On quantum effects in the vicinity of would-be horizons

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Outline of the talk

1. Quantum Field Theory and harmlessness of Black Hole horizons
2. Quantum Field Theory on curved spacetimes: state-independent results
3. Estimate of the ground-state energy densities of Quasi Black Hole Objects

(1) The meaning of Horizons and Black Holes

- Horizons are harmless null surfaces distinguished only asymptotically (non-locally); "one might be passing though this room and we would not notice"
- Schwarzschild horizon is harmless in GRT, but the QFT on the Schwarzschild spacetime distinguishes the horizon (Boulware state).
- It is a problem of the Boulware state (similar to the Fulling state in Rindler space). Other states, representations, (eg. Hartle-Hawking) lead to well defined QFT (away from the central singularity). QFT is regular, all effects finite (and negligible) at the horizon.
- Static observers in Schwarzschild geometry $\neq$ uniformly accelerated observers in Rindler space; before collapse all the matter of a star is assumed to be in the ground state.
- Surface of a star is time-like, whereas the horizon is null-like. The respective ground states have nothing in common. It might happen that the divergence of the energy density of the Boulware state is a sharp-boundary effect.

The meaning of Horizons and Black Holes ...

- But energy densities for ground states of compact objects which only nearly avoid collapse (such as MM-spacetimes, with the surface of the object at 1Angstrom from the would-be horizon) likely possess substantial negative energy densities (explained later).

- Such objects are unlikely to form if matter is a fluid with an equation of state $p = f(\varrho)$. For neutron stars QFT effects are negligible.

- The pressure of quantum systems is not clearly defined once there are external fields. Hydrodynamic formulation of an order-parameter theory ($\sim$Gross-Pitaevski) does not lead to simple EOS. For spherical objects it is more likely that $\varrho = f_1(r)$, $p = f_2(r)$, as in the MM-spacetime. Besides, the Fermi-liquid approximation has been tested in systems with binding energy small w.r.t. mc^2 of fermions.

(2) QFT on curved spacetimes: challenges and results

- (multi-) linear observables of the theory can be determined (eg. electric-field operators and their products at different points)
- No condition for distinguishing the reference state (eg. for the de Sitter space the state is either the ground state or does it share the symmetry of the spacetime)
- Related: non-uniqueness of definition of products of fields (energy density operators); normal ordering abandoned in favor of Hadamard point-splitting regularization
- Search for state-independent results:
  - Ground states for spacetimes with horizons (Schwarschild, de Sitter, Rindler) have: $\varrho_B = -\hbar c/480\,\pi^2 L^4 = -\varrho_{water}$ for $L \approx$ 1Angstrom.
  - Once the reference state is chosen, all other states must fulfill Quantum Energy Inequalities (magnitude times duration like inequalities for non-linear densities).

Quantum Energy Inequalities

- Classically positive, nonlinear quantity ($T_{00}(t, \vec{x}) \equiv \varrho$) can be negative for quantum fields (even arbitrarily negative if not smeared)
- Time-integrated with the Lorentzian function

$$f(t) = \frac{T}{\pi(t^2 + T^2)}$$

the operator fulfills

$$\varrho_\psi - \varrho_\Omega \geqslant -\frac{1}{100\, T^4}$$

for all Hadamard states ψ

- T means the duration of measurement; the essential message of QEIs: large negative energy densities can be present only for short times eg. on Minkowski space energy density equivalent to $-\varrho_{water}$ can last for $10^{-20}s$.

(3) Quasi Black-Hole spacetimes and QFT thereon

- Class of static objects, isometric to Schwarzschild almost up to the horizon
- No local argument (TOV+EOS) for their existence
- Long sampling times, T, are possible close to the horizon:

$$T = 4R_S \ln(L/\ell)$$

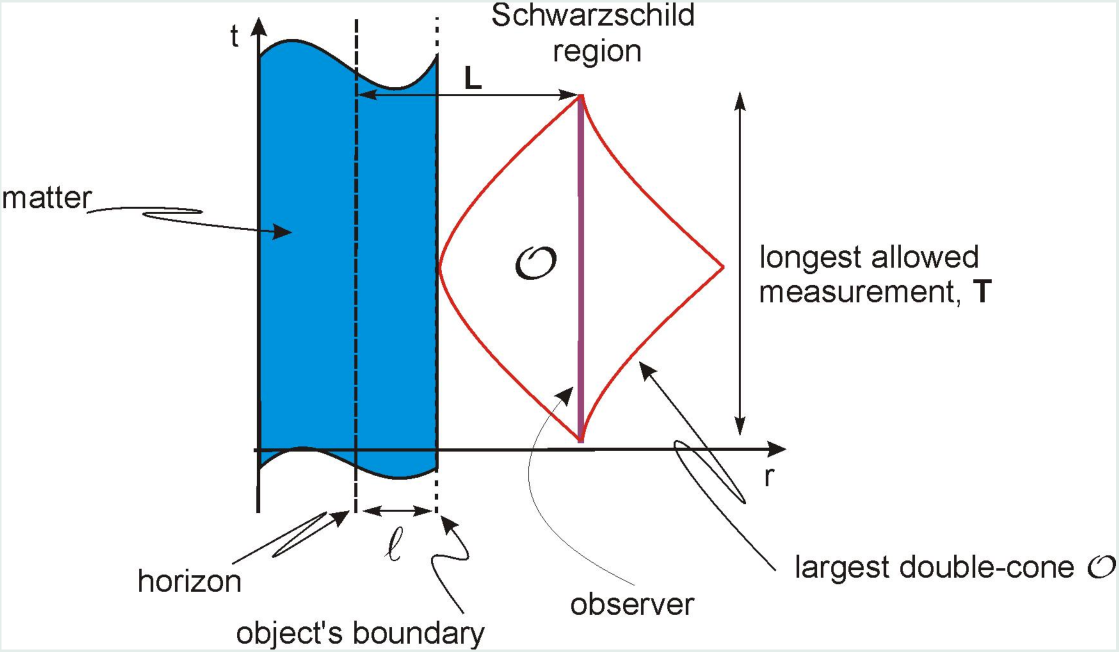

Figure 2: Spacetimes under consideration.

Universal bounds on the energy density of ground states ϱ_G

Close to the horizon we get a bound from below on $\varrho_G(L)$:

$$\varrho_G(L) - \varrho_B(L) \geqslant -\frac{g_1(L)}{T^2}$$

and an anticipated form of the bound from above

$$\varrho_B(L) - \varrho_G(L) \geqslant -\frac{g_2(L)}{T^2}$$

Remarks:

- In the limit of small ℓ (Quasi Black Hole Objects), we get essentially
  $$\varrho_G(L) \approx \varrho_B(L).$$
  because
  $$T = 4R_S \ln(L/\ell)$$
- The function g_1 (related to Boulware mode sums) is known explicitly close to the horizon; the form of g_2 can only be anticipated (without solving the scattering problem for small frequencies)

Summary and outlook

- The ground states of quantum fields in spacetimes of Quasi Black Hole Objects will exhibit substantial negative energy density effects close to the would-be horizons (Boulware-state result does not disappear for horizon-free spacetimes)

- The classical approximations employed for fermions in gravitational fields need a reexamination:

  - Fermionic ground states for QBHOs will likely also exhibit non-negligible energy densities (stress tensors)
  - The Thomas-Fermi approximation appears to contradict the Hadamard form of the ground state of free quantum Dirac-field. (which possesses terms containing derivatives of the external potential, divergent for large Fermi energies)