On the wave equation in spacetimes of Goedel type

Piotr Marecki
Leipzig University
Gödel’s spacetimes:

- Homogeneous, stationary and rotationally symmetric
- Structure: \( ds^2 = (dt + A_\varphi d\varphi)^2 - h_{ij} \, dx^i dx^j - dz^2 \), with flat, spherical or Lobachevsky \( h_{ij} \)
- Solutions of Maxwell-Einstein equations with \( \Lambda \) and a dust
- Dust’s wordlines have homogeneous vorticity
- Geodesics “rotate around every point”

Figure 1: Geometry of \( t = \text{const} \) hypersurfaces.
Wave equation: solutions (spherical case)

- apart from \( \partial_t \) and \( \partial_\varphi \), 3 of 5 Killing vectors fulfill SO(3) relations

\[
\text{Ansatz: } \Psi = e^{-i\omega t} e^{ikz} \psi(\theta, \varphi)
\]

- Complete solution by algebraic methods:

\[
\Box \Psi = [C_2 - (1 + \alpha^2)\omega^2 + k^2] \psi = 0, \quad \text{with } C_2 = \vec{L}^2
\]

- Wave equation as a dispersion relation

\[
\ell(\ell + 1) + k^2 = (1 + \alpha^2)\omega^2
\]
Summary:

• Gödel spacetimes: homogeneous spacetimes with “vorticity”
• Solutions of the wave equation can be determined algebraically
• The spectrum of frequencies is harmonic-oscillator-like
• Details and the Lobachevsky case: gr-qc/0703018