Back-reaction for QFT in static spacetimes

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Introduction

- QFT in external electromagnetic/gravitational fields: reliable approximation (yet - not fundamental)
- Development of local perturbative interacting QFT
- Distribution of interest: $\omega_2(x, y) - H(x, y)$, that is: the two-point function of a state minus the universal Hadamard parametrix

Plan of the talk

1. The tool: Quantum Energy Inequalities
2. The method:
   - long times of measurement possible close to the horizon
   - mutual bounds on energy densities of two ground states
3. The results: energy densities of grounds states of compact objects

(1) Quantum Energy Inequalities

- Classically positive, nonlinear quantity $(T_{00}(t, \vec{x}) \equiv \varrho)$ can be negative for quantum fields

- Smeared in time by the Lorentzian function

$$f(t) = \frac{T}{\pi(t^2 + T^2)}$$

the energy-density operator fulfills (massless scalar fields on Minkowski spacetime)

$$\varrho_\psi - \varrho_\Omega \geqslant -\frac{1}{100\,T^4}$$

for all Hadamard states ψ

- T means the duration of measurement

- the essential message of QEIs: for long measurements, the energy density can not be very negative

QEI facts:

- QEIs can be established for arbitrary smearing functions $f(t)$
- QEIs also hold in a curved-spacetime context; there the RHS becomes a functional $Q_\Omega[f, \vec{x}]$ of similar nature.
- Large negative densities allowed only for very short times: energy density equivalent to $-\varrho_{water}$ can last for $10^{-20}s$.

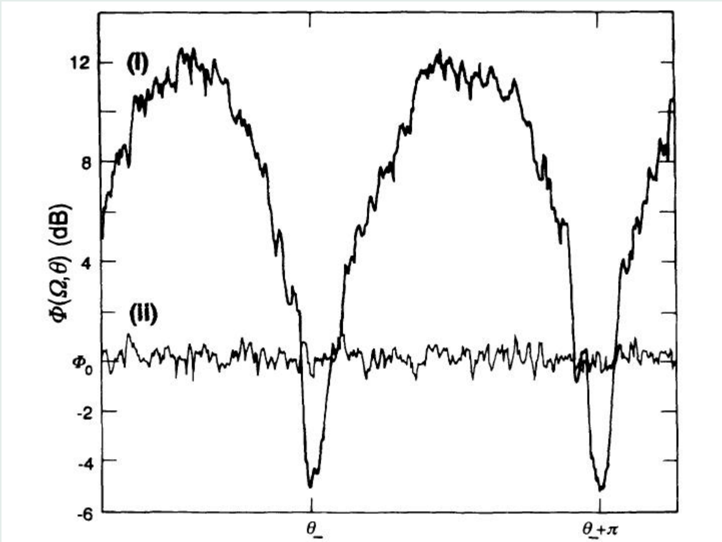

Figure 1: Experimental evidence for negative energy densities, which are compatible with QEIs (Polzik, Carri, Kimble, PRL 1992).

(2) Fundamental geometric observation

Consider two static spacetimes. One of them will contain a compact object ("gravastars"). Both are isometric in the Schwarzschild exterior region. Long sampling times, T, are possible close to the horizon!

