Time Scale of Mass Condensation in Stochastic Transport with Pair Factorized Steady States

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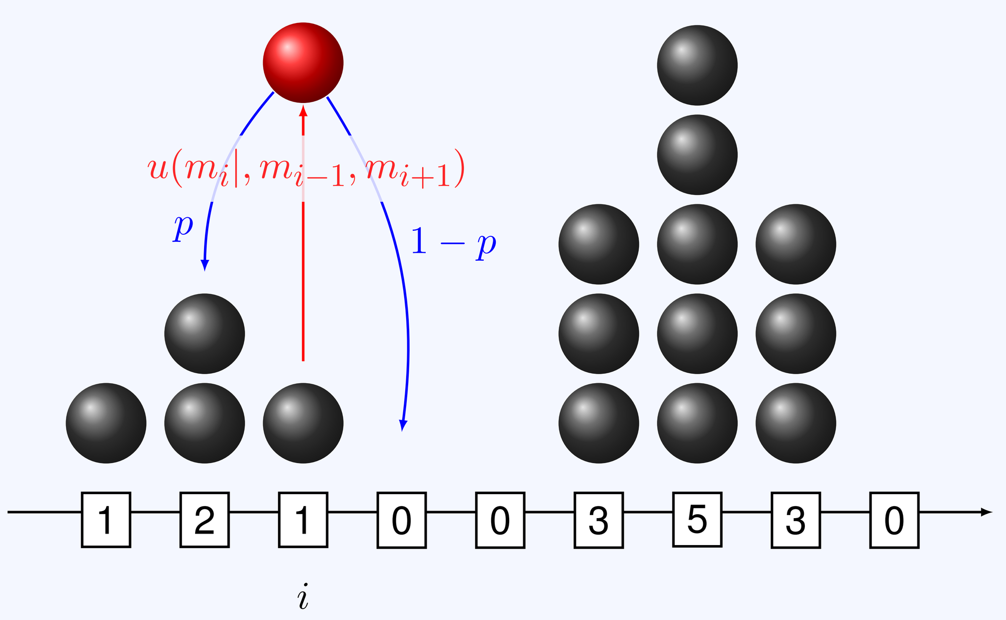
Introduction

Stochastic transport processes such as the one proposed by Evans et al. [1] can be tuned by their generating weight functions to exhibit a steady state with a condensate of particles that is separate from a fluid background phase. We study the dynamics of the relaxation process into the steady state of such driven transport systems using numerical simulations to determine the condensation time scale and discuss the corresponding phenomenological mechanisms.

Despite the existance of short ranged interactions in the studied system, the condensation behavior is found to be quite similar to that of the zero range process on one- and two-dimensional lattices.

Model

Consider M particles initially distributed at random on N sites of a periodic chain. At a time step of the stochastic process, a random particle leaves a site i with probability u proportional to the hopping rate $u(m_i)$ to the left or right with probabilities p or $1-p$ respectively.



The model system is specifically set up to have a known pair-factorized steady state (PFSS) partition function

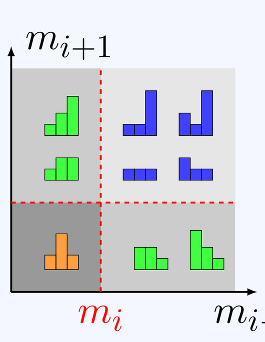
$$Z(M,N) = \sum_{\{\vec{m}\}} \prod_{\langle i,j \rangle} g(m_i, m_j)$$

that is a product over the generating weights $g(m,n)$. The specific choice of these weights

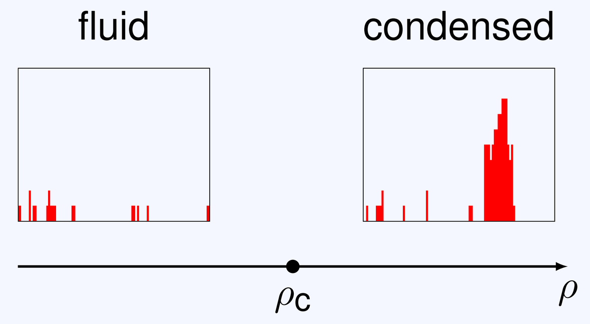
$$g(m,n) = K(|m-n|)\sqrt{p(m)p(n)},\ K(x) = \mathrm{e}^{-Jx},\ p(m) = \mathrm{e}^{\delta_{m,0}}$$

leads to the hopping rates

$$u(m_i|m_{i-1}, m_{i+1}) = \begin{cases} \mathrm{e}^{U_0\delta_{m_i,1}} \\ \mathrm{e}^{-2J+U_0\delta_{m_i,1}} \\ \mathrm{e}^{2J+U_0\delta_{m_i,1}} \end{cases}$$



Above a critical density of particles, the steady state exhibits a background "fluid" background phase at critical density and a condensate containting the excess particles.



