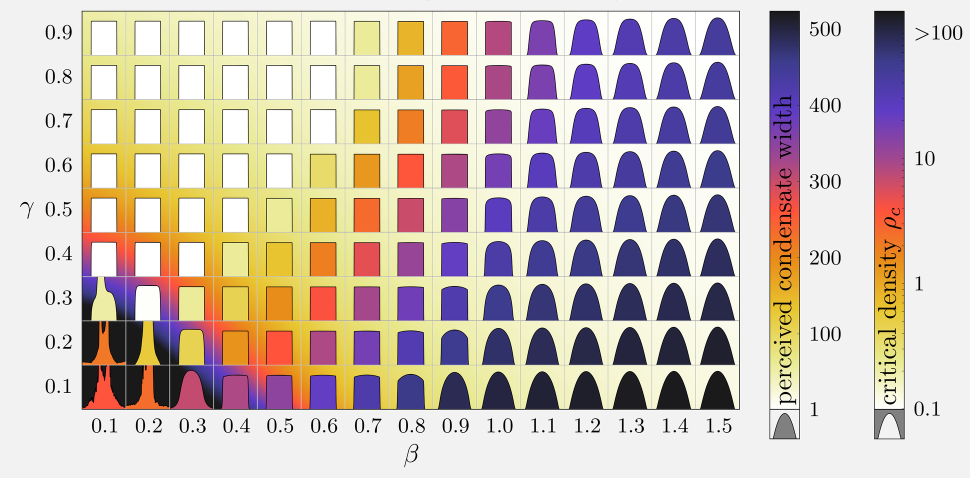
Small systems with about 3000 masses can already be infeasible to simulate, which is why we propose an update method that directly extends or reduces the condensate width, thereby avoiding the transitional states.



Combined with local updates, this method not only reduces the mixing time of the system, but the scaling behavior with an increasing number of masses drastically improves as well. Systems with more than 10^6 masses become feasible to simulate.

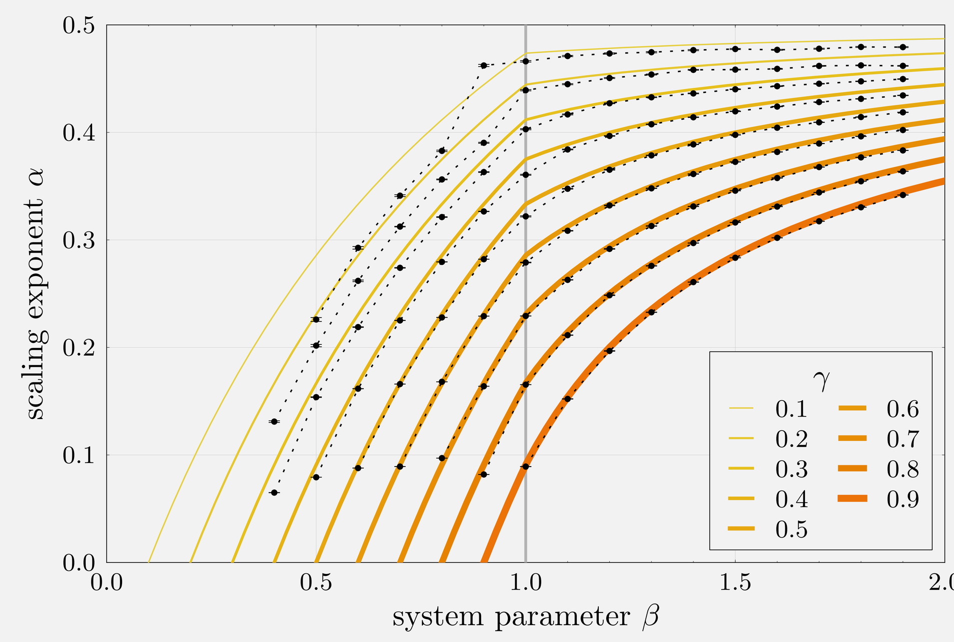
Tunable Condensate Shapes

We measured the characteristic shapes for a system with 10000 masses over a grid of β and γ parameters. We can identify the three distinct condensate shapes in about the same regions as they were expected. The boundary between smooth and rectangular condensates is smeared out with smooth condensates for small γ for $\beta \leq 1$ and condensates with broad tips for large γ and $\beta \geq 1$.



Condensate Width Scaling

We numerically determined the condensate width scaling exponents and compared them with the predicted values for many parameterizations.



Our results confirm the predictions in most parameterizations However, as with the condensate shapes a systematic deviation is observed at the phase boundary $\beta \approx 1$ for decreasing values of γ as well as for the combination of small values of γ and β.

Conclusions

We measured the shape and width scaling of extended condensates in pair-factorized steady states for finite yet large systems using many parameterizations. In the rectangular condensate regime we were only able to simulate large systems by introducing a new collective update method. The predicted regimes are easily identified. For most parameterizations, our data confirms the predictions.

We could not reproduce the sharp regime boundary at $\beta = 1$, which may indicate large finite-size effects or higher-order corrections.

References

[1] F. Spitzer, Adv. Math **5**, 246 (1970).

[2] M. R. Evans, T. Hanney, and S. N. Majumdar, Phys. Rev. Lett. **97**, 010602 (2006).

[3] B. Wacław, J. Sopik, W. Janke, and H. Meyer-Ortmanns, Phys. Rev. Lett. **103**, 080602 (2009).

[4] E. Ehrenpreis, H. Nagel, and W. Janke (unpublished).

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