$$T = 4R_S \ln(L/\ell)$$

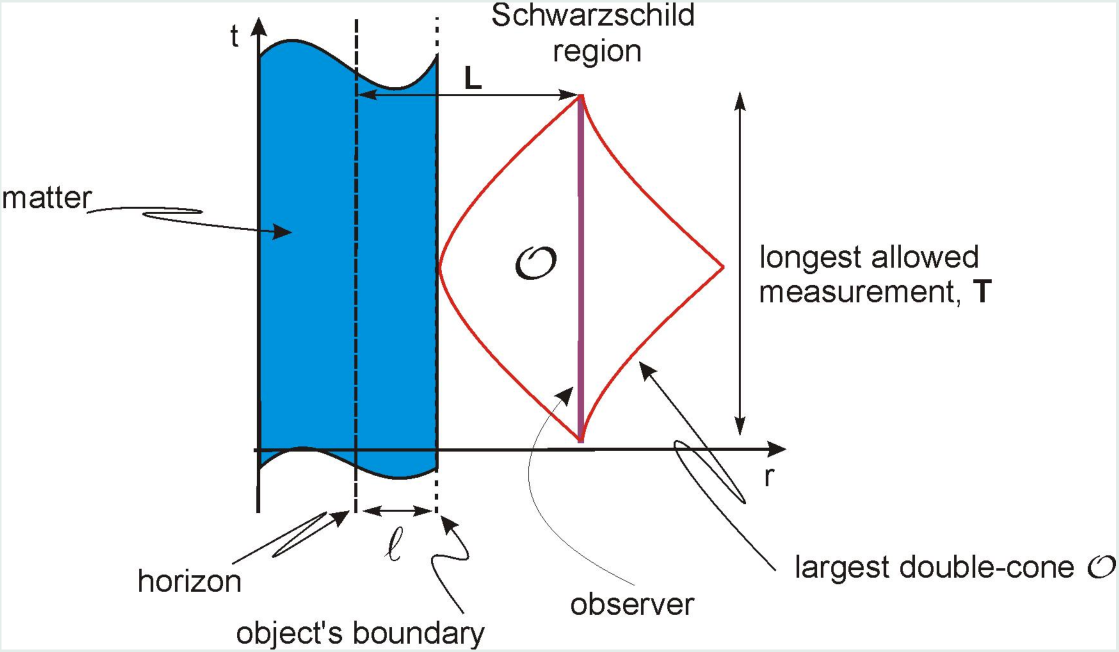

Figure 2: Spacetimes under consideration. The largest double cone cannot cross into the region with matter.

Mutual bounds on energy densities of ground states

- Consider ground states: B for the Schwarzschild spacetime, and G for the gravastar.
- Energy-density operator ϱ (smeared with a test function localized in $\mathcal{O}$) is an element of the algebra $\mathcal{W}(\mathcal{O})$.
- Locality and covariance:

$$\mathcal{W}(\text{Schwarzschild}) \supset \mathcal{W}(\mathcal{O}) \subset \mathcal{W}(\text{gravastar})$$

- QEI with respect to B (bound from below on ϱ_G)

$$\varrho_G - \varrho_B \geqslant Q_B$$

- QEI with respect to G (bound from above on ϱ_G)

$$\varrho_B - \varrho_G \geqslant Q_G$$

(3) Results: bound from below on ϱ_G

Close to the horizon we get:

$$\varrho_G(L) - \varrho_B(L) \geqslant -\frac{I[f]}{\ln^2(L/\ell)}\,|\varrho_B(L)|,$$

where L is the position where we measure, and ℓ is the distance from the gravastar's surface to the (would-be) horizon.

- ℓ appears only in the indicated manner $\ell \longrightarrow 0$ leads to
$$\varrho_G(L) - \varrho_B(L) \geqslant 0.$$
- $I[f]$ is comes from the sampling function $f(t)$ (supported on an interval of unit length); can be optimized over ($I \lesssim 50$).
- Bound comes from a scaling argument, with the maximal allowed time of measurement T; (bound scales as $1/T^2$).
- $\varrho_B = -\hbar c/480\,\pi^2 L^4 = -\varrho_{water}$ for $L \approx 1$Angstrom.

Bound from above (work in progress)

- If KMS and ground states exist for a gravastar, which is the case if $\partial_r(f \cdot h) > 0$, for the metric $(ds)^2 = f(dt)^2 - (dr)^2/h - r^2(d\Omega)^2$, then,

$$\varrho_G(L) - \varrho_B(L) \leqslant -\frac{\text{function}(r)}{T}$$

- The bound at present assures only the required decay with T for a single spacetime.
- The function can in principle depend on ℓ, apart from L
- I expect the limit $\ell \to 0$ to be regular.
- For a concrete bound, only the low-frequency part of the two-point function $\omega_2(t, L, 0, L)$ is necessary.
- Can we say anything more, without considering a concrete spacetime?

Summary

- Quantum energy inequalities put severe bounds on the allowed sub-ground-state energy densities
- If there are horizons, long measurement times are granted
- The energy density for a compact object (a “would-be black hole”) is anchored against the energy density of the Boulware state, which diverges to the minus infinity at the horizon