Condensation Time Scale

The condensation time is the typical time that the relaxation process into the condensed steady state takes, starting in an disordered configuration of the system. Then, the **condensation time scale** is the relation of the condensation time to the size of the system.

From comparison with the zero range process (ZRP, see [2, 3] for discussion), the condensation time scale is expected to have the power law form

$$\tau \propto M'^{\delta},$$

with the scaling exponent δ.

Method

Monte Carlo methods are used to simulate the non-equilibrium dynamics of the described system and study the condensation time scale.

To estimate the scaling exponent, the condensation time is estimated for various system sizes and symmetric as well as total asymmetric dynamics.

To analyze the condensation time scale from the numerical simulations multiple methods were used to complement each other.

- Using the time dependent growth of the largest condensates mass to determine the condensation state.

  The scaling exponent can be estimated by rescaling time to match multiple such curves (see first and third plot in results).

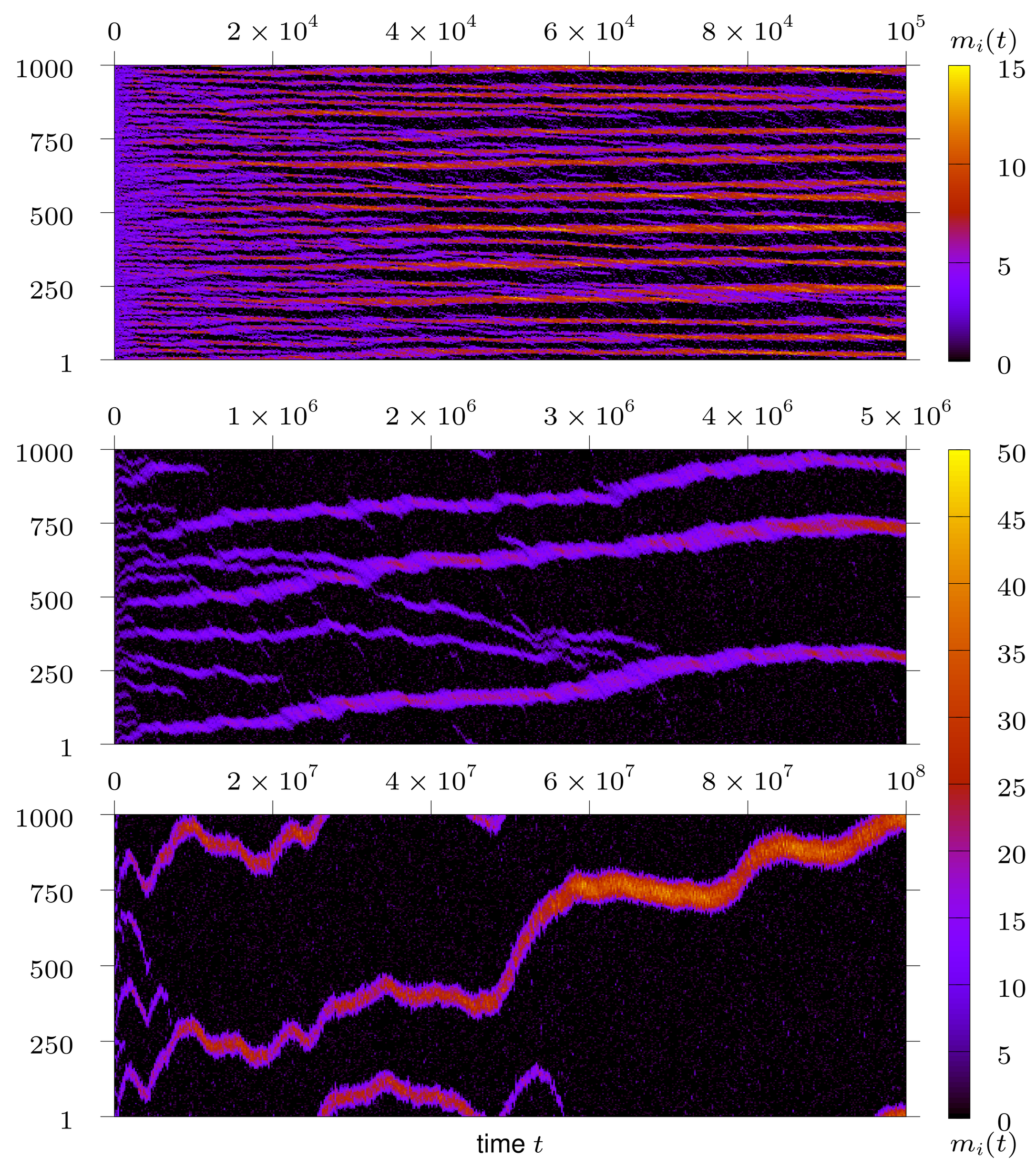
- Tracking the number of condensates in the system versus rescaled time and using this data to get the scaling exponent (similar to above).

