On the wave equation in spacetimes of Goedel type

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23rd Pacific Coast Gravity Meeting, March 16-17 2007, Caltech

Gödel's spacetimes:

- Homogeneous, stationary and rotationally symmetric
- Structure: $ds^2 = (dt + A_\varphi d\varphi)^2 - h_{ij}\, dx^i dx^j - dz^2$, with flat, spherical or Lobachevsky h_{ij}
- Solutions of Maxwell-Einstein equations with Λ and a dust
- Dust's wordlines have homogeneous vorticity
- Geodesics "rotate around every point"

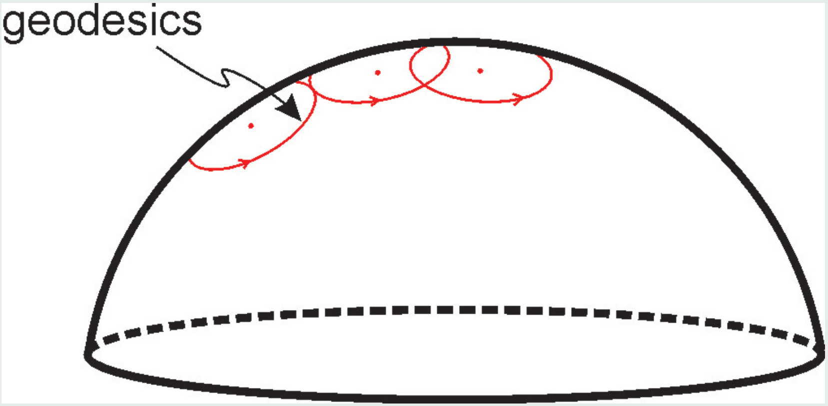


Figure 1: Geometry of $t = const$ hypersurfaces.

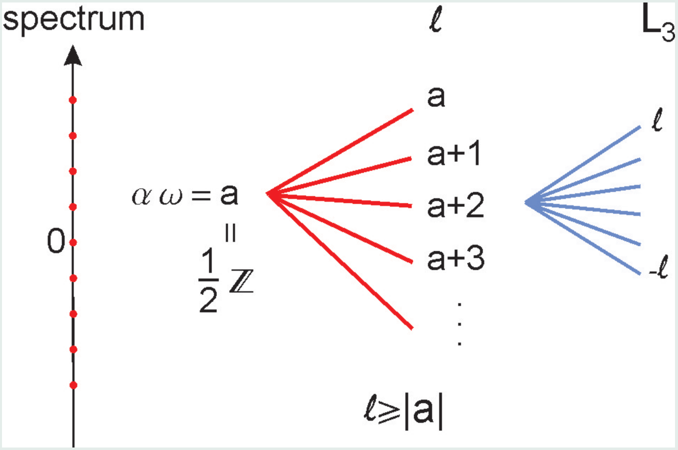
Wave equation: solutions (spherical case)

- apart from ∂_t and ∂_φ, 3 of 5 Killing vectors fulfill SO(3) relations

$$\text{Ansatz:} \qquad \Psi = e^{-i\omega t} e^{ikz} \psi(\theta, \varphi)$$

- Complete solution by algebraic methods:

$$\Box\Psi = [C_2 - (1+\alpha^2)\omega^2 + k^2]\psi = 0, \qquad \text{with } C_2 = \vec{L}^2$$



- Wave equation as a dispersion relation

$$\ell(\ell+1) + k^2 = (1+\alpha^2)\omega^2$$

Summary:

- Gödel spacetimes: homogeneous spacetimes with “vorticity”
- Solutions of the wave equation can be determined algebraically
- The spectrum of frequencies is harmonic-oscillator-like
- Details and the Lobachevsky case: **gr-qc/0703018**