  To count the condensates, one has to distinguish between droplets in the background and stable condensates. This is achieved using a mass threshold of $5/2\sqrt{M}$ (coefficient tuned using data).

- Performing a scaling analysis using a first passage time method using the number of condensates as a threshold.

  Record a histogram of times, when only one large condensate remains in the system. The estimated first passage times are then used to obtain the scaling exponent (see second plot in results).

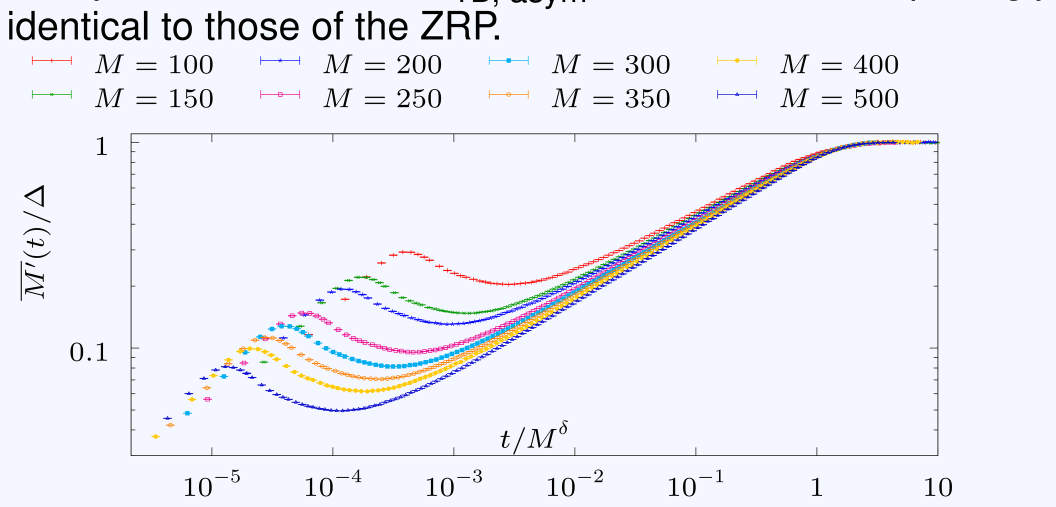
To allow simulations on regular graphs, square lattices were randomly rewired und consecutive runs of the dynamics were performed on different disorder configurations each.

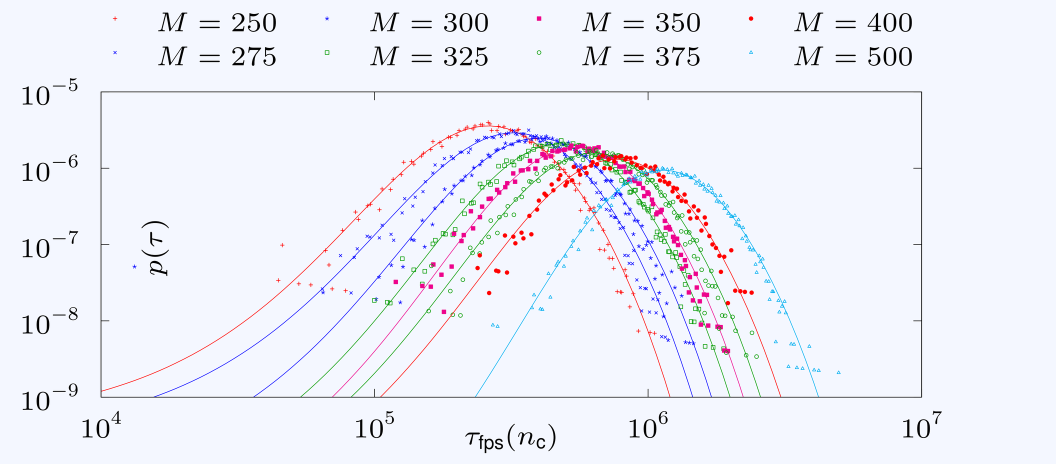


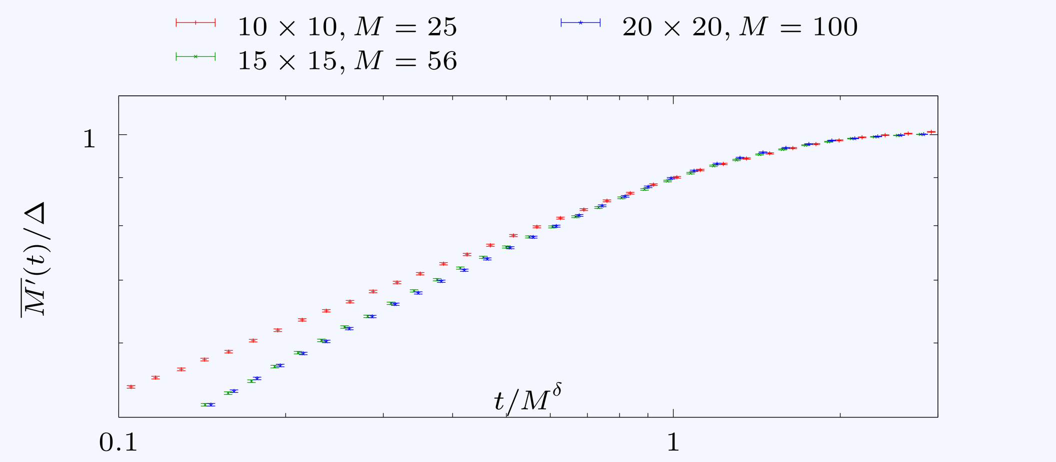
Typical evolution of the condensation process on different time scales for the totally asymmetric ($p=1$, particles hop in negative i direction) model. The displayed time series were recorded in a system with $N = 1000$ sites and $M = 3000$ particles prepared with a homogeneous distribution of particles. In the plot at the top, the emergence of a finite number of small droplets is observed in the early stages of the condensation process, which is refered to as the first regime. These droplets have a range of widths of 10 to 50 sites and mass of 20 to 200 particles at the time of 10^5 MC sweeps. In the middle plot, this situation is seen at the left before the coarsening process begins. In a first stage droplets grow to larger condensates due to the fast evaporation of smaller droplets. However, the coarsening regime is dominated by the last stage, where only two condensates remain. It is also quite notable, that an effective long range interaction between large condensates affecting their movement is observed.

Results: 1D, 2D Lattices

The scaling obtained for 1D (upper) and 2D lattices (lower) yield a good estimate of the scaling exponent $\delta_{\text{sym}} = 2$ in the symmetric case and $\delta_{\text{1D, asym}} = 3$ which are surprisingly identical to those of the ZRP.





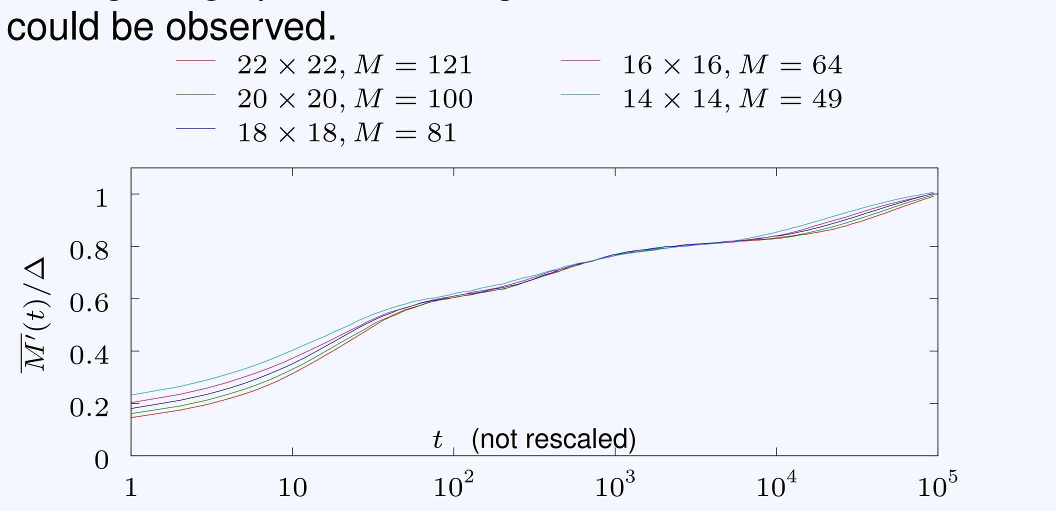


	M'	n_c	$\tau_{\text{fps}}(n_c)$
$\delta_{\text{1D,sym}}$	2.1 ± 0.2	2.0 ± 0.1	2.04 ± 0.07
$\delta_{\text{2D,sym}}$	2.0 ± 0.1	1.9 ± 0.1	

While studying the dynamics of the coarse graining process we could eliminate mechanisms such as movement and collision and mass-dependent evaporation of the condensates as causes of the time scale and remained with the fluctuation of the condensate masses as the main mechanism: The mass of the condensates in the coarsening regime performs a random walk, leading to a time scale of mass exchange as in the first passage time of a random walker to reach a given point.

Results: Regular Graphs

On regular graphs, no scaling of the condensation time scale could be observed.



The observed absence of scaling is possibly a result of shortcuts in the regular graphs. However, this and the peculiarity of the condensation behavior are subject of current research.

References

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